

PURE MATHEMATICS -3

9709

(March, June and November series 2020 – 2023 With marking scheme)

BINOMIAL THEOREM AND RATIONAL FUNCTION

EXERCISE -1

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1) SP-2020 _9709_3 Q2

(a) Expand $(1 + 3x)^{-\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [3]

(b) State the set of values of x for which the expansion is valid. [1]

2) MARCH-2020 _9709_32 Q9

$$\text{Let } f(x) = \frac{2 + 11x - 10x^2}{(1 + 2x)(1 - 2x)(2 + x)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

3) JUNE-2020 _9709_31 Q2

(a) Expand $(2 - 3x)^{-2}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

4) JUNE-2021 _9709_32 Q9

$$\text{Let } f(x) = \frac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

5) JUNE-2021 _9709_33 Q1

Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

6) JUNE-2022 _9709_31 Q2

(a) Expand $(2 - x^2)^{-2}$ in ascending powers of x , up to and including the term in x^4 , simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

7) JUNE-2022_9709_33 Q7

$$\text{Let } f(x) = \frac{5x^2 + 8x - 3}{(x-2)(2x^2 + 3)}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

8) JUNE-2023_9709_31 Q3

Find the coefficient of x^3 in the binomial expansion of $(3+x)\sqrt{1+4x}$. [4]

9) JUNE-2023_9709_33 Q10

$$\text{Let } f(x) = \frac{21 - 8x - 2x^2}{(1+2x)(3-x)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

10) OCT-2020_9709_31 Q9

$$\text{Let } f(x) = \frac{8 + 5x + 12x^2}{(1-x)(2+3x)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

11) OCT-2020_9709_32 Q2

(a) Expand $\sqrt[3]{1+6x}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

12) OCT-2021_9709_31 Q6

When $(a+bx)\sqrt{1+4x}$, where a and b are constants, is expanded in ascending powers of x , the coefficients of x and x^2 are 3 and -6 respectively.

Find the values of a and b . [6]

13) OCT-2021_9709_32 Q4

Express $\frac{4x^2 - 13x + 13}{(2x-1)(x-3)}$ in partial fractions. [5]

14) OCT-2022_9709_31 Q10

$$\text{Let } f(x) = \frac{2x^2 + 7x + 8}{(1+x)(2+x)^2}.$$

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

15) OCT-2022_9709_33 Q2

Expand $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [5]

MARKING SCHEME

1) SP-2020_9709_3 Q2

(a)	State a correct unsimplified version of the x or x^2 term	1	M1	Symbolic coefficients, e.g. $\left(\frac{-1}{2}\right)$, are not sufficient for the M mark.
	State correct first two terms $1 - x$	1	A1	
	State the next term $+ 2x^2$	1	A1	
		3		
(b)	$ x < \frac{1}{3}$	1	B1	OE

2) MARCH-2020_9709_32 Q9

(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{1-2x} + \frac{C}{2+x}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A = -2$, $B = 1$ and $C = 4$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5
(b)	Use correct method to find the first two terms of the expansion of $(1+2x)^{-1}$, $(1-2x)^{-1}$, $(2+x)^{-1}$ or $\left(1+\frac{1}{2}x\right)^{-1}$	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1FT + A1FT + A1FT
	Obtain final answer $1 + 5x - \frac{7}{2}x^2$	A1
		5

3) JUNE-2020_9709_31 Q2

2(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2-3x)^{-2}$ or $\left(1-\frac{3}{2}x\right)^{-2}$	M1
	State correct first term $\frac{1}{4}$	B1
	Obtain the next two terms $\frac{3}{4}x + \frac{27}{16}x^2$	A1 + A1
		4
2(b)	State answer $ x < \frac{2}{3}$, or equivalent	B1
		1

4) JUNE-2021 _9709_32 Q9

(a)	State or imply the form $\frac{A}{2+x} + \frac{B+Cx}{3+x^2}$	B1	
	Use a correct method for finding a constant	M1	SOI
	Obtain one of $A = 4$, $B = 1$ and $C = -2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	ISW
		5	
(b)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(3+x^2)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	M1	Allow unsimplified but not if still including nC_r
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1 FT A1 FT	$2\left(1-\frac{1}{2}x+\left(\frac{1}{2}x\right)^2 \dots\right)$ $+\frac{1}{3}(1-2x)\left(1-\frac{1}{3}x^2 \dots\right)$ The FT is on <i>their</i> A , B and C
	Multiply out, up to the terms in x^2 , by $B+Cx$, where $BC \neq 0$	M1	Allow with B and C as implied in part (b)
	Obtain final answer $\frac{7}{3}-\frac{5}{3}x+\frac{7}{18}x^2$	A1	Or equivalent in form $p+qx+rx^2$. A0 if they multiply through by 18.
		5	

5) JUNE-2021 _9709_33 Q1

State correct first two terms $1+2x$	B1	
State a correct unsimplified version of the x^2 or x^3 term	M1	Symbolic binomial coefficients are not sufficient for the M mark.
Obtain the next term $-x^2$	A1	
Obtain the final term $\frac{4}{3}x^3$	A1	
	4	

6) JUNE-2022 _9709_31 Q2

(a)	State a correct unsimplified version of the x^3 or the x^4 term of the expansion of $(2-x^2)^{-2}$ or $\left(1-\frac{1}{2}x^2\right)^{-2}$	M1	$\frac{1}{4}\left(1+2\frac{x^2}{2}+\frac{-2 \cdot -3}{2}\left(\frac{x^2}{2}\right)^2 \dots\right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	B1	Accept 2^{-2} .
	Obtain the next two terms $\frac{1}{4}x^2 + \frac{3}{16}x^4$	A1 A1	A1 for each one correct ISW. Full marks for $\frac{1}{4}\left(1+x^2+\frac{3}{4}x^4\right)$ ISW.
			SC allow M1 A1 A1 for $\frac{1}{4}$ and $1+x^2+\frac{3}{4}x^4$ SOI. SC allow M1 A1 for $1+x^2+\frac{3}{4}x^4$
		4	
(b)	State answer $ x < \sqrt{2}$	B1	Or $-\sqrt{2} < x < \sqrt{2}$.
		1	

7) JUNE-2022 _9709_33 Q7

(a)	State or imply the form $\frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$	B1	If $1 - \frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$ or $\frac{A}{x-2} + \frac{C}{2x^2+3}$ B0 then M1 A1 (for $A=3$) still possible.
	Use a correct method for finding a constant	M1	
	Obtain one of $A=3$, $B=-1$ and $C=6$	A1	Allow all A marks obtained even if method would give errors if equations solved in a different order.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Use correct method to find the first two terms of the expansion of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2x^2+3)^{-1}$ or $\left(1+\frac{2}{3}x^2\right)^{-1}$	M1	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions, up to the term in x^2 , of each partial fraction	A1 FT A1 FT	The FT is on A , B and C . $-\frac{A}{2} \left[1 - \left(-\frac{x}{2} \right) + \frac{(-1)(-2)}{2} \left(-\frac{x}{2} \right)^2 + \dots \right]$ $\frac{Bx+C}{3} \left[1 - \frac{2x^2}{3} + \dots \right]$
	Extract the coefficient 3 correctly from $(2x^2+3)^{-1}$ with expansion to $1 \pm \frac{2}{3}x^2$ then multiply by $Bx+C$ up to the terms in x^2 , where $BC \neq 0$	M1	$\frac{C}{3} + \frac{Bx}{3} \pm \frac{C}{3} \left(\frac{2}{3} \right) x^2$ or $\frac{1}{3} \left(C + Bx \pm C \left(\frac{2}{3} \right) x^2 \right)$ Allow a slip in multiplication for M1. Allow miscopies in B and C from 7(a).
	Obtain final answer $\frac{1}{2} - \frac{13}{12}x - \frac{41}{24}x^2$	A1	Do not ISW.
		5	

8) JUNE-2023 _9709_31 Q3

	State unsimplified term in x^3 , or its coefficient, in the expansion of $(1+4x)^{\frac{1}{2}}$	B1	$\frac{1}{2} \times \frac{-1}{2} \times \frac{-3}{2} (4x)^3 (=4)$ Must expand binomial coefficient.
	State unsimplified term in x^2 , or its coefficient, in the expansion of $(1+4x)^{\frac{1}{2}}$	B1	$\frac{1}{2} \times \frac{-1}{2} (4x)^2 (= -2)$ Must expand binomial coefficient.
	Multiply by $(3+x)$ and combine terms in x^3 , or their coefficients	M1	$(3 \times 4 - 1 \times 2)$ Allow if they expanded with x rather than $4x$.
	Obtain answer 10	A1	Accept $10x^3$
		4	

9) JUNE-2023 _9709_33 Q10

(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$	B1	Alternative form: $\frac{A}{1+2x} + \frac{Dx+E}{(3-x)^2}$.
	Use a correct method to find a constant	M1	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1+2x)(3-x)^3$.
	Obtain one of $A=2$, $B=2$ and $C=-3$	A1	Alternative form: obtain one of $A=2$, $D=-2$ and $E=3$.
	Obtain a second value	A1	
	Obtain the third value	A1	Do not need to substitute values back into original form.
		5	If $\frac{A}{1+2x} + \frac{B}{3-x} + \frac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for A, A1 for B and A1 for C and D. If $C=0$ then recovers B1 from above.
b)	Use a correct method to obtain the first two terms of one of the unsimplified expansions $(1+2x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}, \left(1-\frac{1}{3}x\right)^{-2}, (3-x)^{-1}, (3-x)^{-2}$	M1	$(1+2x)^{-1} = 1 + (-1)(2x) + \dots$ $\left(1-\frac{1}{3}x\right)^{-1} = 1 + (-1)(-x/3) + \dots$ $\left(1-\frac{1}{3}x\right)^{-2} = 1 + (-2)(-x/3) + \dots$ $(3-x)^{-1} = 3^{-1} + (-1)3^{-2}(-x) \dots$ $(3-x)^{-2} = 3^{-2} + (-2)3^{-3}(-x) + \dots$
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction If correct, should be working with $\frac{2}{1+2x} + \frac{2}{3-x} - \frac{3}{(3-x)^2}$ or $\frac{2}{1+2x} + \frac{-2x+3}{(3-x)^2}$	A1 FT A1 FT A1 FT	Follow through on <i>their</i> A, B, C $A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \dots)$ $\frac{B}{3} (1 + (-1)(-x/3) + ((-1)(-2)/2)(-x/3)^2 + \dots)$ $\frac{C}{3^2} (1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \dots)$. Must be <i>their</i> coefficients from (a) but may be unsimplified expansions for FT marks. If correct, expect to see $2(1 - 2x + (2x)^2)$ or $2 - 4x + 8x^2$ $\frac{2}{3} (1 + \frac{x}{3} + (\frac{x}{3})^2)$ or $\frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2$ $-\frac{1}{3} (1 + \frac{2x}{3} + (3)(\frac{x}{3})^2)$ or $-\frac{1}{3} - \frac{2}{9}x - \frac{x^2}{9}$.
	Obtain final answer $\frac{7}{3} - 4x + \frac{215}{27}x^2$	A1	Accept $2\frac{1}{3} - 4x + 7\frac{26}{27}x^2$. No ISW.
	Alternative Method for Question 10(b)		
(b)	For the form $\frac{A}{1+2x} + \frac{Dx+E}{(3-x)^2}$	M1*	For the first two terms of an expanded partial fraction, following their A, D, E.
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction	A1FT A1FT	$A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \dots) +$ $(Dx + E) \frac{1}{3^2} (1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \dots)$ $2(1 - 2x + (2x)^2 + \dots)$ $+ \frac{-2x+3}{3^2} (1 + \frac{2x}{3} + (3)(\frac{x}{3})^2 + \dots)$.
	Multiply out fully	DM1	Provided $DE \neq 0$. Ignore cubic terms and above. Allow error in one term but all terms must be present. If correct, expect to see $2 - 4x + 8x^2 - \frac{2}{9}x - \frac{4}{27}x^2 + \frac{1}{3} + \frac{2}{9}x + \frac{1}{9}x^2$
	Obtain final answer $\frac{7}{3} - 4x + \frac{215}{27}x^2$	A1	Accept $2\frac{1}{3} - 4x + 7\frac{26}{27}x^2$. No ISW

b) Alternative Method for Question 10(b): Maclaurin's Series		
Correct derivatives for $A(1+2x)^{-1}$, $B(3-x)^{-1}$ and $C(3-x)^{-2}$ $(-1)(2)A(1+2x)^{-2}$, $(-1)(-1)B(3-x)^{-2}$ and $(-2)(-1)C(3-x)^{-3}$	B1 FT	
One of following $(-2)(2)(-1)(2)A(1+2x)^{-3}$, $(-2)(-1)(-1)(-1)B(3-x)^{-3}$ and $(-3)(-1)(-2)(-1)C(3-x)^{-4}$	B1 FT	
All correct	B1 FT	
Substitute in $f(0) + xf'(0) + \frac{x^2}{2} f''(0)$	M1	
Obtain final answer $\frac{7}{3} - 4x + \frac{215}{27}x^2$	A1	Accept $2\frac{1}{3} - 4x + 7\frac{26}{27}x^2$. No ISW
	5	

10) OCT-2020_9709_31 Q9

a)		
State or imply the form $\frac{A}{1-x} + \frac{B}{2+3x} + \frac{C}{(2+3x)^2}$	B1	
Use a correct method for finding a coefficient	M1	
Obtain one of $A = 1$, $B = -1$, $C = 6$	A1	
Obtain a second value	A1	
Obtain the third value	A1	In the form $\frac{A}{1-x} + \frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
	5	
b)		
Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1 $A \left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\dots} \right] A = 1$ $\frac{B}{2} \left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\dots} \right] B = 1$ $\frac{C}{4} \left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\dots} \right] C = 6$
Obtain correct un-simplified expansions up to the term in of each partial fraction	A1 FT + A1 FT + A1 FT	$\left(1+x+x^2\right) + \left(-\frac{1}{2} + \left(\frac{3}{4}\right)x - \left(\frac{9}{8}\right)x^2\right)$ $+ \left(\frac{6}{4} - \left(\frac{18}{4}\right)x + \left(\frac{81}{8}\right)x^2\right)$ [The FT is on A, B, C] $\left(1 - \frac{1}{2} + \frac{6}{4}\right) + \left(1 + \frac{3}{4} - \frac{18}{4}\right)x + \left(1 - \frac{9}{8} + \frac{81}{8}\right)x^2$
Obtain final answer $2 - \frac{11}{4}x + 10x^2$, or equivalent	A1	Allow unsimplified fractions $\frac{(Dx+E)}{4} \left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\dots} \right] D = -3, E = 4$ The FT is on A, D, E.
	5	

11) OCT-2020_9709_32 Q2

a)	State a correct unsimplified version of the x or x^2 or x^3 term	M1	For the given expression
	State correct first two terms $1 + 2x$	A1	
	Obtain the next two terms $-4x^2 + \frac{40}{3}x^3$	A1 + A1	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
b)	State answer $ x < \frac{1}{6}$	B1	OE. Strict inequality
		1	

12) OCT-2021_9709_31 Q6

State or imply $1 + 2x$ as first terms of the expansion of $\sqrt{1+4x}$	B1	Allow for correct unsimplified expression.
State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$	B1	Allow for correct unsimplified expression.
Form an expression for the coefficient of x or coefficient of x^2 in the expansion of $(a+bx)\sqrt{1+4x}$ and equate to given coefficient	M1	All relevant terms considered.
Obtain $2a + b = 3$, or equivalent	A1	One correct equation.
Obtain $-2a + 2b = -6$ or equivalent	A1	Second correct equation.
Obtain answer $a = 2$ and $b = -1$	A1	
	6	

13) OCT-2021_9709_32 Q4

State or imply the form $A + \frac{B}{2x-1} + \frac{C}{x-3}$	B1	$\frac{Dx+E}{2x-1} + \frac{F}{x-3}$ and $\frac{P}{2x-1} + \frac{Qx+R}{x-3}$ are also valid.
Use a correct method for finding a constant	M1	
Obtain one of $A = 2$, $B = -3$ and $C = 2$	A1	Allow maximum M1A1 for one or more 'correct' values after B0 .
Obtain a second value	A1	
Obtain the third value	A1	
Alternative method for Question 4		
Divide numerator by denominator	M1	
Obtain $2\left[+ \frac{Px+Q}{(2x-1)(x-3)} \right]$	A1	$\left[2 + \frac{x+7}{(2x-1)(x-3)} \right]$
State or imply the form $\frac{D}{2x-1} + \frac{E}{x-3}$	B1	
Obtain one of $D = -3$ and $E = 2$	A1	
Obtain a second value	A1	
	5	

14) OCT-2022_9709_31 Q10

(a)	State or imply the form $\frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 3$, $B = -1$ and $C = -2$	A1	SR after B0 can score M1A1 for one correct value
	Obtain a second value	A1	
	Obtain the third value	A1	$\frac{A}{1+x} + \frac{Dx+E}{(2+x)^2}$, where $A = 3$, $D = -1$ and $E = -4$, is awarded B1 M1 A1 A1 A1 as above.
		5	
(b)	Use a correct method to find the first two terms of the expansion of $(1+x)^{-1}$, $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(2+x)^{-2}$ or $\left(1+\frac{1}{2}x\right)^{-2}$	M1	For the A, D, E form of fractions, award M1 A1FT A1FT for the expanded partial fractions, then if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A3 FT	$3(1-x+x^2 \dots)$ $-\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4} \dots\right)$ $-\frac{2}{4}\left(1-x+\frac{3}{4}x^2\right)$
	Obtain final answer $2 - \frac{9}{4}x + \frac{5}{2}x^2$	A1	
		5	

15) OCT-2022_9709_33 Q2

State a correct unsimplified term in x or x^2 of the expansion of either $(1+2x)^{\frac{1}{2}}$ or $(1-2x)^{-\frac{1}{2}}$	B1	
State correct unsimplified expansion of $(1+2x)^{\frac{1}{2}}$ up to the term in x^2	B1	
State correct unsimplified expansion of $(1-2x)^{-\frac{1}{2}}$ up to the term in x^2	B1	
Obtain sufficient terms of the product of the expansions	M1	
Obtain final answer $1+2x+2x^2$	A1	
Alternative method for question 2		
State that the expression equals $(1+2x)(1-4x^2)^{-\frac{1}{2}}$ and state a term of the expansion	B1	
State correct unsimplified expansion of $(1-4x^2)^{-\frac{1}{2}}$ up to the term in x^2	B1 + B1	
Obtain sufficient terms of the product of $(1+2x)$ and the expansion	M1	
Obtain final answer $1+2x+2x^2$	A1	
	5	