PURE MATHEMATICS -3

9709

(June 2022-2023 and November 2020 – 2023 With marking scheme)

DIFFERENTIATION AND INTEGRATION

EXERCISE -1(b)

MANJULA BALAJI

1) JUNE 2022_9709_31 Q6

Let $I = \int_0^3 \frac{27}{(9+x^2)^2}\,dx$.

(a) Using the substitution $x = 3\tan\theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2\theta\,d\theta$. [4]

(b) Hence find the exact value of I. [4]

2) JUNE 2022_9709_31 Q8

The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

(a) Express $\frac{dy}{dx}$ in terms of x and y. [4]

The tangent to the curve at the point where $x = 0$ and the tangent at the point where $y = 0$ intersect at the acute angle α.

(b) Find the exact value of $\tan\alpha$. [5]

3) JUNE 2022_9709_32 Q4

The equation of a curve is $y = \cos^3 x\sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

4) JUNE 2022_9709_32 Q7

The equation of a curve is $x^3 + 3x^2y - y^3 = 3$.

(a) Show that $\frac{dy}{dx} = \frac{x^2+2xy}{y^2-x^2}$. [4]

(b) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

5) JUNE 2022_9709_32 Q8

Let $f(x) = \frac{x^2+9x}{(3x-1)(x^2+3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find $\int_1^3 f(x)\,dx$, giving your answer in a simplified exact form. [5]

6) JUNE 2022_9709_33 Q4

The curve $y = e^{-4x} \tan x$ has two stationary points in the interval $0 \leqslant x < \frac{1}{2}\pi$.

(a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b\sin 2x)e^{-4x}$, where a and b are constants. [4]

(b) Hence find the exact x-coordinates of the two stationary points. [3]

7) JUNE 2022_9709_33 Q6

The parametric equations of a curve are $x = \frac{1}{\cos t}$, $y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = \frac{\cos t}{\sin^2 t}$. [5]

(b) Find the equation of the tangent to the curve at the point where $y = 0$. [3]

8) JUNE 2023_9709_31 Q5

The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

9) JUNE 2023_9709_31 Q8

Let $f(x) = \frac{3 - 3x^2}{(2x+1)(x+2)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^4 f(x)\,dx$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. [5]

10) JUNE 2023_9709_32 Q7

The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x + 2y}{2x + 3y}$. [4]

(b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

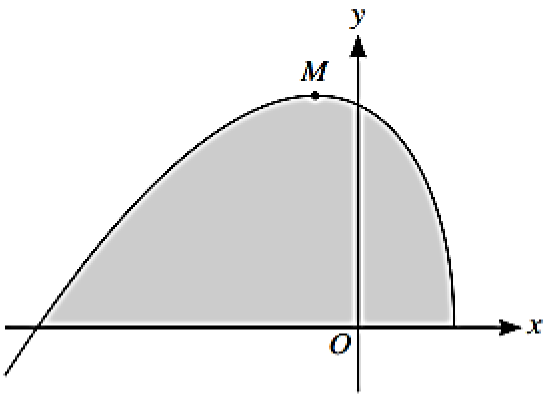
11) JUNE 2023_9709_32 Q9

Let $f(x) = \dfrac{2x^2 + 17x - 17}{(1+2x)(2-x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 f(x)\,dx = \frac{5}{2} - \ln 72$. [5]

12) JUNE 2023_9709_32 Q10



The diagram shows the curve $y = (x+5)\sqrt{3-2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

13) JUNE 2023_9709_33 Q4

The parametric equations of a curve are

$$x = \frac{\cos\theta}{2-\sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\dfrac{dy}{dx} = (2-\sin\theta)^2$. [5]

14) JUNE 2023_9709_33 Q7

(a) Use the substitution $u = \cos x$ to show that

$$\int_0^{\pi} \sin 2x\, e^{2\cos x}\,dx = \int_{-1}^{1} 2ue^{2u}\,du.$$ [4]

(b) Hence find the exact value of $\int_0^{\pi} \sin 2x\, e^{2\cos x}\,dx$. [4]

15) OCT 2020_9709_31 Q3

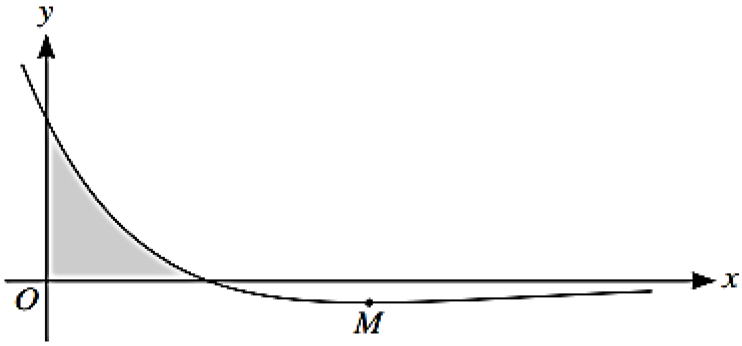
The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$. [5]

16) OCT 2020_9709_31 Q10

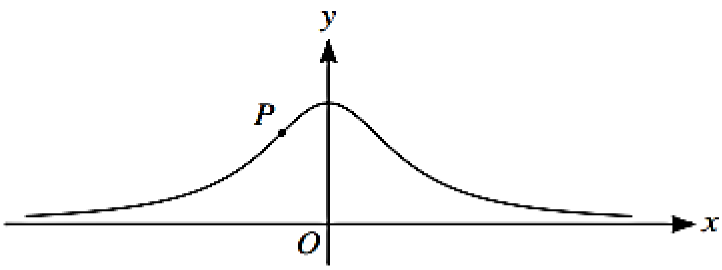


The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

17) OCT 2020_9709_32 Q5



The diagram shows the curve with parametric equations

$$x = \tan \theta, \quad y = \cos^2 \theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

18) OCT 2020_9709_32 Q9

Let $f(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x)\,dx$. [6]

19) OCT 2021_9709_31 Q3

The curve with equation $y = xe^{1-2x}$ has one stationary point.

(a) Find the coordinates of this point. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

20) OCT 2021_9709_31 Q4

Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x + 1)\sqrt{x}}\,dx.$$ [6]

21) OCT 2021_9709_32 Q6

(a) Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\tfrac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$ [3]

(b) Hence show that $\int_0^{\frac{1}{4}\pi} \sin 3x \cos 2x\,dx = \frac{1}{5}(3 - \sqrt{2})$. [3]

22) OCT 2021_9709_32 Q9

The equation of a curve is $ye^{2x} - y^2e^x = 2$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{2ye^x - y^2}{2y - e^x}$. [4]

(b) Find the exact coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

23) OCT 2021_9709_33 Q4

Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \tfrac{1}{2}x\,dx$. [5]

24) OCT 2021_9709_33 Q7

The equation of a curve is $\ln(x+y) = x - 2y$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{x+y-1}{2(x+y)+1}$. [4]

(b) Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

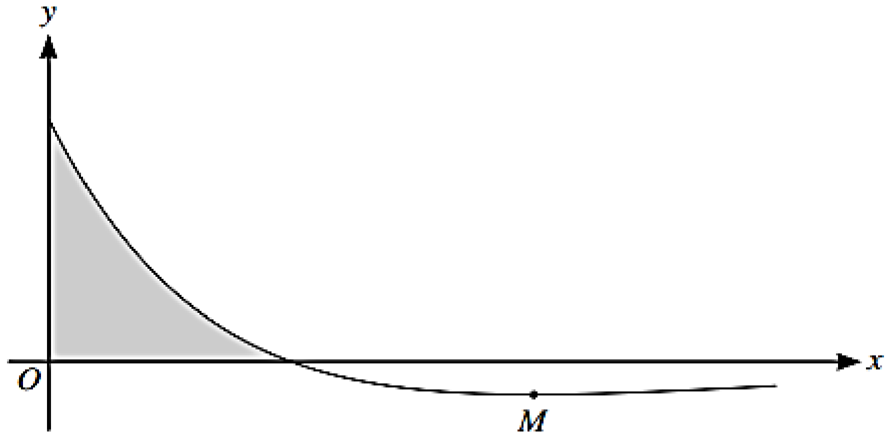
25) OCT 2021_9709_33 Q9

Let $f(x) = \dfrac{1}{(9-x)\sqrt{x}}$.

(a) Find the x-coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x)\,dx = \frac{1}{3}\ln 5$. [6]

26) OCT 2022_9709_31 Q9



The diagram shows part of the curve $y = (3-x)e^{-\frac{1}{3}x}$ for $x \geqslant 0$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

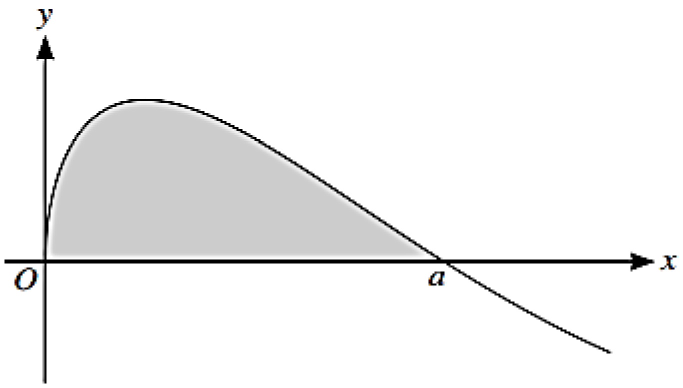
(b) Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e. [5]

27) OCT 2022_9709_32 Q3

The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

28) OCT 2022_9709_32 Q8



The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the x-axis at the point where $x = a$.

(a) State the exact value of a. [1]

(b) Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the x-axis. [7]

29) OCT 2022_9709_32 Q10

Let $f(x) = \dfrac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^4 f(x)\,dx$. Give your answer as a single logarithm. [5]

30) OCT 2022_9709_33 Q3

Find the exact value of $\int_0^{\frac{1}{4}\pi} x \sec^2 x\,dx$. [5]

31) OCT 2022_9709_33 Q4

The parametric equations of a curve are

$$x = 2t - \tan t, \qquad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

Show that $\dfrac{dy}{dx} = \cot t$. [5]

32) OCT 2022_9709_33 Q11

Let $f(x) = \dfrac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

MARKING SCHEME

1) JUNE 2022_9709_31 Q6

(a)	State or imply $dx = 3\sec^2\theta\, d\theta$	B1	
	Substitute throughout for x and dx	M1	
	Obtain any correct form in terms of θ	A1	e.g. $\int \frac{81\sec^2\theta}{(9+9\tan^2\theta)^2} d\theta$
	Justify change of limits and obtain $\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$ correctly	A1	AG
		4	
(b)	Obtain indefinite integral of the form $\int a + b\cos 2\theta\, d\theta$, where $ab \neq 0$	*M1	
	Obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	A1	
	Use correct limits correctly in an expression containing $p\theta$ and $q\sin 2\theta$ where $pq \neq 0$	DM1	$\frac{\pi}{8} + \frac{1}{4}(-0)$
	Obtain answer $\frac{1}{8}(\pi + 2)$	A1	Or exact equivalent e.g. $\frac{1}{8}\pi + \frac{1}{4}$.
		4	

2) JUNE 2022_9709_31 Q8

(a)	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1
	State or imply $2y + 2x\frac{dy}{dx}$ as derivative of $2xy$	B1
	Complete differentiation and equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1
	Obtain answer $-\frac{3x^2+2y}{3y^2+2x}$	A1
		4
(b)	Find gradient at either $(0, -2)$ or $(-2, 0)$	M1
	Obtain answers $\frac{1}{3}$ and 3	A1 A1
	Use $\tan(A \pm B)$ formula to find $\tan\alpha$	M1
	Obtain answer $\tan\alpha = \frac{4}{3}$	A1
		5

3) JUNE 2022_9709_32 Q4

Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$	M1	e.g. two terms with one part of $\frac{dy}{dx} = p\cos^2 x \sin x\sqrt{\sin x} + q\frac{\cos^3 x\cos x}{\sqrt{\sin x}}$.
Obtain correct derivative in any form e.g. $\frac{dy}{dx} = -3\cos^2 x\sin x\sqrt{\sin x} + \frac{\cos^3 x\cos x}{2\sqrt{\sin x}}$	A1 A1	A1 for each correct term substituted in the complete derivative.
Equate their derivative to zero and obtain a horizontal equation with positive integer powers of $\sin x$ and/or $\cos x$ from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra.	M1	e.g. $-3\cos^2 x\sin^2 x + \frac{1}{2}\cos^4 x = 0$
Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function	M1	Can be awarded before the previous M1. May involve more than one trigonometric term.
Obtain $7\cos^2 x = 6$, $7\sin^2 x = 1$, or $6\tan^2 x = 1$, or equivalent, and obtain answer $x = 0.388$	A1	CAO. The question asks for 3 sf. Ignore additional answers outside $\left(0, \frac{\pi}{2}\right)$. 22.2° is A0.
	6	

4) JUNE 2022_9709_32 Q7

(a)	State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$	B1	
	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	Allow B1 B1 for $(3x^2dx+)6xydx + 3x^2dy - 3y^2dy[=0]$
	Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\frac{dy}{dx}$ as subject	M1	Allow if zero implied by subsequent working. Allow if recover from an extra $\frac{dy}{dx} = \ldots$ at the beginning of the left-hand side.
	Obtain $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ correctly	A1	AG Accept y' for $\frac{dy}{dx}$.
		4	
(b)	Equate numerator to zero	*M1	Must be using the given derivative.
	Obtain $x = -2y$, or equivalent	A1	An equation with x or y as the subject SOI.
	Use $x^3 + 3x^2y - y^3 = 3$ to obtain an equation in x or y	DM1	$-8y^3 + 12y^3 - y^3 = 3$ or $x^3 - \frac{3}{2}x^3 + \frac{1}{8}x^3 = 3$ or any equivalent form (do not need to evaluate powers).
	Obtain the point (– 2, 1) and no others from solving their cubic equation	A1	Allow if each component stated separately. ISW.
	State the point $(0, -\sqrt[3]{3})$, or equivalent from correct work	B1	Accept $(0, \sqrt[3]{-3})$, or (0, – 1.44) (-1.44225). Allow if each component stated separately. ISW.
		5	

5) JUNE 2022_9709_32 Q8

8(a)	State or imply the form $\frac{A}{3x-1}+\frac{Bx+C}{x^2+3}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 1$, $B = 0$ and $C = 3$ from correct working	A1	A maximum of M1 A1 is available after B0.
	Obtain a second value from correct working	A1	
	Obtain the third value from correct working	A1	
		5	
8(b)	Integrate and obtain term $\frac{1}{3}\ln(3x-1)$	B1 FT	OE e.g. $\frac{1}{3}\ln(x-\frac{1}{3})$. The FT is on the value of A.
	Obtain term of the form $k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	M1	
	Obtain term $\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	A1 FT	OE. The FT is on the value of C.
	Substitute correct limits in an integral of the form $a\ln(3x-1)+k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$, where $ak\neq 0$, and evaluate trigonometry	M1	Must be subtracted the right way round. $\left(\frac{1}{3}\ln 8-\frac{1}{3}\ln 2+\sqrt{3}\times\frac{\pi}{3}-\sqrt{3}\times\frac{\pi}{6}\right)$ Angles should be in radians. Condone angles as decimals.
	Obtain answer $\frac{2}{3}\ln 2+\frac{\sqrt{3}\pi}{6}$ from correct working in part **8(b)**	A1	Or exact 2-term equivalent e.g. $\frac{1}{3}\ln 4+\frac{\pi}{2\sqrt{3}}$ ISW.
		5	

6) JUNE 2022_9709_33 Q4

(a)	Use correct product rule or quotient rule, and attempt at chain rule	M1	$ke^{-4x}\tan x+e^{-4x}\sec^2 x$ or $\frac{e^{4x}\sec^2 x-\tan x(ke^{4x})}{(e^{4x})^2}$ Need to see $d(\tan x)/dx=\sec^2 x$ (formula sheet) and attempt at ke^{-4x}, where $k\neq 1$.
	Obtain correct derivative in any form	A1	$-4e^{-4x}\tan x+e^{-4x}\sec^2 x$ or $\frac{e^{4x}\sec^2 x-\tan x(4e^{4x})}{(e^{4x})^2}$
	Use trigonometric formulae to express derivative in the form $ke^{-4x}\sin x\cos x\sec^2 x+ae^{-4x}\sec^2 x$ or $ke^{-4x}\frac{\sin x\cos x}{\cos x\cos x}+ae^{-4x}\sec^2 x$ or $\sec^2 x(ke^{-4x}\sin x\cos x+ae^{-4x})$ Allow $\frac{1}{\cos^2 x}$ instead of $\sec^2 x$	M1	Need to use $\frac{\tan x}{\sec^2 x}=\sin x\cos x$ or $\tan x=\frac{\sin x}{\cos x}.\frac{\cos x}{\cos x}$ OE. M1 is independent of previous M1, but expression must be of appropriate form.
	Obtain correct answer with $a = 1$ and $b = -2$	A1	At least one line of trigonometric working is required from $-4e^{-4x}\tan x+e^{-4x}\sec^2 x$ to given answer $\sec^2 x(1-2\sin 2x)e^{-4x}$ with elements in any order. If only error: $4\sin x\cos x = 4\sin 2x$ M1 A1 M1 A0.
		4	
(b)	Equate derivative to zero and use correct method to solve for x	M1	$\sin 2x=\frac{1}{2}$, hence $x=\frac{1}{2}\sin^{-1}\frac{1}{2}$ or $x=\tan^{-1}(2\pm\sqrt{3})$ Allow M1 for correct method for non-exact value.
	Obtain answer, e.g. $x=\frac{1}{12}\pi$	A1	[0.262 M1 A0]
	Obtain second answer, e.g. $\frac{5}{12}\pi$ and no other in the given interval	A1 FT	FT $\frac{\pi-\textit{their}\,2x}{2}$ if exact values; x must be $<\frac{\pi}{2}$. Ignore answers outside the given interval. Treat answers in degrees as a misread. 15°, 75°. SC No values found for a and b in **4(a)** but chooses values in **4(b)**: max **M1** for x.

7) JUNE 2022_9709_33 Q6

6(a)	Use chain rule at least once	M1	Needs $\frac{dy}{dt}=\frac{1}{\tan t}\frac{d}{dt}(\tan t)$ or $\frac{dx}{dt}=(-1)(\cos^{-2}t)\frac{d}{dt}(\cos t)$. BOD if + and (– 1) (– 1) not seen. $\frac{dx}{dt}=\sec t\tan t$ (from List of Formulae MF19) M1 A1. If $\frac{dx}{dt}=-\sec t\tan t$ M1 A0.
	Obtain $\frac{dx}{dt}=\sec t\tan t$	A1	OE e.g. $\sin t(\cos t)^{-2}$. If e.g. $\frac{dx}{dt}=\sec x\tan x$ or $\sec\theta\tan\theta$ or $\sec t\tan x$, condo recovery on next line.
	Obtain $\frac{dy}{dt}=\frac{\sec^2 t}{\tan t}$	A1	OE e.g. $\frac{1}{\sin t\cos t}$. If e.g. $\frac{dy}{dt}=\frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2\theta}{\tan\theta}$, condone recovery on next line. Only penalise notation errors once in $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if no recovery.
	Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1	Allow even if previous M0 scored, but must be using derivatives.
	Obtain given answer $\frac{\cos t}{\sin^2 t}$	A1	AG After $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ used, any notation error A0. Must cancel cos t correctly.
		5	

(b)	State or imply $t=\frac{1}{4}\pi$ when $y=0$	B1	
	Form the equation of the tangent at $y=0$ or find c	M1	$x=\sqrt{2}$, $\frac{dy}{dx}=\sqrt{2}$ and $y=0$, *their* coordinates and gradient used in $y=mx+c$.
	Obtain answer $y=\sqrt{2}x-2$	A1	OE e.g. $y=\sqrt{2}(x-\sqrt{2})$ ISW. Allow $y = 1.41x - 2[.00]$ or $1.41(x - 1.41)$.
		3	

8) JUNE 2023_9709_31 Q5

(a)	State or imply $2xy + x^2\frac{dy}{dx}$ as derivative of x^2y	B1	Accept partial: $\frac{\partial}{\partial x} \to 2xy$.
	State or imply $2ay\frac{dy}{dx}$ as derivative of ay^2	B1	Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain answer $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ from correct working	A1	AG
		4	
(b)	State or imply $2ay - x^2 = 0$	*M1	
	Substitute into equation of curve to obtain equation in x and a or in y and a	DM1	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	A1	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	A1	e.g. $(-2a, 2a)$.
		4	SC: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

9) JUNE 2023_9709_31 Q8

(a)	State or imply the form $\frac{A}{2x+1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$	B1	Accept $\frac{A}{2x+1} + \frac{Dx+E}{(x+2)^2}$.
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 1, B = -2, C = 3$	A1	For alternative form: $A = 1, D = -2, E = -1$.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain one of $\frac{1}{2}\ln(2x+1), -2\ln(x+2), \frac{-3}{x+2}$	B1 FT	The follow through is on *their* A, B, C.
	Obtain a second term	B1 FT	If the alternative form is used, then either need to use integration by parts or split the fraction further.
	Obtain the third term	B1 FT	
	Substitute limits correctly in an integral with at least two terms of the form $\frac{1}{2}\ln(2x+1), -2\ln(x+2)$ and $\frac{-3}{x+2}$ and subtract in correct order	M1	The terms used need to have been obtained correctly. Must be exact values, not decimals.
	Obtain $1 - \ln 3$	A1	
		5	

10) JUNE 2023_9709_32 Q7

(a)	State or imply $6y\frac{dy}{dx}$ as the derivative of $3y^2$	B1	Allow y' for $\frac{dy}{dx}$ throughout. Accept $\frac{\partial f}{\partial x} = 6x + 4y$.
	State or imply $4x\frac{dy}{dx} + 4y$ as the derivative of $4xy$	B1	Accept $\frac{\partial f}{\partial y} = 4x + 6y$.
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	Allow an extra $\frac{dy}{dx}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\frac{dy}{dx} = -\frac{\partial f}{\partial x} \Big/ \frac{\partial f}{\partial y}$
	Obtain $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$	A1	AG – must come from correct working. The position of the negative must be clear.
(b)	Equate $\frac{dy}{dx}$ to –2 and solve for x in terms of y or for y in terms of x	*M1	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	A1	Seen or implied by correct later work.
	Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation	DM1	Allow unsimplified.
	Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$	A1	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	A1	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

11) JUNE 2023_9709_32 Q10

(a)	Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$	*M1	Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted.
	Obtain correct derivative in any form	A1	e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$.
	Equate derivative to zero and obtain a linear equation	DM1	Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}(-(x+5)+(3-2x))=0$.
	Obtain a correct linear equation.	A1	e.g. $-(x+5)+3-2x=0$.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$.	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with x, y stated separately. ISW
	Alternative Method for Question 10(a)		
	Obtain y^2 and differentiate	*M1	Ignore *their* left hand side i.e. *their* $\frac{d}{dx}y^2$.
	Obtain correct derivative in any form	A1	e.g. $-6x^2-34x-20$.
	Equate derivative to zero and solve for x	DM1	
	Obtain $-\frac{2}{3}$	A1	Ignore -5 if seen.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW
		5	
(b)	Use the given substitution and reach $a\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\,du$	*M1	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x}\,dx$ to obtain first term of the line below.
	Obtain correct integral $-\frac{1}{2}\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\,du$	A1	OE e.g. $-\frac{1}{2}\left[\int\frac{3-u}{2}\sqrt{u}\,du+5\int\sqrt{u}\,du\right]$. Ignore limits at this stage. Condone if du missing.
	$x=-5$ and $\frac{3}{2}$	B1	SOI e.g. by $u=13$ and 0. In any order and at any stage.
	Use correct limits the right way round in an integral of the form $a\left(\frac{26}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right)$	DM1	
	Obtain answer $\frac{169}{15}\sqrt{13}$ or $a=\frac{169}{15}$	A1	or exact equivalents.
		5	

12) JUNE 2023_9709_32 Q9

a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1	Alternative form: $\frac{A}{1+2x}+\frac{Dx+E}{(2-x)^2}$
	Use a correct method for finding a coefficient	M1	e.g. $A(2-x)^2+B(1+2x)(2-x)+C(1+2x)$ $=2x^2+17x-17$ and compare coefficients or substitute for x. $A(2-x)^3+B(1+2x)(2-x)^2+C(1+2x)(2-x)$ $=2x^2+17x-17$ scores M0.
	Obtain one of $A=-4$, $B=-3$ and $C=5$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A=-4$, $D=3$ and $E=-1$.
		5	
(b)	Integrate and obtain terms $-2\ln(1+2x)+3\ln(2-x)+\frac{5}{2-x}$	B1FT B1FT B1FT	OE The FT is on correct use of *their* A, B and C; or on A, D and E. If using the A, D, E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\frac{Dx+E}{2-x}-\int\frac{D}{2-x}\mathrm{d}x$ for the 2nd B1 and 3rd B1 for correct completion. B0FT, B0FT, B0FT if they place *their* A, B, C with incorrect denominators.
	Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x)+b\ln(2-x)+\frac{c}{2-x}$, where $abc\neq 0$	M1	Condone minor slips in substitution. Exact substitution required.
	Obtain answer $\frac{5}{2}-\ln 72$ after full and correct working	A1	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9+\ln\frac{1}{8}$ or $-\ln 2^3-\ln 3^2$.
		5	

13) JUNE 2023_9709_33 Q4

State $\frac{\mathrm{d}y}{\mathrm{d}\theta}=1-2\sin\theta$	B1	Ignore left side throughout dx/dt, dy/dt, dx, dy but must see $\frac{\mathrm{d}y}{\mathrm{d}x}$ for final A1.
Use correct quotient rule, or product rule if rewrite x as $\cos\theta(2-\sin\theta)^{-1}$	M1	Incorrect formula seen M0 A0 otherwise BOD.
Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta}=\frac{-(2-\sin\theta)\sin\theta+\cos^2\theta}{(2-\sin\theta)^2}$ o.e.	A1	$-\sin\theta(2-\sin\theta)^{-1}-\cos\theta(2-\sin\theta)^{-2}(-\cos\theta)$ or equivalent.
Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}\theta}\div\frac{\mathrm{d}x}{\mathrm{d}\theta}$	M1	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=(1-2\sin\theta)\div\frac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives.
Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=(2-\sin\theta)^2$.	A1	AG – must see working in above cell to gain final A1. Allow $\cos^2\theta+\sin^2\theta=1$ to be implied. x instead of θ or missing θ more than twice on right side then A0 final mark.
	5	

14) JUNE 2023_9709_33 Q7

(a)	$\frac{du}{dx} = -\sin x$	B1	SOI
	Use double angle formula and substitute for x and dx throughout the integral	M1	All x's must be removed, can be coefficient errors provided 2 seen in working.
	Obtain $\pm\int 2ue^{2u}du$	A1	Limits may be omitted, or left as 0 and π, during the change of variable stage.
	Justify new limits and obtain $\int_{-1}^{1} 2ue^{2u}du$ from correct working	A1	AG Must see $x = 0$, $u = 1$ and $x = \pi$, $u = -1$. Inequalities alone e.g. $0 \leqslant x \leqslant \pi$ and $1 \leqslant u \leqslant -1$ or $-1 \leqslant u \leqslant 1$ for limits are insufficient A0 If sign in expression and order of limits incorrect then A0. If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step.
		4	
(b)	Commence integration and reach $aue^{2u} + b\int e^{2u}du$, where $ab \neq 0$, $b < 0$	M1*	Condone dx.
	Complete integration and obtain $ue^{2u} - \frac{1}{2}e^{2u}$	A1	OE Allow $(2u\frac{1}{2}e^{2u}) - \frac{1}{2}e^{2u}$.
	Use correct limits correctly in $cue^{2u} + d\,e^{2u}$ having integrated twice or in $c\cos x e^{2\cos x} + d e^{2\cos x}$	DM1	1 and −1 for u, 0 and π for x e.g. $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. Not decimals. Allow one sign error at most in going from $cue^{2u} + d\,e^{2u}$ or $c\cos x\, e^{2\cos x} + d\,e^{2\cos x}$ to $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. $[e^2 - ½e^2 - (-e^{-2} - ½e^{-2})]$ Complete reversal of sign by converting back to $\cos x$ and not making $x = 0$ upper limit is DM0 A0.
	Obtain $\frac{1}{2}e^2 + \frac{3}{2}e^{-2}$	A1	ISW Or equivalent 2-term expression e.g. $\frac{e^4+3}{2e^2}$ or $\frac{1}{2}\left(e^2 + \frac{3}{e^2}\right)$.
		4	

15) OCT 2020_9709_31 Q3

State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2 + 2\cos 2\theta$	B1	
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	A1	OE
Use correct double angle formulae	M1	
Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	A1	AG. Must have simplified numerator in terms of $\cos\theta$.
Alternative method for question 3		
Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	M1	
$\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$	B1	
$\frac{dy}{dx} = \frac{(2+2(\cos^2\theta - \sin^2\theta))}{4\cos\theta\sin\theta}$	M1 A1	
Simplify to given answer correctly $\frac{dy}{dx} = \cot\theta$	A1	AG

Alternative method for question 3		
Set $=2\theta$. State $\frac{dx}{dt} = \sin t$ or $\frac{dy}{dt} = 1+\cos t$	B1	
Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
Obtain correct answer $\frac{dy}{dx} = \frac{1+\cos t}{\sin t}$	A1	OE
Use correct double angle formulae	M1	
Obtain the given answer correctly $\frac{dy}{dx} = \cot \theta$	A1	
	5	

16) OCT 2020_9709_31 Q10

(a)	Use correct product or quotient rule	*M1	$\frac{dy}{dx}=\left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x}-e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = 4$	A1	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	A1	
		5	
(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}dx$	*M1	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}dx$	A1	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	*M1 A1	
	$\therefore 2xe^{-\frac{1}{2}x}$	A1	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	A1	ISW
		5	

17) OCT 2020_9709_32 Q5

5(a)	State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$	B1	CWO, AEF.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1	
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	M1	$y = \frac{1}{1+x^2}$
	$\frac{dy}{dx} = \frac{-2x}{(1+x^2)^2}$	A1	OE
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1	AG
		3	
5(b)	Use correct product rule to obtain $\frac{d}{d\theta}(\pm 2\cos^3\theta\sin\theta)$	M1	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	A1	e.g. $\pm(-2\cos^4\theta + 6\sin^2\theta\cos^2\theta)$
	Equate derivative to zero and obtain an equation in one trig ratio	A1	e.g. $3\tan^2\theta = 1$, or $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	A1	Or $-\frac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\frac{d^2y}{dx^2}$	M1	
	Obtain correct derivative in any form	A1	$\frac{-2(1+x^2)^2 + 2\times 2x \times 2x(1+x^2)}{(1+x^2)^4}$
	Equate derivative to zero and obtain an equation in x^2	A1	e.g. $6x^2 = 2$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	A1	
		4	

18) OCT 2020_9709_32 Q9

(a)	State or imply the form $\frac{A}{3x+2}+\frac{Bx+C}{x^2+4}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A=3$, $B=-1$, $C=3$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain $\ln(3x+2)$...	B1 FT	The FT is on A
	State a term of the form $k\ln(x^2+4)$.	M1	From $\int\frac{\lambda x}{x^2+4}dx$
	$...-\frac{1}{2}\ln(x^2+4)...$	A1 FT	The FT is on B
	$...+\frac{3}{2}\tan^{-1}\frac{x}{2}$	B1 FT	The FT is on C
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln(x^2+4)$ and $c\tan^{-1}\left(\frac{x}{2}\right)$, and subtract in correct order	M1	Using terms that have been obtained correctly from completed integrals
	Obtain answer $\frac{3}{2}\ln 2+\frac{3}{8}\pi$, or exact 2-term equivalent	A1	
		6	

19) OCT 2021_9709_31 Q3

(a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx}=e^{1-2x}-2xe^{1-2x}$
	Equate derivative to zero and solve for x	M1	
	Obtain $x=\frac{1}{2}$ and $y=\frac{1}{2}$	A1	
		4	
(b)	Use a correct method for determining the nature of a stationary point	M1	e.g. $\frac{d^2y}{dx^2}=-2e^{1-2x}-2(1-2x)e^{1-2x}$
	Show that it is a maximum point	A1	
		2	

20) OCT 2021_9709_31 Q4

State that $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $du = \frac{1}{2\sqrt{x}}\,dx$	B1	
Substitute throughout for x and dx	M1	
Obtain a correct integral with integrand $\frac{2}{u^2+1}$	A1	
Integrate and obtain term of the form $k\tan^{-1}u$	M1	$(2\tan^{-1}u)$
Use limits $\sqrt{3}$ and ∞ for u or equivalent and evaluate trig.	A1	e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians.
Obtain answer $\frac{1}{3}\pi$	A1	Or equivalent single term.
	6	

21) OCT 2021_9709_32 Q6

(a)	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	B1	
	Substitute expansions in $\frac{1}{2}(\sin 5x+\sin x)$, or equivalent	M1	
	Simplify and obtain $\frac{1}{2}(\sin 5x+\sin x)=\sin 3x\cos 2x$	A1	Obtain the **given identity** correctly.
		3	
(b)	Obtain integral $-\frac{1}{10}\cos 5x-\frac{1}{2}\cos x$, or equivalent	B1	
	Substitute limits correctly in an expression of the form $p\cos 5x+q\cos x$	M1	Correct limits and subtracted the right way around. Do not need values of trig functions for M1. Maximum one slip.
	Obtain $\frac{1}{5}(3-\sqrt{2})$	A1	Substitute values and obtain the **given answer** following full, correct and exact working.
		3	

22) OCT 2021_9709_32 Q9

(a)	State correct derivative of ye^{2x} with respect to x	B1	$2ye^{2x}+e^{2x}\frac{dy}{dx}$
	State correct derivative of y^2e^x with respect to x	B1	$2ye^x\frac{dy}{dx}+y^2e^x$
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	A1	Obtain the **given answer** correctly. Condone multiplication by $\frac{-1}{-1}$ and cancelling of e^x without comment.
	Alternative method for Question 9(a)		
	Rearrange as $y=\frac{2}{e^{2x}-ye^x}$ $\Rightarrow\frac{d}{dx}\left(e^{2x}-ye^x\right)=2e^{2x}-ye^x-e^x\frac{dy}{dx}$	B1	Other rearrangements are possible e.g. $y=2e^{-2x}+y^2e^{-x}$ $\frac{d}{dx}\left(y^2e^{-x}\right)=2ye^{-x}\frac{dy}{dx}-y^2e^{-x}$
	$\frac{dy}{dx}=-\frac{2}{\left(e^{2x}-ye^x\right)^2}\times\left(2e^{2x}-ye^x-e^x\frac{dy}{dx}\right)$	B1	$\Rightarrow\frac{dy}{dx}=-4e^{-2x}+2ye^{-x}\frac{dy}{dx}-y^2e^{-x}$
	Solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	A1	Obtain the **given answer** correctly.
		4	
(b)	Equate denominator to zero and substitute for y or for e^x in the equation of the curve	*M1	
	Obtain equation of the form $ae^{3x}=b$ or $cy^3=d$	DM1	$\left(e^{3x}=8,\ \ y^3=1\right)$ SOI
	Obtain $x=\ln 2$	A1	Accept $\frac{1}{3}\ln 8$ ISW
	Obtain $y=1$	A1	
		4	

23) OCT 2021_9709_33 Q4

Commence integration and reach $ax\cos\frac{1}{2}x+b\int\cos\frac{1}{2}x\,dx$	*M1	
Obtain $-2x\cos\frac{1}{2}x+2\int\cos\frac{1}{2}x\,dx$	A1	OE
Complete integration obtaining $-2x\cos\frac{1}{2}x+4\sin\frac{1}{2}x$	A1	OE
Use limits correctly, having integrated twice	DM1	
Obtain answer $2+\frac{\sqrt{3}}{3}\pi$, or exact equivalent	A1	
	5	

24) OCT 2021_9709_33 Q7

(a)	Use chain rule to differentiate LHS	*M1
	Obtain $\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$	A1
	Equate derivative of LHS to $1-2\frac{dy}{dx}$ and solve for $\frac{dy}{dx}$	DM1
	Obtain the given answer correctly	A1
		4
(b)	State $x+y=1$	B1
	Obtain or imply $x-2y=0$	B1
	Obtain coordinates $x=\frac{2}{3}$ and $y=\frac{1}{3}$	B1

25) OCT 2021_9709_33 Q9

(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and solve for x	M1
	Obtain answer $x=3$	A1
		4
(b)	State $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$, or $dx=2\sqrt{x}du$, or $2u\,du=dx$	B1
	Substitute and obtain integrand $\frac{2}{9-u^2}$	B1
	Use given formula for the integral or integrate relevant partial fractions	M1
	Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$	A1
	Use limits $u=0$ and $u=2$ correctly	M1
	Obtain the given answer correctly	A1
		6

26) OCT 2022_9709_31 Q9

(a)	Use correct product or quotient rule	*M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx}=-e^{-\frac{x}{3}}-\frac{1}{3}(3-x)e^{-\frac{x}{3}}$
	Equate their derivative to zero and solve for x	DM1	
	Obtain $x=6$	A1	
	Obtain $y=-3e^{-2}$	A1	Or exact equivalent.
		5	
(b)	Commence integration and reach $a(3-x)e^{-\frac{1}{3}x}+b\int e^{-\frac{1}{3}x}\,dx$, where $ab\neq 0$	*M1	
	Obtain $-3(3-x)e^{-\frac{1}{3}x}-3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	
	Complete integration and obtain $3xe^{-\frac{1}{3}x}$, or equivalent	A1	$-3e^{-\frac{x}{3}}(3-x)+9e^{-\frac{x}{3}}$
	Substitute limits $x=0$ and $x=3$, having integrated twice	DM1	
	Obtain answer $\frac{9}{e}$, or exact equivalent	A1	
		5	

27) OCT 2022_9709_32 Q3

Use correct product rule on given expression	*M1	
Obtain correct derivative in any form	A1	e.g. $\cos x\sin 2x+2\sin x\cos 2x$
Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$	*M1	
Equate derivative to zero and obtain an equation in one trig variable	DM1	dependent on the 2 previous M Marks.
Obtain $3\sin^2 x=2$, $3\cos^2 x=1$ or $\tan^2 x=2$	A1	OE
Solve and obtain $x=0.955$	A1	3 sf only. Final answer in degrees is A0. Ignore any attempt to find the corresponding value of y.
Alternative method for the first three marks		
Use correct double angle formula to obtain $y=2\cos x-2\cos^3 x$	*M1	or $y=2\sin^2 x\cos x$
Use chain rule and / or product rule	*M1	
Obtain derivative $y'=-2\sin x+6\sin x\cos^2 x$	A1	$y'=-2\sin^3 x+4\sin x\cos^2 x$
Alternative method for the second and third M marks		
Equate derivative to zero and obtain an equation in $\tan x$ and $\tan 2x$	*M1	
Use correct double angle formula to obtain an equation in $\tan x$	DM1	
	6	

28) OCT 2022_9709_32 Q8

(a)	State $(a =)\ \pi^2$	B1	Allow 32400, 180^2. Accept $(x=)\pi^2$.
		1	
(b)	State or imply $dx = 2u\,du$ or equivalent	B1	e.g. $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ Incorrect statements e.g. $du = \frac{1}{2\sqrt{x}}$ is B0.
	Substitute for x and dx throughout the integral	M1	
	Obtain $\int 2u\sin u\,du$	A1	Allow with missing du.
	Commence integration of $\int ku\sin u\,du$ by parts and reach $\mp ku\cos u \pm \int k\cos u\,du$	*M1	
	Obtain integral $-ku\cos u + k\sin u$	A1	
	Substitute limits $u = 0$ and $u = \sqrt{their\ a}$, $a \neq 0$, a in radians or $x = 0$ and *their a* in the complete integral	DM1	$-2\pi\cos\pi + 2\sin\pi(+0-2\sin 0)$ Need limits stated but condone if zeros not shown in substitution.
	Obtain answer 2π	A1	
		7	

29) OCT 2022_9709_32 Q10

(a)	State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 2$, $B = -1$ and $C = 0$	A1	SC: A maximum of M1A1 is available for obtaining $A = 2$ after scoring B0.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain term $2\ln(1+x)$	B1 FT	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	*M1	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	A1 FT	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x)+b\ln(2+x^2)$, where $ab \neq 0$	DM1	Ignore working relating to $C \neq 0$. $(2\ln 5 - 2\ln 1 - \frac{1}{2}\ln 18 + \frac{1}{2}\ln 2)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	A1	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	

30) OCT 2022_9709_33 Q3

Commence integration by parts and reach $x\tan x \pm \int \tan x.1\,\mathrm{d}x$	*M1
Use a correct method to integrate tan x	M1
Obtain integral $x\tan x - \ln\sec x$, or equivalent	A1
Use limits correctly, having integrated twice	DM1
Obtain answer $\frac{1}{4}\pi - \frac{1}{2}\ln 2$, or exact equivalent	A1
	5

31) OCT 2022_9709_33 Q4

State or imply $\frac{\mathrm{d}x}{\mathrm{d}t} = 2 - \sec^2 t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\frac{\cos 2t}{\sin 2t}$	B1	
Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
Obtain correct answer in any form	A1	
Use double angle formula to express derivative in terms of cos x and sin x	M1	
Obtain the given answer correctly	A1	AG
	5	

32) OCT 2022_9709_33 Q11

(a)	State or imply the form $\frac{A}{3-x} + \frac{Bx+C}{1+3x^2}$	B1
	Use a correct method to find a constant	M1
	Obtain one of $A = 2$, $B = 0$ and $C = 1$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5
(b)	Integrate and obtain term $-2\ln(3-x)$	B1 FT
	Obtain term of the form $b\tan^{-1}(\sqrt{3}x)$	M1
	Obtain term $\frac{1}{\sqrt{3}}\tan^{-1}(\sqrt{3}x)$	A1 FT
	Substitute limits correctly in an integral with terms $a\ln(3-x)$ and $b\tan^{-1}(\sqrt{3}x)$, where $ab \neq 0$	M1
	Obtain answer $2\ln\frac{3}{2} + \frac{1}{3\sqrt{3}}\pi$, or equivalent	A1
		5