PURE MATHEMATICS -3

9709

(March, June and November series 2020 – 2023 With marking scheme)

NUMBERICAL SOLUTION AND ITERATION

EXERCISE -1

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1) SP-2020 _9709_3 Q4

The parametric equations of a curve are

$$x = e^{2t-3}, \quad y = 4\ln t,$$

where $t > 0$. When $t = a$ the gradient of the curve is 2.

(a) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [4]

(b) Verify by calculation that this equation has a root between 1 and 2. [2]

(c) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

2) MARCH-2020 _9709_32 Q3

(a) By sketching a suitable pair of graphs, show that the equation $\sec x = 2 - \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \frac{1}{2}\pi$. [2]

(b) Verify by calculation that this root lies between 0.8 and 1. [2]

(c) Use the iterative formula $x_{n+1} = \cos^{-1}\left(\frac{2}{4 - x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3) MARCH-2021 _9709_32 Q9

Let $f(x) = \dfrac{e^{2x} + 1}{e^{2x} - 1}$, for $x > 0$.

(a) The equation $x = f(x)$ has one root, denoted by a.

Verify by calculation that a lies between 1 and 1.5. [2]

(b) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(c) Find $f'(x)$. Hence find the exact value of x for which $f'(x) = -8$. [6]

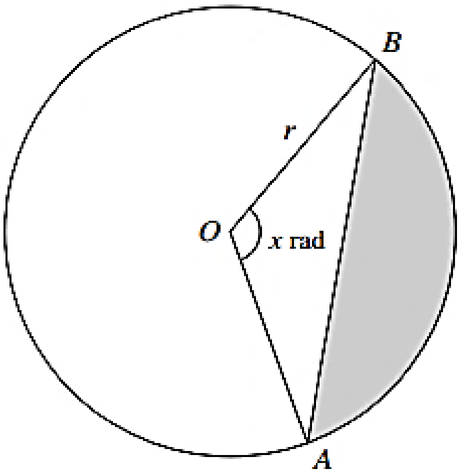
4) MARCH-2022 _9709_32 Q7

(a) By sketching a suitable pair of graphs, show that the equation $4 - x^2 = \sec\frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 2. [2]

(c) Use the iterative formula $x_{n+1} = \sqrt{4 - \sec\frac{1}{2}x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5) MARCH-2023 _9709_32 Q7



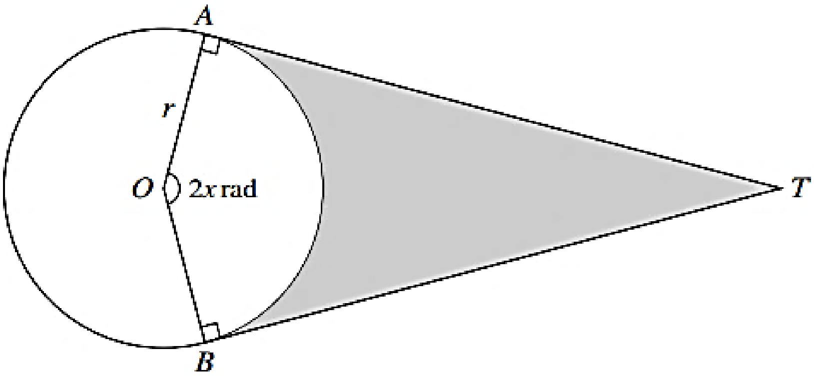
The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a) Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$. [4]

(b) Show by calculation that the root of the equation in **(a)** lies between 2 and 2.5. [2]

(c) Use an iterative formula based on the equation in **(a)** to calculate this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6) JUNE-2020 _9709_31 Q6



The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The area of the shaded region is equal to the area of the circle.

(a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

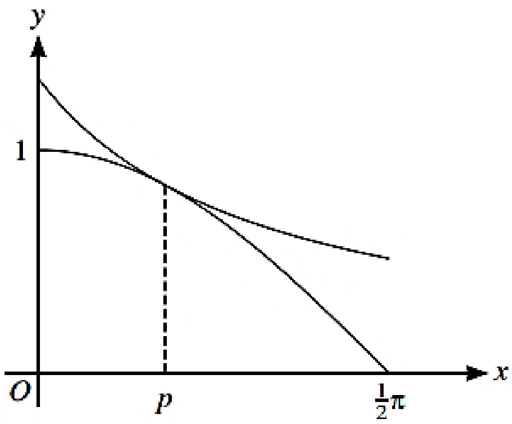
(b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

(c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7) JUNE-2020 _9709_32 Q9



The diagram shows the curves $y = \cos x$ and $y = \dfrac{k}{1+x}$, where k is a constant, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = p$.

(a) Show that p satisfies the equation $\tan p = \dfrac{1}{1+p}$. [5]

(b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\dfrac{1}{1+p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(c) Hence find the value of k correct to 2 decimal places. [2]

8) JUNE-2020 _9709_33 Q6

(a) By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

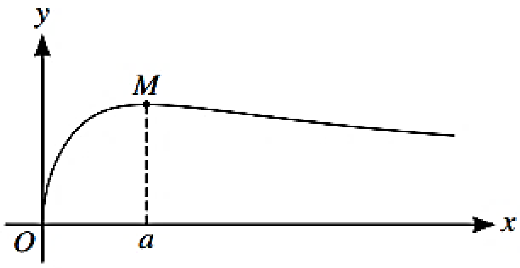
(b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part (a). [2]

(c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9) JUNE-2021 _9709_31 Q7



The diagram shows the curve $y = \dfrac{\tan^{-1} x}{\sqrt{x}}$ and its maximum point M where $x = a$.

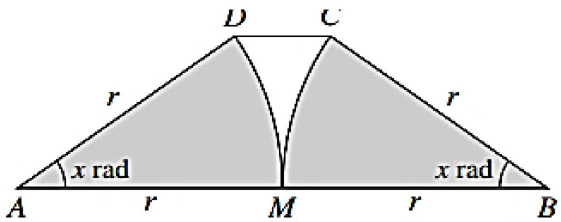
(a) Show that a satisfies the equation

$$a = \tan\left(\frac{2a}{1+a^2}\right).$$ [4]

(b) Verify by calculation that a lies between 1.3 and 1.5. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

10) JUNE-2021 _9709_32 Q10



The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M, the midpoint of AB.

(a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x)\sin x$. [3]

(b) Verify by calculation that x lies between 0.5 and 0.7. [2]

(c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(2 - \frac{x_n}{0.9\sin x_n}\right)$$

converges, then it converges to the root of the equation in part **(a)**. [2]

(d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

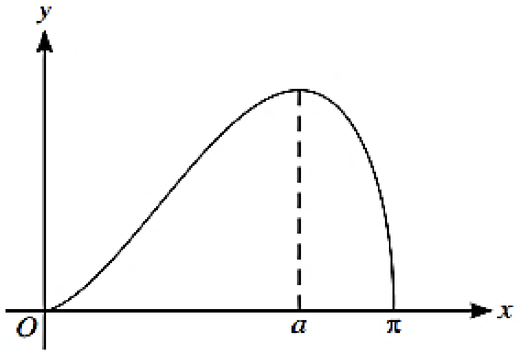
11) JUNE-2021 _9709_33 Q6

(a) By sketching a suitable pair of graphs, show that the equation $\cot\frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leqslant \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 1.5. [2]

(c) Use the iterative formula $x_{n+1} = 2\tan^{-1}\left(\frac{1}{1+e^{-x_n}}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

12) JUNE-2022 _9709_31 Q10



The curve $y = x\sqrt{\sin x}$ has one stationary point in the interval $0 < x < \pi$, where $x = a$ (see diagram).

(a) Show that $\tan a = -\frac{1}{2}a$. [4]

(b) Verify by calculation that a lies between 2 and 2.5. [2]

(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi - \tan^{-1}(\frac{1}{2}x_n)$ converges, then it converges to a, the root of the equation in part **(a)**. [2]

(d) Use the iterative formula given in part **(c)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

13) JUNE-2022 _9709_32 Q5

(a) By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

(b) Verify by calculation that the root lies between 2 and 2.8. [2]

(c) Use the iterative formula $x_{n+1} = \sqrt{3x_n - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

14) JUNE-2022 _9709_33 Q10

The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$. [5]

(b) Verify by calculation that a lies between 2.4 and 2.8. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. **[3]**

15) JUNE-2023 _9709_31 Q9

The constant a is such that $\int_0^a xe^{-2x}\,dx = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2}\ln(4a+2)$. **[5]**

(b) Verify by calculation that a lies between 0.5 and 1. **[2]**

(c) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. **[3]**

16) JUNE-2023 _9709_32 Q6

The equation $\cot\frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. **[2]**

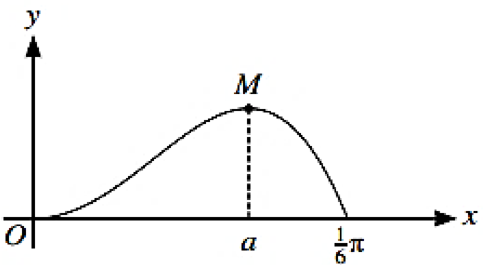
(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\left(\frac{1}{3x_n}\right)\right)$$

converges, then it converges to α. **[2]**

(c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. **[3]**

17) JUNE-2023 _9709_33 Q5



The diagram shows the part of the curve $y = x^2\cos 3x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$, and its maximum point M, where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. **[3]**

(b) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. **[3]**

18) OCT 2020_9709_31 Q5

(a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

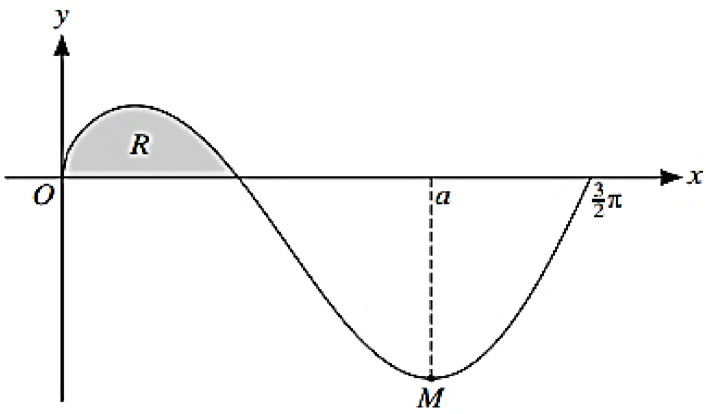
(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

19) OCT 2020_9709_32 Q10



The diagram shows the curve $y = \sqrt{x}\cos x$, for $0 \leqslant x \leqslant \frac{3}{2}\pi$, and its minimum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Show that a satisfies the equation $\tan a = \frac{1}{2a}$. [3]

(b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\frac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a.

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(c) Find the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π. [6]

20) OCT 2021_9709_31 Q8

The constant a is such that $\int_1^a \frac{\ln x}{\sqrt{x}}\,dx = 6$.

(a) Show that $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$. [5]

[$\exp(x)$ is an alternative notation for e^x.]

(b) Verify by calculation that a lies between 9 and 11. [2]

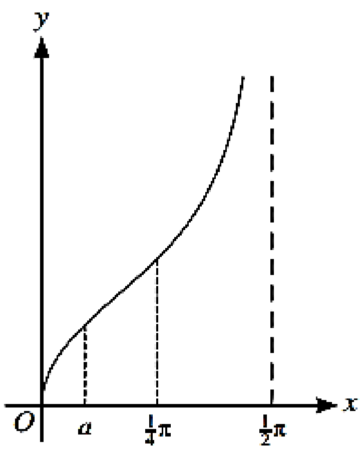
(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

21) OCT 2021_9709_32 Q11

The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leqslant x < \frac{1}{2}\pi$.

(a) Express $\frac{dy}{dx}$ in terms of $\tan x$, and verify that $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$. [4]

The value of $\frac{dy}{dx}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.



(b) Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

(c) Use the iterative formula

$$a_{n+1} = \tan^{-1}\left(\tfrac{1}{3}(1 - \tan^2 a_n - \tan^3 a_n)\right)$$

to determine a correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

22) OCT 2021_9709_33 Q10

A large plantation of area $20\,\text{km}^2$ is becoming infected with a plant disease. At time t years the area infected is $x\,\text{km}^2$ and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}.$$ [2]

(b) Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

(c) Use an iterative formula based on the equation in part (b), with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(d) Calculate the value of t at which the entire plantation becomes infected. [1]

23) OCT 2022-9709_31 Q7

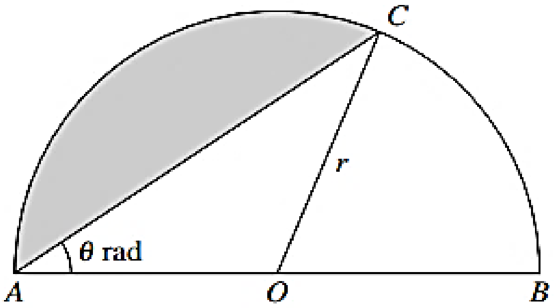
The equation of a curve is $y = \dfrac{x}{\cos^2 x}$, for $0 \leqslant x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1}\left(\sqrt[3]{\dfrac{\cos a + 2a \sin a}{12}}\right)$. [3]

(b) Verify by calculation that a lies between 0.9 and 1. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

24) OCT 2022_9709_32 Q9



The diagram shows a semicircle with diameter AB, centre O and radius r. The shaded region is the minor segment on the chord AC and its area is one third of the area of the semicircle. The angle CAB is θ radians.

(a) Show that $\theta = \frac{1}{3}(\pi - 1.5 \sin 2\theta)$. [4]

(b) Verify by calculation that $0.5 < \theta < 0.7$. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine θ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

25) OCT 2022_9709_33 Q8

The curve with equation $y = \dfrac{x^3}{e^x - 1}$ has a stationary point at $x = p$, where $p > 0$.

(a) Show that $p = 3(1 - e^{-p})$. [3]

(b) Verify by calculation that p lies between 2.5 and 3. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

MARKING SCHEME

1) SP-2020 _9709_3 Q4

(a)	State or imply $\frac{dx}{dt} = 2e^{2t-3}$ or $\frac{dy}{dt} = \frac{4}{t}$	1	B1
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	1	M1
	Obtain $\frac{dy}{dx} = \frac{4}{2te^{2t-3}}$, or equivalent	1	A1
	Set $t = a$, equate gradient to 2 and obtain the given answer	1	A1
		4	
(b)	Calculate $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	1	M1
	Complete the argument by considering the signs of the correct calculated values	1	A1
		2	
(c)	Use the iterative formula correctly at least once	1	M1
	Obtain final answer 1.35	1	A1
	Show sufficient iterations to 4 dp to justify 1.35 to 2 dp, **OR** show there is a sign change in the interval (1.345, 1.355)	1	A1
		3	

2) MARCH-2020 _9709_32 Q3

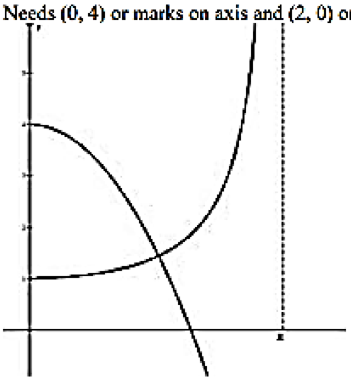
(a)	Sketch the graph $y = \sec x$	M1
	Sketch the graph $y = 2 - \frac{1}{2}x$, and justify the given statement	A1
		2
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 0.88	A1
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	A1
		3

3) MARCH-2021 _9709_32 Q9

(a)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(b)	Use the iterative formula $x_{n+1} = \frac{e^{2x_n}+1}{e^{2x_n}-1}$, or equivalent, correctly at least once	M1
	Obtain final answer 1.20	A1
	Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195,1.205)	A1
		3

(c)	Use quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to – 8 and obtain a quadratic in e^{2x}	M1
	Obtain $2(e^{2x})^2 - 5e^{2x} + 2 = 0$	A1
	Solve a 3-term quadratic in e^{2x} for x	M1
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	A1
	Alternative method for question 9(c)	
	Use quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to – 8, take square roots and obtain a quadratic in e^x	M1
	Obtain $\sqrt{2}e^{2x} - e^x - \sqrt{2} = 0$	A1
	Solve a 3-term quadratic in e^x for x	M1
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	A1
		6

4) MARCH-2022 _9709_32 Q7

a)	Sketch a relevant graph, e.g. $y = 4 - x^2$	B1	Needs (0, 4) or marks on axis and (2, 0) or (π, 0) 
	Sketch a second relevant graph, e.g. $y = \sec\frac{1}{2}x$, and justify the given statement	B1	Needs (0, 1) or mark on axis and (π, 0) Asymptote NOT required, but must NOT reach $x = \pi$. Sec graph must exist over at least interval $\left[0, \frac{3\pi}{4}\right]$ and quadratic graph over [0, 2.5].
		2	
b)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 1$ and $x = 2$.	M1	Need all 4 values or the 2 values correct for M1. Angles in degrees score M0.
	Complete the argument with correct calculated values	A1	
		2	

c)	Use the iterative process correctly at least twice	M1
	Obtain final answer 1.60	A1
	Show sufficient iterations to 4 d.p.to justify 1.60 to 2 d.p. or show there is a sign change in the interval (1.595, 1.605)	A1
		3

5) MARCH-2023 _9709_32 Q7

(a)	State or imply area of major sector = $\frac{1}{2}r^2(2\pi - x)$	B1	OE
	State or imply area of shaded segment = $\frac{1}{2}r^2x - \frac{1}{2}r^2\sin x$	B1	OE $r^2\sin(x/2)\cos(x/2)$ B0 until changed to $(1/2)r^2\sin x$.
	State $\frac{1}{2}r^2(2\pi - x) = 3\left(\frac{1}{2}r^2x - \frac{1}{2}r^2\sin x\right)$	M1	OE Area of major sector = 3 times (area of minor sector – area of triangle). Allow $r^2\sin(x/2)\cos(x/2)$.
	Obtain the given answer $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$ after full and correct working	A1	AG Allow rectified slip if before penultimate line.
		4	
(b)	Calculate the values of a relevant expression or pair of expressions at x = 2 and x = 2.5	M1	$x = 2$ $x = 2.5$ $(3/4)\sin x + (1/2)\pi$ 2.2(5277) 2.0(197) $2 < 2.2$ or 2.3 $2.5 > 2.0$ $x - (3/4)\sin x - (1/2)\pi$ $-0.2(5277) < 0$ $+0.4(803) > 0$ or change of sign Attempt both values and one correct for M1.
	Complete the argument correctly with correct calculated values	A1	Degrees award 0/2
		2	
(c)	Use the iterative formula correctly at least twice	M1	
	Obtain final answer 2.18	A1	
	Show sufficient iterations to 4 d.p. to justify 2.18 to 2 d.p. or show there is a sign change in the interval (2.175, 2.185)	A1	2, 2.2528, 2.1530, 2.1972, 2.1784, 2.1866, 2.1830, 2.1846 2.25, 2.1543(5), 2.1967, 2.1786, 2.1865, 2.1831, 2.1845 2.5, 2.0196(5), 2.2465, 2.1560, 2.1960, 2.1789, 2.1863, 2.1831, 2.1845 Degrees award 0/3
		3	

6) JUNE-2020 _9709_31 Q6

(a)	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1
	Use correct area formula and form an equation in r and x	M1
	Rearrange in the given form	A1
		3
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.35	A1
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	A1
		3

7) JUNE-2020 _9709_32 Q9

(a)	State $\cos p = \frac{k}{1+p}$	B1
	Differentiate both equations and equate derivatives at $x = p$	M1
	Obtain a correct equation in any form, e.g. $-\sin p = -\frac{k}{(1+p)^2}$	A1
	Eliminate k	M1
	Obtain the given answer showing sufficient working	A1
		5
(b)	Use the iterative formula correctly at least once	M1
	Obtain final answer $p = 0.568$	A1
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	A1
		3
(c)	Use a correct method to find k	M1
	Obtain answer $k = 1.32$	A1
		2

8) JUNE-2020 _9709_33 Q6

(a)	Sketch a relevant graph, e.g. $y = x^5$	B1
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	B1
		2
(b)	State a suitable equation, e.g. $x = \frac{4x^5 + 2}{5x^4 - 1}$	B1
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	B1
		2
(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.267	A1
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	A1
		3

9) JUNE-2021 _9709_31 Q7

(a)	Use correct quotient rule or correct product rule	M1	e.g. $\frac{dy}{dx} = \frac{\sqrt{x}.\frac{1}{1+x^2} - \tan^{-1}x.\frac{1}{2\sqrt{x}}}{x}$
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and remove inverse tangent	M1	
	Obtain $a = \tan\left(\frac{2a}{1+a^2}\right)$ from correct working	A1	AG. Accept with x in place of a.
		4	

(b)	Calculate the value of a relevant expression or pair of expressions at $a = 1.3$ and $a = 1.5$	M1	Must be using radians
	Complete the argument correctly with correct calculated values	A1	e.g. $1.3 < 1.448$, $1.5 > 1.322$ $(0.148, -0.178)$
		2	
(c)	Use the iterative process $a_{n+1} = \tan\left(\frac{2a_n}{1+a_n^2}\right)$ correctly at least twice	M1	
	Obtain final answer 1.39	A1	
	Show sufficient iterations to at least 4 d.p. to justify 1.39 to 2 d.p. or show there is a sign change in the interval (1.385, 1.395)	A1	Allow recovery
		3	

10) JUNE-2021 _9709_32 Q10

(a)	State or imply $CD = 2r - 2r\cos x$	B1	
	Using correct formulae for area of sector and trapezium, or equivalent, form an equation in r and x	M1	e.g. $2\times\frac{1}{2}r^2x = \frac{0.9}{2}(2r+2r-2r\cos x)r\sin x$
	Obtain $x = 0.9(2-\cos x)\sin x$	A1	AG, NFWW
		3	
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$	M1	Calculated for both values and correct for one value is sufficient for M1. Must be working in radians.
	Complete the argument correctly with correct values	A1	Must have sufficient accuracy to support the answer e.g. $\begin{matrix}0.5 > 0.484\\0.7 < 0.716\end{matrix}$ or $\begin{matrix}0.016 > 0\\-0.016 < 0\end{matrix}$ or $\begin{matrix}0.96\ldots < 1\\1.02\ldots > 1\end{matrix}$
		2	
(c)	State a suitable equation, e.g. $\cos x = \left(2 - \frac{x}{0.9\sin x}\right)$	B1	If working in reverse, the first B1 is for $\frac{x}{0.9\sin x} = 2 - \cos x$
	Rearrange this as $x = 0.9\sin x(2-\cos x)$	B1	Need to see the complete sequence of changes.
		2	

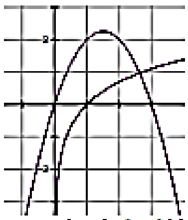
11) JUNE-2021 _9709_33 Q6

(a)	Sketch a relevant graph, e.g. $y = \cot\frac{1}{2}x$	B1
	Sketch a second relevant graph, e.g. $y = 1 + e^{-x}$, and justify the given statement	B1
		2
(b)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(c)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.34	A1
	Show sufficient iterations to 4 d.p. to justify 1.34 to 2 d.p. or show there is a sign change in the interval (1.335, 1.345)	A1
		3

12) JUNE-2022 _9709_31 Q10

(a)	Use correct product rule	M1	Condone incorrect / missing chain rule
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx}=\sqrt{\sin x}+\frac{x\cos x}{2\sqrt{\sin x}}$ or $2y\frac{dy}{dx}=2x\sin x+x^2\cos x$
	Equate derivative to zero and obtain an equation in tan x or tan a	M1	
	Obtain $\tan a=-\frac{1}{2}a$ correctly	A1	AG
		4	
(b)	Calculate the value of a relevant expression or pair of expressions at $a=2$ and $a=2.5$	M1	Must be working in radians At least one correct
	Complete the argument correctly with correct calculated values	A1	e.g. $-1>-2.18$ and $-1.25<-0.747$
		2	
(c)	State a suitable equation, e.g. $x=\pi-\tan^{-1}\left(\frac{1}{2}x\right)$	B1	A correct equation without subscripts or quote $\tan\theta=-\tan(\pi-\theta)$
	Using $\tan(A\pm B)$ formula, or otherwise, rearrange this as $\tan x=-\frac{1}{2}x$	B1	Complete argument correctly
		2	
(d)	Use the iterative process correctly at least once	M1	Must be working in radians
	Obtain answer $a=2.29$	A1	
	Show sufficient iterations to 4 dp to justify 2.29 to 2 dp or show there is a sign change in the interval (2.285, 2.295)	A1	e.g. 2.25, 2.2974, 2.2871, 2.2893, 2.2888, …
		3	

13) JUNE-2022 _9709_32 Q5

(a)	Sketch a relevant graph, e.g. $y=\ln x$	B1	
	Sketch a second relevant graph, e.g. $y=3x-x^2$, and justify the given statement by marking the root on the sketch or by use of a suitable comment	B1	$\ln(x)$: sketch should imply y-axis is an asymptote. Through (1, 0) if marked. Correct shape. $3x-x^2$: Symmetrical. Through (0, 0) and (3, 0) if marked. If ln(x) correct accept parabola for +ve y only. If ln(x) incorrect then need parabola in 3 quadrants.
		2	
(b)	Calculate the values of a relevant expression or pair of expressions at $x=2$ and $x=2.8$	M1	Allow for a smaller interval. At least one value correct if comparing with 0. If using pairs then the pairing must be clear.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.693<2$ and $1.03>0.56$ or $1.307>0,-0.47<0$ using $\sqrt{3x-\ln x}$ $0.304>0,-0.085<0$. Need to have calculated values to at least 2 sf.
		2	
(c)	Use the iterative process correctly at least once	M1	
	Obtain final answer 2.63	A1	
	Show sufficient iterations to at least 4 dp to justify 2.63 to 2 dp or show there is a sign change in the interval (2.625, 2.635)	A1	SC Allow M1 A1 A0 to a candidate who starts at a point in the interval and reaches a premature conclusion
		3	

14) JUNE-2022 _9709_33 Q10

a)	Commence integration and reach $ax^3 \ln x + b\int x^3 . \frac{1}{x}\, dx$	*M1	OE Allow omission of dx.
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 . \frac{1}{x}\, dx$	A1	OE Allow omission of dx.
	Complete integration and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$	A1	Allow $-\frac{1}{3}\left(\frac{1}{3}x^3\right)$.
	Use limits correctly and equate to 4, having integrated twice	DM1	$\frac{1}{3}a^3 \ln a - \frac{1}{9}a^3 - (0 - \frac{1}{9}) = 4$ allow one sign error OR one numerical error, but 0 may be absent or expressed as $\frac{a^3}{3}\ln 1$. Allow $-\frac{1}{3}\left(\frac{1}{3}ax^3\right)$ and $-\frac{1}{3}\left(\frac{1}{3}\right)$.
	Obtain given result correctly	A1	$a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$ AG After substitution, any errors even if corrected A0. Need to see at least one line of working between substitution and the given answer.
		5	
b)	Calculate the values of a relevant expression or pair of expressions at $a = 2.4$ and $a = 2.8$ All values must be correct for M1 (numerical question)	M1	
	Justify the given statement with correct calculated values	A1	$2.4 < 2.7(8)$ and $2.8 > 2.5(6)$ sign change here insufficient OR $-0.3(8)$ and $0.2(4)$ $< 0, > 0$ or change of sign.
		2	

c)	Use the iterative process $a_{n+1} = \left(\frac{35}{3\ln a_n - 1}\right)^{\frac{1}{3}}$ correctly at least twice	M1	
	Obtain final answer $a = 2.64$	A1	Must be 2 dp.
	Show sufficient iterations to 4 dp to justify 2.64 to 2 dp, or show there is a sign change in (2.635, 2.645)	A1	2.635 $(35/(3\ln a - 1))^{1/3} - a = 0.0029(4) > 0$ 2.645 $(35/(3\ln a - 1))^{1/3} - a = -0.012 < 0$
		3	

15) JUNE-2023 _9709_31 Q9

(a)	Commence integration and reach $pxe^{-2x}+q\int e^{-2x}dx$	*M1	OE
	Obtain $-\frac{1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}dx$	A1	OE
	Complete integration and obtain $-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}$	A1	
	Use limits correctly and equate to $\frac{1}{8}$, having integrated twice	DM1	$-\frac{1}{2}ae^{-2a}-\frac{1}{4}e^{-2a}+\frac{1}{4}=\frac{1}{8}$.
	Obtain $a=\frac{1}{2}\ln(4a+2)$ correctly	A1	AG
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 0.5$ and $a = 1$	M1	
	Justify the given statement with correct calculated values	A1	e.g. $0.5 < 0.69\ldots$, $1 > 0.89\ldots$ $0.193 > 0$, $-1.105 < 0$ $0.066 < 0.125$, $0.148 > 0.125$ if put limits in the integral. Condone if they use calculator for the definite integral.
		2	
(c)	Use the iterative process $a_{n+1}=\frac{1}{2}\ln(4a_n+2)$ correctly at least once.	M1	
	Obtain final answer 0.84	A1	
	Show sufficient iterations to at least 4 d.p. to justify 0.84 to 2 d.p. or show that there is a sign change in (0.835, 0.845)	A1	e.g. 0.75, 0.8047, 0.8261, 0.8343, 0.8373, 0.8385 1, 0.8959, 0.8599, 0.8469, 0.8420, 0.8402 .
		3	

16) JUNE-2023 _9709_32 Q6

(a)	Calculate the values of a relevant expression or pair of expressions at $x =$ 0.5 and $x = 1$	M1	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using $g(x) - f(x)$, there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	A1	e.g. $3.92 > 1.5$, $1.83 < 3$ or $2.42 > 0$, $-1.17 < 0$.
		2	
(b)	State $x=\frac{1}{3}\left(x+4\tan^{-1}\frac{1}{3x}\right)$	M1	Or rearrange $\cot\left(\frac{x}{2}\right)=3x$ as far as $2x=4\tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right)=3x$ Need intermediate step between $\frac{x}{2}=\tan^{-1}\frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right)=3x$	A1	Or continue rearrangement to $x=\frac{1}{3}\left(x+4\tan^{-1}\frac{1}{3x}\right)$ and state iterative formula of $x_{n+1}=\frac{1}{3}\left(x_n+4\tan^{-1}\frac{1}{3x_n}\right)$ AG
		2	
(c)	Use the iterative process correctly at least once	M1	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	A1	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	A1	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in (b) but restate the conclusion here, no marks in (b) but could score 3/3 for (c).

17) JUNE-2023 _9709_33 Q5

(a)	Use correct product rule	M1	$\frac{d}{dx}(x^2)\cos(3x)+x^2\frac{d}{dx}(\cos 3x)$.
	Obtain correct derivative in any form	A1	e.g. $2x\cos 3x-3x^2\sin 3x$.
	Equate derivative to zero and obtain $a=\frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$.	A1	AG Condone $a=\frac{1}{3}\tan^{-1}\frac{2}{3a}$. Must at least reach expression $2x=3x^2\tan(3x)$ or better <u>before</u> final answer to gain A1. Final answer must be in terms of a. Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	
(b)	Use the iterative process $a_{n+1}=\frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	M1	Degrees 0/3.
	Obtain final answer 0.36	A1	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$	A1	Allow small errors in 4[th] d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 $\pi/6$ $\pi/12$ 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	3	

18) OCT 2020_9709_31 Q5

(a)	Sketch a relevant graph, e.g. $y=\operatorname{cosec} x$	B1	cosec x, U shaped, roughly symmetrical about $x=\frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right)=1$ and domain at least $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y=1+e^{-\frac{1}{2}x}$, and justify the given statement	B1	Exponential graph needs $y(0)=2$, negative gradient, always increasing, and $y(x)>1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
(b)	Use the iterative formula correctly at least twice	M1	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	A1	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	A1	
		3	

19) OCT 2020_9709_32 Q10

a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}\cos x - \sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a = \frac{1}{2a}$	A1	Obtain given answer from correct working. The question says 'show that ..' so there should be an intermediate step e.g. $\cos x = 2x\sin x$. Allow $\tan x = \frac{1}{2x}$
		3	
b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	M1	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	A1	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	A1	3, 3.3067, 3.2917, 3.2923 Allow more than 4d.p. Condone truncation.
		3	
(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2 dx$	B1	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos 2*A* formula, commence integration by parts and reach $x(ax + b\sin 2x) \pm \int ax + b\sin 2x dx$	*M1	Alternative: $\frac{x^2}{4} + \frac{x}{4}\sin 2x - \int \frac{1}{4}\sin 2x dx$
	Obtain $x(\frac{1}{2}x + \frac{1}{4}\sin 2x) - \int \frac{1}{2}x + \frac{1}{4}\sin 2x dx$, or equivalent	A1	
	Complete integration and obtain $\frac{1}{4}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x$	A1	OE
	Substitute limits $x = 0$ and $x = \frac{1}{2}\pi$, having integrated twice	DM1	$\frac{\pi}{2}\left[\frac{\pi^2}{8} + 0 - \frac{1}{4} - 0 - 0 - \frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2 - 4\right)$, or exact equivalent	A1	CAO
		6	

20) OCT 2021_9709_31 Q8

(a)	Commence integration and reach $a\sqrt{x}\ln x + b\int\sqrt{x}\,.\,\frac{1}{x}\,dx$, or equivalent	*M1	
	Obtain $2\sqrt{x}\ln x - \int 2\sqrt{x}\,.\,\frac{1}{x}\,dx$, or equivalent	A1	
	Obtain integral $2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits and equate result to 6	DM1	
	Rearrange and obtain $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$	A1	Obtain given answer from full and correct working.
		5	
(b)	Calculate the values of a relevant expression or pair of expressions at $a = 9$ and $a = 11$	M1	e.g. $\begin{cases} 9 < 10.31 \\ 11 > 9.99 \end{cases}$ or $1.31 > 0, -1.01 < 0$
	Complete the argument correctly with correct values	A1	
		2	
(c)	Use the iterative process $a_{n+1} = \exp(\frac{1}{\sqrt{a_n}} + 2)$ correctly at least once	M1	
	Obtain answer 10.12	A1	
	Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125)	A1	e.g. 10, 10.1374, 10.1156, 10.1190, ... 9, 10.3123, 10.0886, 10.1233, 10.1178, ... 11, 9.9893, 10.1391, 10.1153, 10.1191, ...
		3	

21) OCT 2021_9709_32 Q11

(a)	Use chain rule	M1	Allow if not starting with the correct index.
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of $\tan x$	A1	e.g. $\frac{dy}{dx} = \frac{1+\tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	B1	Confirm the given statement from correct work. Should see at least $\frac{2}{2} = 1$.
		4	
(b)	Equate answer to part (a) to 1 and obtain a quartic equation in t or $\tan x$	*M1	At least as far as $(1+\tan^2 x)^2 = 4\tan x$.
	Obtain correct answer, i.e. $t^4 + 2t^2 - 4t + 1 = 0$	A1	Or equivalent horizontal form.
	Commence division by $t - 1$	DM1	As far as $t^3 + t^2 + \ldots$ by long division or inspection. Allow verification by multiplying given answer by $t - 1$.
	Obtain the given answer	A1	
		4	
(c)	Use the iterative process correctly with the given formula at least once	M1	Obtain one value and use that to obtain the next. Must be working in radians.
	Obtain final answer $a = 0.29$	A1	
	Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295)	A1	e.g. 0.3, 0.2854, 0.2894, 0.2883, ... 0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ... 0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ...
		3	

22) OCT 2021_9709_33 Q10

(a)	State or imply equation of the form $\frac{dx}{dt} = k\frac{x}{20-x}$	M1	
	Obtain $k = 19$	A1	AG
		2	
(b)	Separate variables and integrate at least one side	M1	
	Obtain terms $20 \ln x - x$ and $19t$, or equivalent	A1 A1	
	Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms $a \ln x$ and bt	M1	
	Substitute $t = 1$ and rearrange the equation in the given form	A1	AG
		5	
(c)	Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once	M1	
	Obtain final answer $x = 2.83$	A1	
	Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835)	A1	
		3	
(d)	Set $x = 20$ and obtain answer $t = 2.15$	B1	
		1	

23) OCT 2022-9709_31 Q7

a)	Use correct product or quotient rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\cos^2 x + 2x\sin x\cos x}{\cos^4 x}$ or $\frac{dy}{dx} = \sec^2 x + 2x\sec^2 x\tan x$
	Equate derivative at $x = a$ to 12 and obtain $a = \cos^{-1}\left(\sqrt[3]{\frac{\cos a + 2a\sin a}{12}}\right)$	A1	AG
		3	
b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	M1	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	A1	e.g. $\cos 0.9 = 0.622 > 0.553$, $\cos 1 = 0.540 < 0.570$ or $0.9 < 0.985$, $1 > 0.964$ or $0.0846 > 0$, $-0.0358 < 0$ or could be looking at values of the gradient 8.46 & 14.1
		2	
c)	Use the process $a_{n+1} = \cos^{-1}\left(\sqrt[3]{\frac{\cos a_n + 2a_n\sin a_n}{12}}\right)$ correctly at least once	M1	Must be working in radians.
	Obtain final answer 0.97	A1	
	Show sufficient iterations to 4 d.p. to justify 0.97 to 2 d.p., or show there is a sign change in the interval (0.965, 0.975)	A1	e.g. 0.95, 0.9743, 0.9694, 0.9704
		3	

24) OCT 2022_9709_32 Q9

(a)	State or imply angle $AOC = \pi - 2\theta$	B1	Might be seen on the printed diagram.
	Use correct formulae for the area of a sector and triangle, or of a segment, and find the area of the shaded region	M1	$\frac{1}{2}r^2(\pi-2\theta)-\frac{1}{2}r^2\sin(\pi-2\theta)$ or $\frac{1}{2}\pi r^2-\left[\frac{1}{2}r^2(2\theta)+\frac{1}{2}r^2\sin(\pi-2\theta)\right]$ M0 if subtraction the wrong way round.
	Equate to $\frac{1}{6}\pi r^2$ and obtain a correct equation in any form	A1	e.g. $\frac{1}{6}\pi r^2=\frac{1}{2}r^2(\pi-2\theta)-\frac{1}{2}r^2\sin(\pi-2\theta)$.
	Obtain $\theta=\frac{1}{3}(\pi-1.5\sin 2\theta)$ correctly	A1	AG Condone if state / imply $\sin(\pi-2\theta)=\sin 2\theta$.
		4	
(b)	Evaluate a relevant expression or pair of expressions at $\theta=0.5$ and $\theta=0.7$	M1	Allow work on a smaller interval. Need to evaluate for both limits, with at least one correct. When using $x=f(x)$ embedded values are not sufficient e.g. f(0.5) .... is accepted but $\frac{1}{3}(\pi-1.5\sin 2\times 0.5)=...$ is not.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.5<0.626, 0.7>0.554$ or $0.126>0, -0.146<0$ If using pairs then the pairing must be clear. Need to see the inequalities or an appropriate comment. Need to see values calculated to at least 2 sf.
		2	
(c)	Use the iterative process $\theta_{n+1}=\frac{1}{3}(\pi-1.5\sin 2\theta_n)$ correctly at least once	M1	i.e obtain one value and use that value to obtain a second value. Must be working in radians.
	Obtain final answer 0.586	A1	
	Show sufficient iterations to 5 d.p. to justify 0.586 to 3 d.p., or show there is a sign change in the interval (0.5855, 0.5865).	A1	e.g. 0.5, 0.62646, 0.57225, 0.59195, 0.58416, 0.58715, 0.58599, 0.58644 0.6, 0.58118, 0.58833, 0.58553, 0.58661, 0.58619, 0.58636 0.7, 0.55447, 0.59958, 0.58133, 0.58827, 0.58556, 0.58661, 0.58620, 0.58636 Allow working to more than 5 dp, but not less.
		3	

25) OCT 2022_9709_33 Q8

(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative at $x=p$ to zero and obtain the given equation	A1
		3
(b)	Evaluate a relevant expression or pair of relevant pair of expressions at $p=2.5$ and $p=3$	M1
	Complete the argument with correct calculated values	A1
		2
(c)	Use the iterative formula $p_{n+1}=3\left(1-e^{-p_n}\right)$ correctly at least once	M1
	Obtain final answer $p=2.82$	A1
	Show sufficient iterations to 4 d.p.to justify 2.82 to 2 d.p., or show there is a sign change in the interval (2.815, 2.825)	A1
		3