P-3

Pure Maths - 3

Exponential and Logarithmic Function

Exercise-1. Solution (Revision)

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Exponential and Logarithmic Functions Revision

Example 1: Find the set of values of x for which $3(2^{3x+1}) < 8$. Give your answer in a simplified exact form. [SP-20/03/Q1] [3]

Solution:

$$3(2^{3x+1}) < 8$$

$$\Rightarrow \ln\left(3(2^{3x+1})\right) < \ln 8$$

$$\Rightarrow \ln 3 + \ln 2^{3x+1} < \ln 8$$

$$\ln 3 + (3x+1)\ln 2 < \ln 8$$

$$\ln 3 + \ln 2 + 3x \ln 2 < \ln 8$$

$$\ln 6 + x \ln 2^3 < \ln 8$$

$$x \ln 8 < \ln 8 - \ln 6$$

$$x < \frac{\ln 8/6}{\ln 8}$$

or $x < \dfrac{\ln 4/3}{\ln 8}$ ✓

Example 2: Solve the equation $\ln 3 + \ln(2x+5) = 2\ln(x+2)$. Give your answer in a simplified exact form. ---[4] [M-20/32/Q2]

Solution: $\ln 3 + \ln(2x+5) = 2\ln(x+2)$

$$\Rightarrow \ln 3(2x+5) = \ln(x+2)^2$$

$$\Rightarrow 3(2x+5) = (x+2)^2$$

$$\Rightarrow 6x + 15 = x^2 + 4x + 4$$

or $x^2 - 2x - 11 = 0$

(Using quadratic formula) $b^2 - 4ac = 4 + 44 = 48$

$$x = \frac{2 \pm \sqrt{48}}{2}$$

$$= 1 \pm 2\sqrt{3}$$

$= 1 + 2\sqrt{3}$ ✓ but not $(1 - 2\sqrt{3})$

$1 - 2\sqrt{3} = -2.46$ — ①

$\ln(x+2)$ is defined for, $(x+2) > 0$

$x > -2$ — ②

$x = -2.46 > -2$ is false.

$\therefore x = 1 + 2\sqrt{3}$ ✓

3 Solve the equation: $\ln(x^3-3) = 3\ln x - \ln 3$, Give your answer to 3 s.f.

[M-21 | 32 | Q1] ...[3]

Solution: $\ln(x^3-3) = 3\ln x - \ln 3$

$\Rightarrow \ln(x^3-3) = \ln x^3 - \ln 3$

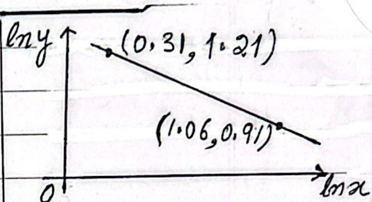
$\Rightarrow \ln(x^3-3) = \ln \frac{x^3}{3}$

$\Rightarrow x^3 - 3 = \frac{x^3}{3}$

$\Rightarrow x^3 - \frac{x^3}{3} = 3$

$\Rightarrow \frac{2}{3}x^3 = 3 \Rightarrow x^3 = \frac{9}{2} \Rightarrow x = \sqrt[3]{4.5} = \underline{1.65}$ ✓

4. The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points $(0.31, 1.21)$ and $(1.06, 0.91)$, as shown in the diagram.



...[5]

Find the value of n and of C, correct to 2 d.p.

[M-22 | 32 | Q 3]

Solution: $x^n \cdot y^2 = C \Rightarrow n\ln x + 2\ln y = \ln C$ --①

Now $(0.31, 1.21)$ lies on ① $(\ln x, \ln y)$

$\Rightarrow \quad 0.31n + 2\times 1.21 = \ln C$ --②

$(1.06, 0.91)$ lies on ① $\Rightarrow 1.06n + 2\times 0.91 = \ln C$ --③

from ② & ③ $\quad 0.75n = 0.6 \Rightarrow n = 0.8$ ✓

from ② $\quad \ln C = 0.31\times 0.8 + 2.42 = 2.668$

$C = e^{2.668} = 14.41$ ✓

$\therefore \; n = 0.8$ and $C = 14.41$ ✓

5. It is given that $x = \ln(2y-3) - \ln(y+4)$.
Express y in terms of x.

...[3]

[M-23 | 32 | Q1]

Solution: $x = \ln(2y-3) - \ln(y+4) \Rightarrow x = \ln\left(\frac{2y-3}{y+4}\right)$ $[\ln a - \ln b = \ln \frac{a}{b}]$

$\Rightarrow \frac{2y-3}{y+4} = e^x \Rightarrow 2y - 3 = e^x(y+4)$

$\Rightarrow 2y - 3 = ye^x + 4e^x$

$\Rightarrow y(2 - e^x) = 3 + 4e^x \Rightarrow y = \frac{3+4e^x}{2-e^x}$ ✓

Example 6: Find the set of values of x for which $2(3^{1-2x}) < 5^{x}$. Give your answer in a simplified exact form. [S-20/31/Q1] --[4]

Solution: $2(3^{1-2x}) < 5^{x}$

$\Rightarrow \ln\left(2(3^{1-2x})\right) < \ln 5^{x}$

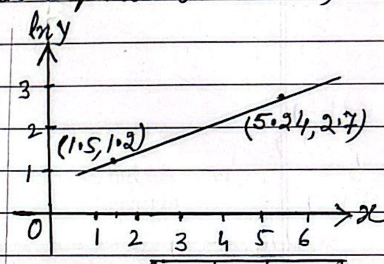
$\ln 2 + (1-2x)\ln 3 < x\ln 5$

$\ln 2 + \ln 3 < x\ln 5 + 2x\ln 3$

$\Rightarrow \ln 6 < x(\ln 5 + \ln 9)$

$x \cdot \ln 45 > \ln 6 \Rightarrow$ $\underline{x = \dfrac{\ln 6}{\ln 45}}$ ✓

Example 7: The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is straight line passing through the points $(1.5, 1.2)$ and $(5.24, 2.7)$ as shown in the diagram. Find the values of A and k correct to two decimal places. ---[5]



[S-20/32/Q2]

Solution: $y^2 = Ae^{kx}$

$\ln y^2 = \ln Ae^{kx}$

$2\ln y = \ln A + kx$ — ① $\qquad (x, \ln y)$

Given line passes through a point $(1.5, 1.2)$

for ① $\quad 2 \times 1.2 = \ln A + k \times 1.5 \Rightarrow \ln A + 1.5k = 2.4$ — ②

and $(5.24, 2.7)$ lies on ①.

$2 \times 2.7 = \ln A + k \times 5.24 \Rightarrow \ln A + 5.24k = 5.4$ — ③

Subtract ② fr ③ $\Rightarrow 3.74k = 3$

$\Rightarrow k = 3/3.74 = 0.80$ ✓

Put $k = 0.8$ in ② $\quad \ln A + 1.5 \times 0.8 = 2.4$

$\Rightarrow \ln A = 2.4 - 1.2 = 1.2$

$A = e^{1.2} = 3.32$ ✓

$\therefore$ $\underline{A = 3.32}$ and $\underline{k = 0.80}$ ✓

Example 8 (a) Show that the equation, $\ln(1+e^{-x}) + 2x = 0$ can be expressed as a quadratic equation in e^x. --[2]

(b) Hence solve the equation $\ln(1+e^{-x}) + 2x = 0$, giving your answer correct to 3 d.p. [S-20/33/Q.3] --[4]

Solution: $\ln(1+e^{-x}) + 2x = 0$ —— ①

(a) $\Rightarrow 1 + e^{-x} = e^{-2x}$

$\Rightarrow e^{2x} + e^{x} = 1$ [dividing by e^{-2x}]

$\rightarrow (e^x)^2 + e^x - 1 = 0$ ✓

(b) from part (a) $(e^x)^2 + e^x - 1 = 0$

$u^2 + u - 1 = 0$ [Let $u = e^x$]

$u = \dfrac{-1 \pm \sqrt{5}}{2}$ [using quad formula $b^2 - 4ac = 1 + 4 = 5$]

$u = \dfrac{-1+\sqrt{5}}{2}$ or $\cancel{\left(\dfrac{-1-\sqrt{5}}{2}\right)}$ $\because e^x > 0$

$\Rightarrow e^x = 0.618$

$x = \ln 0.618 = \underline{-0.481}$ ✓

9. Find the real roots of the equation $\dfrac{2e^{x}+e^{-x}}{2+e^{x}}=3$, giving your answer correct to 3 decimal places. Show that the equation has only one root. ---[5]
[S21/31/Q2]

Solution: $\dfrac{2e^{x}+e^{-x}}{2+e^{x}}=3 \Rightarrow 2e^{x}+e^{-x}=3(2+e^{x})$

$\Rightarrow 2e^{x}+\dfrac{1}{e^{x}}=6+3e^{x}$

$\Rightarrow e^{x}-\dfrac{1}{e^{x}}+6=0 \Rightarrow (e^{x})^{2}+6e^{x}-1=0$

$\Rightarrow u^{2}+6u-1=0$ (Let $e^{x}=u$)

$u=\dfrac{-6\pm\sqrt{40}}{2}=(\sqrt{10}-3), (-\sqrt{10}-3)^{*}$

$\therefore e^{x}=\sqrt{10}-3=0.162277$ ($\because e^{x}>0$)

$x=\ln 0.162277=-1.818 \Rightarrow x=-1.818$

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10 The variable x and y satisfy the equation $x=A\cdot 3^{-y}$, where A is constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. ---[3]

It is given that the line intersects the y-axis at the point where $y=1.3$

(b) Calculate the value of A, correct to 2 decimal places. [S-21/32/Q3] --[2]

Solution: $x=A\cdot 3^{-y} \Rightarrow \ln x=\ln A+(-y)\ln 3 \Rightarrow y=-\dfrac{1}{\ln 3}\cdot\ln x+\dfrac{\ln A}{\ln 3}$ --- (1)

(a) Equation (1) is a linear equation of y against $\ln x$.

$\therefore$ (1) represents a straight line graph of y against $\ln x$

Gradient of line (1) $=\dfrac{-1}{\ln 3}$

(b) In Eqn (1) we put $\ln x=0 \Rightarrow y=\dfrac{\ln A}{\ln 3}=1.3$ (Given)

$\Rightarrow \ln A=1.3\ln 3$

$=1.3\times 1.0986=1.42819$

$\therefore A=e^{1.42819}=4.1711$

$\therefore A=4.17$ (2 d.p)

11. Solve the equation $4^{x} = 3 + 4^{-x}$. Giving your answer correct to 3 decimal places.
$\boxed{S-21/33/Q2}$ ---[5]

Solution: $4^{x} = 3 + 4^{-x} \Rightarrow 4^{x} = 3 + \frac{1}{4^{x}}$

$\Rightarrow u = 3 + \frac{1}{u}$ (Let $4^{x} = u$)

$\Rightarrow u^{2} - 3u - 1 = 0$

$\Rightarrow u = \frac{3 \pm \sqrt{13}}{2} = \frac{3+\sqrt{13}}{2};\ \cancel{\left(\frac{3-\sqrt{13}}{2}\right)} < 0$

$\Rightarrow 4^{x} = \frac{3+\sqrt{13}}{2} = 3.30$ $[\because 4^{x} > 0]$

$\Rightarrow x \ln 4 = \ln 3.30277$

$\Rightarrow x = \frac{\ln 3.30277}{\ln 4} = 0.86183$

$\therefore x = 0.862$ (3 d.p)

12. Solve the equation $2(3^{2x-1}) = 4^{x+1}$, give answer to 2 d.p. ---[4]
[S-22/31/Q1]

Solution: Solve: $2\cdot 3^{2x-1} = 4^{x+1}$

$\Rightarrow \ln(2\cdot 3^{2x-1}) = \ln 4^{x+1}$

$\Rightarrow \ln 2 + (2x-1)\ln 3 = (x+1)\ln 4$

$\Rightarrow \ln 2 + 2\ln 3\cdot x - \ln 3 = \ln 4\cdot x + \ln 4$

$\Rightarrow x(2\ln 3 - \ln 4) = \ln 4 + \ln 3 - \ln 2$

$x = \dfrac{\ln 6}{\ln 9 - \ln 4} = \dfrac{1.79176}{0.8109} = 2.2095 = \underline{2.21}$ (2 d.p)

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13. Solve the equation $\ln(e^{2x}+3) = 2x + \ln 3$; give answer to 2 d.p.
[S-22/32/Q1]

Solution: $\ln(e^{2x}+3) = 2x + \ln 3 \Rightarrow \ln(e^{2x}+3) - \ln 3 = 2x$

$\Rightarrow \ln\left(\dfrac{e^{2x}+3}{3}\right) = 2x \Rightarrow \dfrac{e^{2x}+3}{3} = e^{2x} \Rightarrow e^{2x}+3 = 3e^{2x}$

$\Rightarrow 2e^{2x} = 3 \Rightarrow e^{2x} = 3/2 \Rightarrow 2x = \ln 3/2 \Rightarrow x = \frac{1}{2}\ln\frac{3}{2} = 0.203$

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14. (a) Show that the equation $\log_3(2x+1) = 1 + 2\log_3(x-1)$ can be written as a quadratic equation in x. ---[3]

(b) Hence solve the equation $\log_3(4y+1) = 1 + 2\log(2y-1)$, giving your answer to 2 d.p. ---[2]
[S-22/33/Q3]

Solution: $\log_3(2x+1) = 1 + 2\log_3(x-1)$

$\Rightarrow \log_3(2x+1) = \log_3 3 + \log_3(x-1)^2$

$\Rightarrow \log_3(2x+1) = \log_3 3(x-1)^2$

$\Rightarrow (2x+1) = 3(x-1)^2$

$\Rightarrow 2x+1 = 3(x^2-2x+1)$

$\Rightarrow 3x^2 - 8x + 2 = 0$

a quadratic equation in x.

Solve $\log_3(4y+1) = 1 + 2\log(2y-1)$

Put $2y = x$ —— (1)

$\log_3(2x+1) = 1 + 2\log(x-1)$

$\Rightarrow 3x^2 - 8x + 2 = 0$ [using part (a)]

$x = \dfrac{8 \pm \sqrt{40}}{6}$ [$b^2 - 4ac = 64 - 4\times3\times2 = 40$]

$x = \dfrac{4\pm\sqrt{10}}{3} = 2.3874$ or $x = 0.2792$

from (1) $y = x/2 = 1.1937$ or $y = 0.1396$

$\therefore y = 1.19$ (2dp)

as $2y-1 > 0$ or $y > \frac{1}{2}$ for $\log(2y-1)$ to be defined.

15. Solve the equation, $3e^{2x} - 4e^{-2x} = 5$, Give the answer to 3 d.p. ---[3]

$\boxed{S-23 | 31 | Q1}$

Solution: $3e^{2x} - 4e^{-2x} = 5 \Rightarrow 3(e^{2x})^2 - 5e^{2x} - 4 = 0$ $\left[e^{-2x} = \frac{1}{e^{2x}}\right]$

$\Rightarrow 3u^2 - 5u - 4 = 0$ [Let $e^{2x} = u$]

$u = \frac{5 \pm \sqrt{73}}{6}$ $\Big\{ a = 3, b = -5, c = -4;\ b^2 - 4ac = (-5)^2 - 4 \times 3 \times (-4) = 73$

$\Rightarrow e^{2x} = 2.2573$, $e^{2x} = -0.59^{\times}$ $[e^{2x} > 0]$

$\Rightarrow 2x = \ln(2.2573) \Rightarrow x = \frac{1}{2}\ln(2.2573) = 0.407 \Rightarrow x = 0.407$

16. Solve the equation, $\ln(2x^2 - 3) = 2\ln x - \ln 2$, give the answer in exact form. ---[3]

$\boxed{S-23 | 32 | Q2}$

Solution: $\ln(2x^2 - 3) = 2\ln x - \ln 2 \Rightarrow$

$\Rightarrow \ln(2x^2 - 3) = \ln x^2 - \ln 2$

$\Rightarrow \ln(2x^2 - 3) = \ln \frac{x^2}{2}$

$\Rightarrow 2x^2 - 3 = \frac{x^2}{2} \Rightarrow \frac{3}{2}x^2 = 3 \Rightarrow x^2 = 2 \Rightarrow x^2 = \sqrt{2}, \cancel{-\sqrt{2}} \Rightarrow x = \sqrt{2}$ ✓

17. Solve the equation, $\ln(x+5) = 5 + \ln x$. Give answer to 3 d.p. ---[4]

$\boxed{S-23 | 33 | Q1}$

Solution: $\ln(x+5) = 5 + \ln x$

$\Rightarrow \ln(x+5) - \ln x = 5$

$\Rightarrow \ln\left(\frac{x+5}{x}\right) = 5$

$\Rightarrow \frac{x+5}{x} = e^5$

$\Rightarrow x + 5 = xe^5 \Rightarrow$

$xe^5 - x = 5$

$\Rightarrow x(e^5 - 1) = 5$

$\Rightarrow x = \frac{5}{e^5 - 1} = 0.0339$

$x = 0.034$ ✓

18. Solve the equation. $\log_{10}(2x+1) = 2\log_{10}(x+1) - 1$
Give your answers correct to 3 decimal places. ---[6]
[W-20/31/Q4]

Solution: $\log_{10}(2x+1) = 2\log_{10}(x+1) - 1$ $\quad [\because \log_{10}10 = 1]$

$\Rightarrow \log_{10}(2x+1) = \log_{10}(x+1)^2 - \log_{10}10$

$\Rightarrow \log_{10}(2x+1) + \log_{10}10 = \log_{10}(x^2+2x+1)$

$\Rightarrow \log_{10}(10(2x+1)) = \log_{10}(x^2+2x+1)$

$\Rightarrow x^2+2x+1 = 20x+10 \Rightarrow x^2 - 18x - 9 = 0$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\left[\begin{array}{l} b^2-4ac \\ = 18^2 - 4\times1\times(-9) \\ = 324+36 \\ = 360 \end{array}\right.$

$$= \frac{18 \pm \sqrt{360}}{2}$$

$$= \frac{18 \pm 18.9736}{2}$$

$$x = 18.487 \,;\, -0.487 ✓$$

19. Solve the equation, $\ln(1+e^{-3x}) = 2$
Give the answer correct to 3 decimal places. ---[3]
[W-20/32/Q1]

Solution: $\ln(1+e^{-3x}) = 2 \Rightarrow 1+e^{-3x} = e^2$

$\Rightarrow 1+e^{-3x} = 7.389$

$\Rightarrow e^{-3x} = 6.389$

$\Rightarrow -3x \cdot \ln e = \ln 6.389$

$\Rightarrow -3x = 1.8545$ $\quad [\ln e = 1$

$x = -\frac{1.8545}{3} = -0.618$

$\therefore x = -0.618 ✓$

20. The variables x and y satisfy the relation $2^y = 3^{1-2x}$.
(a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. ---[3]
(b) Find the exact x-coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [W-20/32/Q3] ---[2]

Solution:

$$2^y = 3^{1-2x}$$

(a)

$$\Rightarrow \ln 2^y = \ln 3^{1-2x} \Rightarrow y\ln 2 = (1-2x)\ln 3$$

$$\Rightarrow y\ln 2 = -2\ln 3\, x + \ln 3$$

$$\Rightarrow y = -\frac{2\ln 3}{\ln 2}x + \frac{\ln 3}{\ln 2} \quad \text{——} ①$$

it represent a straight line as (of the form $y = mx + c$)

Gradient of the line $= -2\frac{\ln 3}{\ln 2}$

(b) Now to find the point of intersection of this line ①

and $y = 3x$ —— ②

from ① & ②

$$3x = -2\frac{\ln 3}{\ln 2} + \frac{\ln 3}{\ln 2}$$

$$\Rightarrow x\left[3 + \frac{2\ln 3}{\ln 2}\right] = \frac{\ln 3}{\ln 2}$$

$$\Rightarrow x\left[\frac{3\ln 2 + 2\ln 3}{\ln 2}\right] = \frac{\ln 3}{\ln 2}$$

$$\Rightarrow x \times \frac{\ln 72}{\ln 2} = \frac{\ln 3}{\ln 2} \Rightarrow x = \frac{\ln 3}{\ln 72}$$

21. Solve the equation $4|5^x - 1| = 5^x$, giving your answer correct to 3 decimal places. ---[4]

[W-21/31/Q1]

Solution: $4|5^x-1| = 5^x \Rightarrow 4(5^x-1) = \pm 5^x \Rightarrow 4.5^x - 4 = \pm 5^x$

$\Rightarrow 5.5^x = 4$ or $3.5^x = 4$

$\Rightarrow 5^{x+1} = 4$ or $5^x = 4/3$

$\Rightarrow (x+1)\ln 5 = \ln 4$; $x \ln 5 = \ln \frac{4}{3}$

$\Rightarrow x+1 = \frac{\ln 4}{\ln 5}$; $x = \frac{(\ln 4 - \ln 3)}{\ln 5} = 0.1787$

$= 0.86135$;

$x = 0.86135 - 1$; $\underline{x = 0.179}$ ✓ (3 dp)

$x = -0.1386$;

$\underline{x = -0.139}$ (3dp) ✓ ;

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22. Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. ---[4]
[W-21/32/Q1]

Solution: $3(2^{1-x}) = 7^x \Rightarrow \ln(3 \cdot 2^{1-x}) = \ln 7^x$

$\Rightarrow \ln 3 + (1-x)\ln 2 = x \ln 7$

$\Rightarrow x(\ln 7 + \ln 2) = \ln 3 + \ln 2$

$\Rightarrow x \cdot \ln 14 = \ln 6 \Rightarrow x = \frac{\ln 6}{\ln 14}$ ✓

23. Solve the equation $4^{x-2} = 4^x - 4^2$, give your answer correct to 3 decimal places
[W-21/33/Q3] -- [4]

Solution: $4^{x-2} = 4^x - 4^2 \Rightarrow 4^x \cdot 4^{-2} = 4^x - 16 \Rightarrow \frac{4^x}{16} = 4^x - 16$

$\Rightarrow 4^x = 16 \cdot 4^x - 256$

$\Rightarrow 15 \cdot 4^x = 256 \Rightarrow 4^x = \frac{256}{15} \Rightarrow \ln 4^x = \ln \frac{256}{15}$

$\Rightarrow x \cdot \ln 4 = \ln 256 - \ln 15$

$\Rightarrow x = \frac{\ln 256 - \ln 15}{\ln 4} = 2.04655\ldots$

$\therefore x = 2.047$ ✓ (3dp)

24 Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. ---[4]

[W-22/31/Q3]

Solution: $2^{3x-1} = 5(3^{-x}) \Rightarrow \ln 2^{3x-1} = \ln(5 \cdot 3^{-x})$

$\Rightarrow (3x-1)\ln 2 = \ln 5 - x\ln 3$

$\Rightarrow 3x\ln 2 - \ln 2 = \ln 5 - x\ln 3 \Rightarrow 3x\ln 2 + x\ln 3 = \ln 5 + \ln 2$

$\Rightarrow x[3\ln 2 + \ln 3] = \ln 5 \times 2 \Rightarrow x(\ln 2^3 + \ln 3) = \ln 10$

$\Rightarrow x = \frac{\ln 10}{\ln 24}$

25. Solve the equation $2^{3x-1} = 5(3^{1-x})$, give answer in form $\frac{\ln a}{\ln b}$, where a and b are integers.

[W-22/32/Q1]

Solution: $2^{3x-1} = 5 \cdot 3^{1-x} \Rightarrow \ln 2^{3x-1} = \ln(5 \cdot 3^{1-x})$

$\Rightarrow (3x-1)\ln 2 = \ln 5 + (1-x)\ln 3$

$\Rightarrow 3x\ln 2 + x\ln 3 = \ln 5 + \ln 2 + \ln 3$

$\Rightarrow x(\ln 2^3 + \ln 3) = \ln 5 \times 2 \times 3 \Rightarrow x = \frac{\ln 30}{\ln 24}$

26 Solve the equation $\ln(2x-1) = 2\ln(x+1) - \ln x$. Give your answer correct to 3 d.p. ---[4]

[W-22/33/Q1]

Solution: $\ln(2x-1) = 2\ln(x+1) - \ln x$

$\Rightarrow \ln(2x-1) + \ln x = 2\ln(x+1)$

$\Rightarrow \ln x(2x-1) = \ln(x+1)^2$

$\Rightarrow x(2x-1) = (x+1)^2$

$\Rightarrow 2x^2 - x = x^2 + 2x + 1$

$\Rightarrow x^2 - 3x - 1 = 0$

$x = \frac{3 \pm \sqrt{13}}{2}$

$= \underline{3.303}$ or $x = -0.303$

($\ln x$ is not defined for $x < 0$)