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Pure Math - 3

Binomial Theorem and Rational Functions

Ex. 1 Solution (Revision)

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Binomial Expansion and Rational Functions Revision

Example 1 (a) Expand $(1+3x)^{-\frac{1}{3}}$ in ascending powers of x, upto and including the term in x^2, simplifying the coefficients. ---[3]

(b) State the set of values of x for which the expansion is valid. --[1]

[SP-20/03/Q2]

Solution:

(a) $(1+3x)^{-1/3} = 1 + (-\frac{1}{3})(3x) + \frac{-\frac{1}{3}(-\frac{1}{3}-1)}{2!}(3x)^2 + \cdots$

$= 1 - x + \frac{1}{2}\times\frac{1}{3}\times\frac{4}{3}\times 9x^2 + \cdots$

$= 1 - x + 2x^2 + \cdots$ ✓

(b) $|3x| < 1 \Rightarrow |x| < \frac{1}{3}$ ✓

Example 2 Let $f(x) = \dfrac{2+11x-10x^2}{(1+2x)(1-2x)(2+x)}$

(a) Express $f(x)$ in partial fractions. ---[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, upto and including powers of x, upto and including the term in x^2. ---[5]

[M-20/32/Q9]

Solution: Consider $\dfrac{2+11x-10x^2}{(1+2x)(1-2x)(2+x)} = \dfrac{a}{(1+2x)} + \dfrac{b}{(1-2x)} + \dfrac{c}{(2+x)}$ —— ①

(a) multiply ① by $(1+2x)$,

$\dfrac{2+11x-10x^2}{(1+2x)(2+x)} = a + \dfrac{b(1+2x)}{1-2x} + \dfrac{c(1+2x)}{(2+x)}$

$(2x+1)=0$, Put $x = -\frac{1}{2} \Rightarrow \frac{-6}{3} = a \Rightarrow a = -2$ ✓

multiple ① by $(1-2x) \Rightarrow \dfrac{2+11x-10x^2}{(1+2x)(2+x)} = \dfrac{a(1-2x)}{(1+2x)} + b + \dfrac{c(1-2x)}{(2+x)}$

$(1-2x)=0$ Put $x = \frac{1}{2} \Rightarrow \dfrac{5}{2\times 5/2} = b \Rightarrow b = 1$ ✓

multiply ① by $(2+x)$

$\Rightarrow \dfrac{2+11x-10x^2}{(1+2x)(1-2x)} = \dfrac{a(2+x)}{(1+2x)} + \dfrac{b(2+x)}{(1-2x)} + c$

Put $2+x=0 \Rightarrow x=-2 \Rightarrow \dfrac{-60}{-15} = c \Rightarrow c = 4$ ✓

Hence from ① Required Partial fraction $= \dfrac{-2}{(1+2x)} + \dfrac{1}{(1-2x)} + \dfrac{4}{(2+x)}$ ✓ —— ②

(b) $-2(1+2x)^{-1} = -2[1-2x+(2x)^2$ $\rightarrow$ $\left(\because (1+x)^{-1} = 1-x+x^2 - \cdots \quad (1-x)^{-1} = 1+x+x^2+\cdots\right)$

$= -2+4x-8x^2$ —— ③

$(1-2x)^{-1} = 1+2x+(2x)^2+\cdots$

$= 1+2x+4x^2+\cdots$ —— ④

and $4(2+x)^{-1} = 4\times 2^{-1}(1+\frac{x}{2})^{-1} = 2[1-\frac{x}{2}+\frac{x^2}{4}\cdots]$

$= (2-x+\frac{x^2}{2}\cdots)$ —— ⑤

add ③, ④ and ⑤, Req. Expansion

$= (-2+1+2) + x(4+2-1) + x^2(-8+4+\frac{1}{2})$

$= [1+5x-\frac{7}{2}x^2\cdots]$ ✓

Example 3 (a) Expand $(2-3x)^{-2}$ in ascending powers of x, upto and including the term in x^2, simplifying the coefficients. ---[4]

(b) State the set of values of x for which the expansion is valid. ---[1]

[S-20/31/Q 2]

Solution (a) $(2-3x)^{-2} = 2^{-2}\left(1-\frac{3}{2}x\right)^{-2} = \frac{1}{4}\left[1+(-2)\left(-\frac{3}{2}x\right)+\frac{(-2)(-3)}{2!}\left(-\frac{3}{2}x\right)^2+\cdots\right]$

$= \frac{1}{4}\left[1+3x+\frac{27x^2}{4}+\cdots\right]$

$= \frac{1}{4}+\frac{3}{4}x+\frac{27}{16}x^2$ ✓

(b) $\left|-\frac{3}{2}x\right| < 1 \Rightarrow x < \frac{2}{3}$ ✓

4. Let $f(x) = \dfrac{14-3x+2x^2}{(2+x)(3+x^2)}$

(a) Express $f(x)$ in partial fractions. ---[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. ---[5]

S-21/32/Q9

Solution (a) $f(x) = \dfrac{14-3x+2x^2}{(2+x)(3+x^2)} = \dfrac{a}{(2+x)} + \dfrac{bx+c}{(3+x^2)}$ ---①

multiply ① by $(2+x)$

$\Rightarrow \dfrac{14-3x+2x^2}{(3+x^2)} = a + \dfrac{(bx+c)(2+x)}{(3+x^2)}$ ---②

Put $2+x=0 \Rightarrow x=-2$ in ② $\Rightarrow \dfrac{28}{7} = a \Rightarrow a = 4$ ✓

multiply by $(2+x)(3+x^2)$ to ①

$14-3x+2x^2 = 4(3+x^2)+(bx+c)(x+2)$ — ③

Comparing constant terms in ③ $\Rightarrow 14 = 12+2c \Rightarrow c = 1$ ✓

Comparing the coeff of x^2 in ③ $\Rightarrow 2 = 4+b \Rightarrow b = -2$

∴ from ① the Req. Partial fractions are.

$$f(x) = \frac{4}{(2+x)} + \frac{(-2x+1)}{(3+x^2)}$$

(b) $f(x) = \dfrac{4}{2(1+\frac{x}{2})} + \dfrac{(1-2x)}{3(1+\frac{x^2}{3})}$

[$(1+x)^{-1} = 1-x+x^2+\cdots$]

[(b) $(1+x)^n = 1+nx+\dfrac{n(n-1)x^2}{2!}+\cdots$]

$= 2\left(1+\frac{x}{2}\right)^{-1} + \frac{1}{3}(1-2x)\left(1+\frac{x^2}{3}\right)^{-1}$

$= 2\left[1-\frac{x}{2}+\frac{x^2}{4}+\cdots\right] + \frac{1}{3}(1-2x)\left[1-\frac{x^2}{3}+\cdots\right]$

$= 2+\frac{1}{3} - x - \frac{2}{3}x + \frac{1}{2}x^2 - \frac{1}{9}x^2 + \cdots$

$= \frac{7}{3} - \frac{5}{3}x + \frac{7}{9}x^2 + \cdots$ ✓

Binomial theorem:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3}x^3 + \cdots$$

for $|x| < 1$

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5. Expand $(1+3x)^{2/3}$ in ascending powers of x, upto and including the term in x^3, simplifying the coefficients. --- [4]

S-21	33	Q1

Solution: $(1+3x)^{2/3} = 1 + \frac{2}{3} \times 3x + \frac{\frac{2}{3}\left(\frac{2}{3}-1\right)}{2!} \times (3x)^2 + \frac{\frac{2}{3}\left(\frac{2}{3}-1\right)\left(\frac{2}{3}-2\right)}{3!} \times (3x)^3 + \cdots$

$\left[(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots\right]$

$= 1 + 2x - x^2 + \frac{4}{3}x^3 + \cdots$

Binomial theorem:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \cdots \text{ for } |x|<1$$

6 (a) Expand $(2-x^2)^{-2}$ in ascending powers of x, upto and including the term in x^4, simplify the coefficients. ---[4]

(b) State the set of values of x for which the expansion is valid. ---[1]

[S-22 | 31 | Q2]

Solution: $(2-x^2)^{-2} = \left[2\left(1-\frac{x^2}{2}\right)\right]^{-2} = 2^{-2}\left(1-\frac{x^2}{2}\right)^{-2} = \frac{1}{4}(1-x^2)^{-2}$

Now $\frac{1}{4}(1-x^2)^{-2} = \frac{1}{4}\left[1+(-2)\left(-\frac{x^2}{2}\right) + \frac{(-2)(-3)}{2}\left(\frac{x^2}{2}\right)^2\right]$ [$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\cdots$ (for $|x|<1$)]

$= \frac{1}{4}\left[1 + x^2 + \frac{3}{4}x^4 + \cdots\right] = \frac{1}{4} + \frac{1}{4}x^2 + \frac{3}{16}x^4 + \cdots$ ✓

(b)

Expansion is valid for $\left|-\frac{x^2}{2}\right| < 1 \Rightarrow |x|^2 < 2 \Rightarrow |x| < \sqrt{2}$ ✓

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7. Let $f(x) = \frac{5x^2+8x-3}{(x-2)(2x^2+3)}$

(a) Express $f(x)$ in partial fractions. --- [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. ---[5]

[S-22 | 33 | Q7]

Solution: (a) $f(x) = \frac{5x^2+8x-3}{(x-2)(2x^2+3)} = \frac{a}{x-2} + \frac{bx+c}{(2x^2+3)}$ --- (1)

multiply (1) by $(x-2) \Rightarrow \frac{5x^2+8x-3}{(2x^2+3)} = a + \frac{(bx+c)}{(2x^2+3)} \times (x-2)$

$(x-2)=0 \Rightarrow x=2$ put $\Rightarrow \frac{33}{11} = a \Rightarrow a = 3$ ✓

Now from (1) $\frac{5x^2+8x-3}{(x-2)(2x^2+3)} = \frac{3}{(x-2)} + \frac{bx+c}{(2x^2+3)}$

multiply by $D^r \Rightarrow 5x^2+8x-3 = 3(2x^2+3) + (bx+c)(x-2)$ --- (2)

Comparing the coeff of $x^2 \Rightarrow 5 = 6+b \Rightarrow b=-1$ ✓

Comparing the constant term $\Rightarrow -3 = 9-2c \Rightarrow c = 6$ ✓

Hence from (1) the Required partial fractions

$f(x) = \frac{3}{(x-2)} + \frac{-x+6}{(2x^2+3)}$ --- (3)

(b) $\frac{3}{-2\left(1-\frac{x}{2}\right)} + \frac{(6-x)}{3\left(1+\frac{2}{3}x^2\right)}$

$= -\frac{3}{2}\left(1-\frac{x}{2}\right)^{-1} + \frac{1}{3}(6-x)\left(1+\frac{2}{3}x^2\right)^{-1}$

$= -\frac{3}{2}\left(1+\frac{x}{2}+\frac{(-1)(-2)}{2}\left(-\frac{x}{2}\right)^2 + \cdots\right) + \frac{1}{3}(6-x)\left(1-\frac{2}{3}x^2+\cdots\right)$

$= -\frac{3}{2}\left(1+\frac{x}{2}-\frac{x^2}{4}\cdots\right) + \frac{1}{3}\left(6-4x^2-x\cdots\right)$

$= -\frac{3}{2} + 2 - \frac{3}{4}x - \frac{1}{3}x - \frac{3}{8}x^2 - \frac{4}{3}x^2 + \cdots$

$= \frac{1}{2} - \frac{13}{12}x - \frac{41}{24}x^2 + \cdots$ ✓

8. Find the coefficient of x^3 in the binomial expansion of $(3+x)\sqrt{1+4x}$ ---[4]
S-23/31/Q3

Solution: $(1+4x)^{1/2} = 1+\frac{1}{2}\times 4x + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2!}(4x)^2 + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{3!}(4x)^3$ — ①

Hence
$(3+x)\sqrt{1+4x}$ from ①
$=(3+x)(1+2x-2x^2+4x^3+\cdots)$ --- ②

$\left[(1+x)^n = 1+nx+\frac{n(n-1)}{2}x^2 + \frac{n(n-1)(n-2)}{3!}x^3+\cdots\right.$

$\Rightarrow$ Coeff of x^3 in ② $= 3\times 4 + 1\times(-2) = 12-2 = 10$ ✓

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9. Let $f(x) = \frac{21-8x-2x^2}{(1+2x)(3-x)^2}$

(a) Express $f(x)$ in partial fractions.
(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, upto and including the term in x^2. ---[5]
S-23/33/Q10

Solution(a) $f(x) = \frac{21-8x-2x^2}{(1+2x)(3-x)^2} = \frac{a}{1+2x} + \frac{b}{(3-x)^2} + \frac{c}{(3-x)}$ --- ①

multiply ① by $(1+2x)(3-x)^2 \Rightarrow 21-8x-2x^2 = a(3-x)^2 + b(1+2x) + c(3-x)(1+2x)$ --- ②

Put $(1+2x)=0 \Rightarrow x=-\frac{1}{2}$ in ② $\Rightarrow 21+4-2\times(-\frac{1}{2})^2 = a(3+\frac{1}{2})^2+0+0 \Rightarrow \frac{49}{4}a = \frac{49}{2} \Rightarrow \underline{a=2}$ ✓

Put $(3-x)=0 \Rightarrow x=3$ in ② $\Rightarrow 21-24-18 = 0+b\times 7+0 \Rightarrow 7b=-21 \Rightarrow \underline{b=-3}$ ✓

Put the values of a and b in ②.
$\Rightarrow 21-8x-2x^2 = 2(3-x)^2 - 3(1+2x) + c(3-x)(1+2x)$ --- ③

Put $x=0$ in ③ $\Rightarrow 21 = 18-3+3c \Rightarrow 3c=6 \Rightarrow$ $\underline{c=2}$ ✓

Put the values of a, b and c in ①: $f(x) = \frac{2}{(1+2x)} - \frac{3}{(3-x)^2} + \frac{2}{(3-x)}$ — ④
Req. Partial fractions.

(b) $f(x) = \frac{2}{(1+2x)} + \frac{2}{(3-x)} - \frac{3}{(3-x)^2}$

$= 2(1+2x)^{-1} + \frac{2}{3}(1-\frac{x}{3})^{-1} - \frac{3}{3^2}(1-\frac{x}{3})^{-2}$

$= 2\left[1+(-1)(2x)+\frac{(-1)(-2)}{2!}(2x)^2\cdots\right] + \frac{2}{3}\left[1+(-1)(-\frac{x}{3}) + \frac{(-1)(-1-1)}{2!}(-\frac{x}{3})^2+\cdots\right]$
$- \frac{1}{3}\left[1+(-2)(-\frac{x}{3}) + \frac{(-2)(-3)}{2!}(-\frac{x}{3})^2+\cdots\right]$

$= 2[1-2x+4x^2\cdots] + \frac{2}{3}[1+\frac{x}{3}+\frac{x^2}{9}\cdots] - \frac{1}{3}[1+\frac{2x}{3}+\frac{x^2}{3}+\cdots]$

$= (2+\frac{2}{3}-\frac{1}{3}) + (-4+\frac{2}{9}-\frac{2}{9})x + (8+\frac{2}{27}-\frac{1}{9})x^2+\cdots$

$= \frac{7}{3} - 4x + \frac{215}{27}x^2 + \cdots$

10. Let $f(x) = \dfrac{8+5x+12x^2}{(1-x)(2+3x)^2}$

(a) Express $f(x)$ in partial fractions. ---[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, upto and including the term in x^2. ---[5]

W-20/31/Q9

Solution: (a) $f(x) = \dfrac{8+5x+12x^2}{(1-x)(2+3x)^2} = \dfrac{a}{(1-x)} + \dfrac{b}{2+3x} + \dfrac{c}{(2+3x)^2}$ —— ①

$\Rightarrow \dfrac{8+5x+12x^2}{(2+3x)^2} = a + \dfrac{b(1-x)}{2+3x} + \dfrac{c(1-x)}{(2+3x)^2}$ [multiply ① by $(1-x)$]

let $(1-x)=0 \Rightarrow x=1$; put $x=1 \Rightarrow \dfrac{25}{25} = a+0+0 \Rightarrow \underline{a=1}$ ✓

multiply ① by $(2+3x)^2 \rightarrow \dfrac{8+5x+12x^2}{(1-x)} = \dfrac{a(2+3x)^2}{(1-x)} + b(2+3x) + c$ —— ②

In ② let $2+3x=0 \Rightarrow x=-\frac{2}{3} \Rightarrow \dfrac{10}{5/3} = c \Rightarrow \underline{c=6}$ ✓

multiply ① by D^r

$8+5x+12x^2 = 1\cdot(2+3x)^2 + b(2+3x)(1-x) + 6(1-x)$ $\left[\begin{matrix} a=1 \\ b=6 \end{matrix}\right]$

Put $x=0 \Rightarrow 8 = 4+2b+6 \Rightarrow \underline{b=-1}$ ✓

$\therefore$ from ① $f(x) = \dfrac{1}{(1-x)} + \dfrac{-1}{(2+3x)} + \dfrac{6}{(2+3x)^2}$

(b) $f(x) = (1-x)^{-1} - \frac{1}{2}\left(1+\frac{3}{2}x\right)^{-1} + \frac{6}{4}\left(1+\frac{3}{2}x\right)^{-2}$

$\left[\begin{aligned} &(1+x)^n = 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots \\ &(1+x)^{-1} = 1 - x + x^2 + \cdots \\ &(1-x)^{-1} = 1 + x + x^2 + \cdots \\ &(1+x)^{-2} = 1 - 2x + 3x^2 + \cdots \end{aligned}\right.$

$= (1+x+x^2+\cdots) - \frac{1}{2}\left(1 - \frac{3}{2}x + \frac{9}{4}x^2 \cdots\right) + \frac{3}{2}\left[1 - 2\times\frac{3}{2}x + 3\times\frac{9}{4}x^2 \cdots\right]$

$= (1+x+x^2+\cdots) + \left(-\frac{1}{2} + \frac{3}{4}x - \frac{9}{8}x^2 + \cdots\right) + \left(\frac{3}{2} - \frac{9}{2}x + \frac{81}{8}x^2 + \cdots\right)$

$= \underline{2 - \frac{11}{4}x + 10x^2 + \cdots}$

11 (a) Expand $\sqrt[3]{1+6x}$ in ascending powers of x, upto and including the term x^3, simplifying the coefficients. --[4]

(b) State the set of values of x for which the expansion is valid. --[1]

W-20/32/Q2

Solution: $\sqrt[3]{1+6x}$

(a) $= (1+6x)^{1/3}$

$\left[\because (1+x)^n = 1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots\right.$ for $|x|<1$

$= 1+\frac{1}{3}\cdot 6x+\frac{\frac{1}{3}(\frac{1}{3}-1)}{2}(6x)^2+\frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)}{6}(6x)^3$

$= 1+2x-4x^2+\frac{40}{3}x^3+\cdots$ ✓

(b) Expansion is valid for $|6x|<1 \Rightarrow |x|<\frac{1}{6}$ ✓

12. When $(a+bx)\sqrt{1+4x}$, where a and b are constants, is expanded in ascending power of x, the coefficient of x and x^2 are 3 and -6 respectively. Find the values of a and b. ---[6]

[W-21|31|Q6]

Solution: $(a+bx)(1+4x)^{1/2} = (a+bx)\left[1+\frac{1}{2}\times 4x + \frac{\frac{1}{2}\cdot(\frac{1}{2}-1)}{2!}\cdot(4x)^2 - - -\right]$

$= a+(b+2a)x+(2b-2a)x^2 + - -]$

$\therefore$ Coeff of x ; $2a+b=3$ --- (i)

and Coeff of x^2, $-2a+2b=-6$ --- (ii)

solving (i) and (ii)

$a=2$ and $b=-1$

13. Express: $\dfrac{4x^2-13x+13}{(2x-1)(x-3)}$ in partial fractions. ---[5]

[W-31|32|Q4]

Solution: $\dfrac{4x^2-13x+13}{(2x-1)(x-3)} = \dfrac{4x^2-13x+13}{2x^2-7x+3} = 2+\dfrac{x+7}{(2x-1)(x-3)}$ --- (1)

consider $\dfrac{x+7}{(2x-1)(x-3)} = \dfrac{a}{(2x-1)}+\dfrac{b}{(x-3)}$ --- (2)

(2) multiply by $(2x-1) \Rightarrow \dfrac{x+7}{x-3} = a+\dfrac{(2x-1)}{x-3}$

Now $(2x-1)=0 \Rightarrow$ put $x=\frac{1}{2} \Rightarrow \dfrac{15/2}{-5/6}=a \Rightarrow a=-3$

Again multiply by $(x-3)$ to (2) $\Rightarrow \dfrac{x+7}{2x-1} = \dfrac{a(x-3)}{(2x-1)}+b$

Now $x-3=0 \Rightarrow$ put $x=3 \Rightarrow \dfrac{10}{5}=b \Rightarrow b=2$

Put $a=-3$ and $b=2$ in (1)

Required partial fractions are: $2-\dfrac{3}{(2x-1)}+\dfrac{2}{(x-3)}$

14. Let $f(x) = \dfrac{2x^2+7x+8}{(1+x)(2+x)^2}$

(a) Express $f(x)$ in partial fractions. ---[5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. ---[5]

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Solution: $f(x) = \dfrac{2x^2+7x+8}{(1+x)(2+x)^2} = \dfrac{a}{(1+x)} + \dfrac{b}{(2+x)} + \dfrac{c}{(2+x)^2}$ --- ①

multiply ① by $(1+x) \Rightarrow \dfrac{2x^2+7x+8}{(2+x)^2} = a + \dfrac{b}{(2+x)} \times (1+x) + \dfrac{c}{(2+x)^2} \times (1+x)$

Put $x+1=0 \Rightarrow x=-1 \Rightarrow \dfrac{3}{1} = a+0+0 \Rightarrow \underline{a=3}$

multiply ① by $(2+x)^2 \Rightarrow \dfrac{2x^2+7x+8}{(1+x)} = \dfrac{a}{(1+x)} \times (2+x)^2 + b(2+x) + c$

Put $2+x=0 \Rightarrow x=-2 \Rightarrow \dfrac{8-14+8}{-1} = c \Rightarrow \underline{c=-2}$

Now multiply ① by D^r and (put $a=3$, $b=-2$)

$\Rightarrow 2x^2+7x+8 = 3(2+x)^2 + b(1+x)(2+x) - 2(1+x)$

Put $x=0 \Rightarrow 8 = 12 + 2b - 2 \Rightarrow b=-1$

Hence from ① the required partial fractions are:

$f(x) = \dfrac{3}{(1+x)} - \dfrac{1}{(2+x)} - \dfrac{2}{(2+x)^2}$

$= 3(1+x)^{-1} - \frac{1}{2}(1+x/2)^{-1} - \frac{1}{2}(1+x/2)^{-2}$

$= 3\left(1 - x + \frac{(-1)(-2)}{2}x^2 - -\right) - \frac{1}{2}\left(1 - \frac{x}{2} + \frac{(-1)(-2)}{2}\cdot\left(\frac{x}{2}\right)^2\right) - \frac{1}{2}\left(1 - 2\times\frac{x}{2} + \frac{(-2)(-3)}{2}\cdot\left(\frac{x}{2}\right)^2\right)$

$= \left(3 - 3x + 3x^2 + -\right) - \frac{1}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4}\right) - \frac{1}{2}\left(1 - x - \frac{3}{4}x^2 + -\right)$

$= \left(3 - \frac{1}{2} - \frac{1}{2}\right) + \left(-3 + \frac{1}{4} + \frac{1}{2}\right)x + \left(3 - \frac{1}{8} - \frac{3}{8}\right)x^2$

$= \underline{2 - \frac{9}{4}x + \frac{5}{2}x^2 + --}$

$\left[(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + --\right] \quad : |x| < 1$

15 Expand $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x, upto and including the terms in x^2, simplify the coefficients. ---[5]

W-22/33/Q2

Solution: $\sqrt{\frac{1+2x}{(1-2x)}} = \sqrt{\frac{(1+2x)}{(1-2x)} \times \frac{(1+2x)}{(1+2x)}} = \frac{(1+2x)}{\sqrt{1-4x^2}}$

$= (1+2x)(1-4x^2)^{-\frac{1}{2}} = (1+2x)\cdot\left(1+\left(-\frac{1}{2}\right)\cdot(-4x^2)\right) + \cdots$

$= (1+2x)(1+2x^2) = 1+2x^2+2x+\cancel{4x^3}+\cdots$

$= 1+2x+2x^2+\cdots$ ✓ $\left\{(1+x)^n = 1+nx+\frac{n(n-1)}{2}x^2 \cdots\right\}$