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P-3

Pure Maths-3

Polynomials and Moduls Functions

Exercise-1: Solutions (Revision)

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Example 1(a) Sketch the graph of $y = |2x-3|$ --[1]
(b) Solve the inequality $3x-1 > |2x-3|$ [SP-20/03/Q3] --[3]

Solution: (a) $y = |2x-3| = \begin{cases} 2x-3; & 2x-3 \geq 0 \text{ or } x \geq 3/2 \\ -(2x-3) \text{ or } (3-2x); & x < 3/2 \end{cases}$

(b) for the point of intersect

$3x-1 = 3-2x$ (on the left of $x = 3/2$)

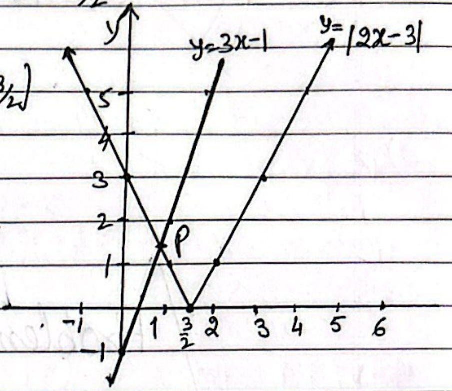
$5x = 4$

$x = 4/5$

Two curves intersect at P, $x = 4/5$.

$3x-1 > |2x-3|$ --- (1)

for $x > \frac{4}{5}$ ✓



(b) Alternatively:

$3x-1 > |2x-3|$ --(1) $\left\{ |a| > |b| \Rightarrow a^2 = b^2 \right.$ (here we will check the answers.)

$(3x-1)^2 > (2x-3)^2$

$9x^2 - 6x + 1 > 4x^2 - 12x + 9$

$5x^2 + 6x - 8 > 0$ (Critical values are $\frac{4}{5}, -2$)

$(5x-4)(x+2) > 0$

-2 $4/5$

$\Rightarrow x > \frac{4}{5}$ or $x \leq -2$ ×

[for $x = 1$, in (1) $\Rightarrow 2 > 1$ True]

for $x < -2$

$3x-1 > |2x-3|$ -- (1)

for $x = -3$

in (1) $\Rightarrow -10 > 9$ fal.

$\therefore$ $x \leq -2$ is false.

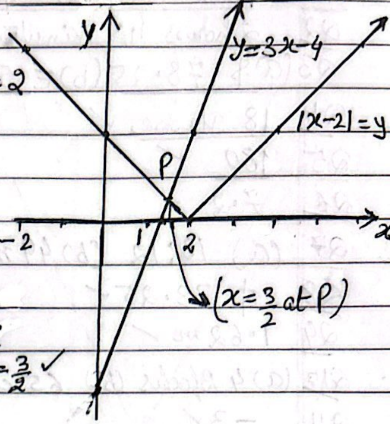
$\therefore$ Required Solution is

$x > \frac{4}{5}$ ✓

Example 2(a) Sketch the graph of $y = |x-2|$ ---(1) [M-20/32/Q1]

(b) Solve the inequality $|x-2| < 3x-4$ ---(3)

Solution: $|x-2| = \begin{cases} x-2 ; & \text{for } x-2 \geq 0 \text{ for } x \geq 2 \\ -(x-2) = (2-x) & \text{for } x < 2 \end{cases}$



(b) To solve $|x-2| < 3x-4$ —— ①

consider $y = 3x-4$ —— ②

and $y = |x-2|$ --- ③

② & ③ Intersect for $3x-4 = 2-x$

$\Rightarrow 4x = 6 \Rightarrow x = \frac{3}{2}$ ✓

① is True for; $x > \frac{3}{2}$ ✓

(b) Alternate method:

$|x-2| < 3x-4$ —— ①

$\Rightarrow (x-2)^2 < (3x-4)^2$ ⊛

$\Rightarrow x^2 - 4x + 4 < 9x^2 - 24x + 16$

$8x^2 - 20x + 12 > 0$

$2x^2 - 5x + 3 > 0$

$(2x-3)(x-1) > 0$

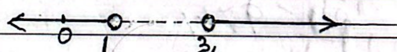
$x > 3/2$ or $x < 1$ ×

[for $x=2$ in ① $\Rightarrow 0 < 2$, True)

$\therefore x > 3/2$ ✓

{ $|a| < |b|$ (is not given) $\Rightarrow a^2 < b^2$ }

(⊛ check the answer)



{ at $x=0$ for ①

$2 < -4$

$\therefore x \nless 1$ false.

3. The polynomial ax^3+5x^2-4x+b, where a and b are constants, is denoted by $p(x)$. It is given that $(x+2)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x+1)$ the remainder is 2. Find the values of a and b. ---[5]

$\boxed{M\text{-}21/32/Q2}$

Solution $p(x) = ax^3+5x^2-4x+b$ --- ①

Given $(x+2)$ is a factor of $p(x)$; $p(-2)=0$

$\Rightarrow p(-2) = a(-2)^3+5(-2)^2-4(-2)+b=0$ (from ①)

$\Rightarrow -8a+b=-28$ --- ②

also given that $p(x)$ divided by $(x+1)$ the remainder is $2 \Rightarrow p(-1)=2$ (from ①)

$\Rightarrow a(-1)^3+5(-1)^2-4(-1)+b = 2$

$\Rightarrow -a+5+4+b= 2$

$\Rightarrow -a+b=-7$ --- ③

Solving ② and ③

$\left.\begin{array}{l} a=3 \\ b=-4 \end{array}\right\}$ ✓

4. Solve the inequality $|2x+3| > 3|x+2|$ ---[4]

$\boxed{M\text{-}22/32/Q1}$

Solution: $|2x+3| > 3|x+2|$

$(2x+3)^2 > 3^2(x+2)^2$ $\left\{\begin{array}{l} |a| > |b| \\ \Rightarrow a^2 > b^2 \end{array}\right.$

$4x^2+12x+9 > 9(x^2+4x+4)$

$\Rightarrow 9x^2+36x+36 < 4x^2+12x+9$

$5x^2+24x+27<0$

$(5x+9)(x+3)<0$ (Critical values are -3, $-9/5$)

$\Rightarrow -\frac{9}{5} < x < 3$ ✓

5. Find the quotient and remainder when: $8x^3+4x^2+2x+7$ is divided by $4x^2+1$, ---[3]

$\boxed{M\text{-}22/32/Q8(a)}$

Solution: $4x^2+1 \,\big)\, \cancel{8x^3}+4x^2+\cancel{2x}+7 \,\big(\, 2x+1$

$-\cancel{8x^3} \quad \pm\cancel{2x}$

$\cancel{4x^2}+7$

$-\cancel{4x^2} \pm 1$

6

$\left\{\begin{array}{l} \text{Quotient} = 2x+1 \text{ ✓} \\ \text{Remainder} = 6 \text{ ✓} \end{array}\right.$

6. The polynomial $2x^4+ax^3+bx-1$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by x^2-x+1 the remainder is $3x+2$. Find the values of a and b. --- [5]

M-23 | 32 | Q3

Solution:

$x^2-x+1 \,)\, 2x^4+ax^3+bx-1 \,(\, 2x^2+(a+2)x+a$

$\quad -\cancel{2x^4} \mp 2x^3 \quad \pm 2x^2$

$\quad (a+2)\cancel{x^3}-2x^2+bx-1$

$\quad -\cancel{(a+2)x^3} \mp (a+2)x^2 \pm (a+2)x$

$\quad a\cancel{x^2}+(b-a-2)x-1$

$\quad -\cancel{ax^2} \mp ax \pm a$

$\quad (b-2)x-(1+a)$

$\therefore$ Remainder $(b-2)x-(1+a) \equiv 3x+2$ (Given)

$\Rightarrow b-2=3$ and $-1-a=2$

$\Rightarrow b=5$ ✓ and $a=-3$ ✓

7. Find the quotient and remainder when, $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [S-20/31/Q5(a)] --[3]

Solution:

$$
\begin{array}{r l}
x^2+3 \,\big)\, & 2x^3 - x^2 + 6x + 3 \;\big(\, 2x - 1 \\
 & \underline{-2x^3 \qquad\quad \underset{-}{+} 6x} \\
 & \qquad -x^2 + 3 \\
 & \qquad \underline{\mp x^2 \mp 3} \\
 & \qquad\qquad 6
\end{array}
$$

$\therefore$ quotient $= (2x-1)$ and remainder $= 6$.

8. Find the quotient and remainder when, $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. --[3] [S-20/32/Q1]

Solution:

$$
\begin{array}{r l}
2x^2 - x + 1 \,\big)\, & 6x^4 + x^3 - x^2 + 5x - 6 \;\big(\, 3x^2 + 2x - 1 \\
 & \underline{-6x^4 \mp 3x^3 \pm 3x^2} \\
 & \qquad 4x^3 - 4x^2 + 5x \\
 & \qquad \underline{-4x^3 \mp 2x^2 \pm 2x} \\
 & \qquad\qquad -2x^2 + 3x - 6 \\
 & \qquad\qquad \underline{\mp 2x^2 \pm x \mp 1} \\
 & \qquad\qquad\quad 2x - 5
\end{array}
$$

hence Quotient $= 3x^2 + 2x - 1$ and remainder $= 2x - 5$ ✓

9. Solve the inequality $|2x-1| > 3|x+2|$ [S-20/33/Q1] -[4]

Solution: $|2x-1| > 3|x+2|$

$\Rightarrow \quad (2x-1)^2 > 9(x+2)^2 \qquad\qquad (|a| > |b| \Rightarrow a^2 > b^2)$

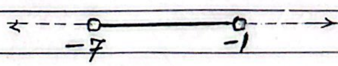
$\Rightarrow \quad 4x^2 - 4x + 1 > 9(x^2 + 4x + 4)$

or $\quad 5x^2 + 40x + 35 < 0$

$x^2 + 8x + 7 < 0 \qquad\qquad$ (critical values are, -7 & -1)

$(x+1)(x+7) < 0$

$\Rightarrow \quad -7 < x < -1$ ✓



10. Solve the inequality: $2|3x-1| < |x+1|$ ---[4]

[S-21|31|Q1]

Solution: Given $2|3x-1| < |x+1|$

Squaring $\rightarrow 4(3x-1)^2 < (x+1)^2$ $(\because |x|<|y| \Rightarrow x^2<y^2)$

$\Rightarrow 4(9x^2-6x+1) < x^2+2x+1$

$\Rightarrow 36x^2-24x+4 < x^2+2x+1$

$\Rightarrow 35x^2-26x+3<0$

$\Rightarrow (5x-3)(7x-1)<0$ (critical values are $\frac{1}{7}$ and $3/5$)

$\Rightarrow 1/7 < x < 3/5$. ✓

11. Solve the inequality $|2x-1| < 3|x+1|$ ---[4]

[S-21|32|Q1]

Solution: $|2x-1| < 3|x+1| \Rightarrow (2x-1)^2 < 3^2(x+1)^2$

$\Rightarrow (4x^2-4x+1) < 9(x^2+2x+1)$

$\Rightarrow 4x^2-4x+1 < 9x^2+18x+9$

$\Rightarrow 5x^2+22x+8>0$

$(5x+2)(x+4)>0$ (critical values are $-4, -2/5$

$\Rightarrow x<-4$ or $x>-2/5$ ✓

12. The polynomial ax^3-10x^2+bx+8, where a and b are constants, is denoted by $p(x)$. It is given that $(x-2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b. ---[5]

(b) When a and b have these values, factorise $p(x)$ completely. ---[3]

[S-22|31|Q5]

Solution:
(a) $p(x) = ax^3-10x^2+bx+8$ ----(1)

and $p'(x) = 3ax^2-20x+b$ ——(2)

$(x-2)$ is a factor of $p(x) \Rightarrow p(2)=0$

from (1) $p(2) = 8a-40+2b+8=0$ —(3)

Also $(x-2)$ is a factor of $p'(x) \Rightarrow p'(2)=0$

from (2) $p'(2) = 12a-40+b=0$ ---(4)

Solving (3) & (4): $a=3$ and $b=4$ ✓

(b) for $a=3$, $b=4$

$p(x) = 3x^3-10x^2+4x+8$ --(5)

$(x-2)$ is a factor of $p(x)$

$$
\begin{array}{r|l}
x-2 & 3x^3-10x^2+4x+8 \;(3x^2-4x-4 \\
 & \underline{-3x^3 \mp 6x^2} \\
 & -4x^2+4x \\
 & \underline{\mp 4x^2 \pm 8x} \\
 & -4x+8 \\
 & \underline{-4x+8} \\
 & \times
\end{array}
$$

$\therefore p(x) = (x-2)(3x^2-4x-4)$

$= (x-2)(x-2)(3x+2)$ ✓

13. The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x-1)$ and when $p(x)$ is divided by $(x+2)$ the remainder is 5. Find the value of a and b. ---[5]

S-22/32/Q3

Solution: $p(x) = ax^3 + x^2 + bx + 3$ ---①

$p(x)$ is divisible by $(2x-1) \Rightarrow p(\frac{1}{2}) = 0$ $\quad [2x-1=0 \Rightarrow x = \frac{1}{2}]$

from ① $a(\frac{1}{2})^3 + (\frac{1}{2})^2 + b \times \frac{1}{2} + 3 = 0$

$\frac{1}{8}a + \frac{1}{4} + \frac{1}{2}b + 3 = 0 \Rightarrow a + 4b = -26$ ---②

when $p(x)$ is divided by $(x+2)$, Remainder is $5 \Rightarrow p(-2) = 5$

from ① $p(-2) = a(-2)^3 + (-2)^2 + b(-2) + 3 = 5 \Rightarrow -8a + 4 - 2b + 3 = 5$

$\Rightarrow 8a + 2b = 2$ ---③

Solving ② & ③ $a = 2$ ✓, $b = -7$ ✓

14. Find in terms of a, the set of values of x satisfying the inequality: $2|3x+a| < |2x+3a|$ where a is a positive constant. ---[4]

S-22/33/Q1

Solution: $2|3x+a| < |2x+3a|$

$\Rightarrow 4(3x+a)^2 < (2x+3a)^2$

$\Rightarrow 4(9x^2 + 6ax + a^2) < (4x^2 + 12ax + 9a^2)$

$\Rightarrow 32x^2 + 12ax - 5a^2 < 0$

$32x^2 + 20ax - 8ax - 5a^2 < 0$

$\Rightarrow (8x+5a)(4x-a) < 0$

$\left\{(8x+5a)(4x-a) = 0 \right.$

$\Rightarrow$ Critical Values are, $\frac{a}{4}$ & $-\frac{5a}{8}$

$\therefore$ Solution of Inequation ① is

$-\frac{5a}{8} < x < \frac{a}{4}$ ✓

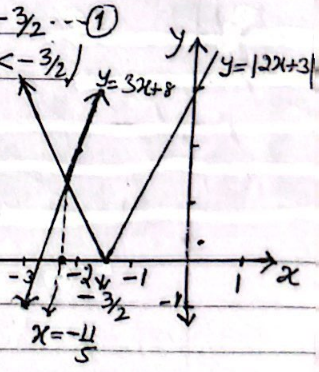
15. (a) Sketch the graph of $y = |2x+3|$ ---[1]

(b) Solve the inequality $3x + 8 > |2x+3|$ ---[3]

S-23/31/Q2

Solution (a) $y = |2x+3| = \begin{cases} 2x+3, & 2x+3 \geq 0 \ (x \geq -3/2) \text{ ---①} \\ -(2x+3), & 2x+3 < 0 \ (x < -3/2) \end{cases}$

Sketch the graph:



(b) Solve $3x + 8 > |2x+3|$

consider $y = 3x + 8$ ---②

$(-2, 2)$, $(-3, -1)$

Solving ① & ② $3x + 8 = -(2x+3)$

$\Rightarrow 5x = -11 \Rightarrow x = -\frac{11}{5}$ ---③

$\therefore 3x + 8 > |2x+3| \Rightarrow x > -\frac{11}{5}$ ✓

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16. Solve the inequality. $|5x-3| < 2|3x-7|$. ---[4]

S-23 | 32 | Q 1

Solution: $|5x-3| < 2|3x-7|$

$\Rightarrow (5x-3)^2 < 2^2(3x-7)^2$ $\quad (\because |x| < |y| \Rightarrow x^2 < y^2)$

$\Rightarrow 25x^2 - 30x + 9 < 4(9x^2 - 42x + 49)$

$\Rightarrow -11x^2 + 138x - 187 < 0$

$\Rightarrow 11x^2 - 138x + 187 > 0$ $\quad (11 \times 187 = 11 \times 11 \times 17 = 121 \times 17$

$11x^2 - 121x - 17x + 187 > 0$

$11x(x-11) - 17(x-11) > 0$

$(x-11)(11x-17) > 0$ $\quad$ (Critical values are $x = 11, \frac{17}{11}$

$\Rightarrow x < \frac{17}{11}$ or $x > 11$ ✓

17. Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$.

S-23 | 33 | Q2

Solution:

$$\begin{array}{r|l} x^2+x+3 & 2x^4 - 27 \quad (2x^2 - 2x - 4 \checkmark \\ & \underline{-\cancel{2x^4} \quad \pm 2x^3 \pm 6x^2} \\ & -\cancel{2x^3} - 6x^2 - 27 \\ & \underline{\pm \cancel{-2x^3} \mp 2x^2 \quad \mp 6x} \\ & -\cancel{4x^2} + 6x - 27 \\ & \underline{\mp 4x^2 \mp 4x \mp 12} \\ & 10x - 15 \checkmark \end{array}$$

$\therefore$ Quotient $= 2x^2 - 2x - 4$ ✓

and

Remainder $= x - 15$ ✓

18. Solve the inequality $2 - 5x > 2|x-3|$ ---[4]

W-20 | 31 | Q 1

Solution: $2 - 5x > 2|x-3|$ —— ①

$\Rightarrow (2-5x)^2 > 2(x-3)^2$

$4 + 25x^2 - 20x > 4(x^2 - 6x + 9)$

$25x^2 - 20x + 4 - 4x^2 + 24x - 36 > 0$

$21x^2 + 4x - 32 > 0$

$21x^2 + 28x - 24x - 32 > 0$ $\quad (-21 \times 32 = (28 \times (-24))$

$7x(3x+4) - 8(3x+4) > 0$

$(3x+4)(7x-8) > 0$ $\quad$ Critical values: $-\frac{4}{3}; \frac{8}{7}$ ×

$\Rightarrow x < -\frac{4}{3}$ ✓ , $x > \frac{8}{7}$ ×

(does not satisfy ①)

19. Solve the inequality $|3x-a| > 2|x+2a|$, where a is a positive constant.

[W-21/32/Q2]--[4]

Solution: To Solve, $|3x-a| > 2|x+2a|$

$$\Rightarrow (3x-a)^2 > 2^2(x+2a)^2$$

$$\Rightarrow 9x^2-6ax+a^2 > 4(x^2+4ax+4a^2)$$

$$\Rightarrow 5x^2-22ax-15a^2 > 0$$

$$(x-5a)(5x+3a) > 0$$

[critical values are $5a, -\frac{3}{5}a$]

$$\Rightarrow x < -\frac{3}{5}a \text{ or } x > 5a$$

20. Find the quotient and remainder when $2x^4+1$ is divided by x^2-x+2 ---[3]

[W-21/33/Q1]

Solution:

$$x^2-x+2 \,\big)\, 2x^4+1 \,\big(\, 2x^2+2x-2$$

$$\underline{-2x^4 \mp 2x^3 \pm 4x^2}$$

$$2x^3-4x^2+1$$

$$\underline{-2x^3 \mp 2x^2 \pm 4x}$$

$$-2x^2-4x+1$$

$$\underline{\mp 2x^2 \pm 2x \mp 4}$$

$$\underline{-6x+5}$$

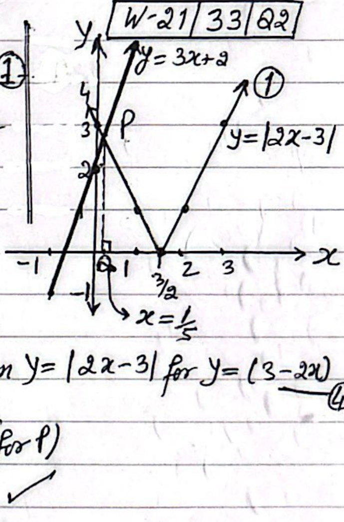
$$\left.\begin{array}{l}\text{Quotient} = 2x^2+2x-2 \\ \text{Remainder} = -6x+5\end{array}\right\}$$

21. (a) Sketch the graph of $y=|2x-3|$ ---[1]

(b) Solve the inequality $|2x-3|<3x+2$. ---[3]

[W-21/33/Q2]

Solution: Graph; $y=|2x-3|=\begin{cases}2x-3; & x\geq 3/2 \\ -(2x-3); & x<3/2 \\ (\text{or } (3-2x) & \end{cases}$ ①



(b) Solve; $|2x-3|<3x+2$ ---②

Now consider $y=|2x-3|$ ---①

and $y=3x+2$ ---③

Let ① & ② intersect at P; P lies on $y=|2x-3|$ for $y=(3-2x)$ ---④

from ③ & ④ $3x+2=3-2x$

$\Rightarrow 5x=1 \Rightarrow x=\frac{1}{5}$ (for P)

$\therefore |2x-3|<3x+2$ for $x>\frac{1}{5}$ ✓

(b) Alternate method:

$|2x-3|<3x+2$ ---②

$\Rightarrow (2x-3)^2<(3x+2)^2$ {for this check the answers}

$\Rightarrow 4x^2-12x+9<9x^2+12x+4$

$\Rightarrow 5x^2+24x-5>0$

$(x+5)(5x-1)>0$ (critical values are, $\frac{1}{5}, -5$)

$x>\frac{1}{5}$ or $x<-5$

{for ② $x>\frac{1}{5} \Rightarrow 1<5$ (Let $x=1$) True)

{for $x<-5$ ✗ for ② (Let $x=-6$) $\Rightarrow 15<-16$ false

$\therefore$ Ans is $x>\frac{1}{5}$ ✓

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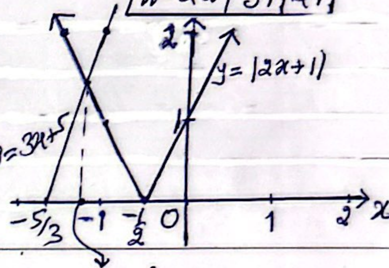
2.2 (a) Sketch the graph of $y = |2x+1|$. --- [1]

(b) Solve the inequality $3x+5 < |2x+1|$ --- [3]

W-22 / 31 / Q1

Soln (a) $y = |2x+1| = \begin{cases} 2x+1, & x \geq -\frac{1}{2} \\ -(2x+1), & x < -\frac{1}{2} \end{cases}$



(b) Solve $3x+5 < |2x+1|$ --- ①

consider $y = 3x+5$ ----- ②

for $x < -\frac{1}{2}$; (from graph we find $y = 3x+5$ & $y = |2x+1|$ intersect when $x < -\frac{1}{2}$) $\Rightarrow |2x+1| = -(2x+1)$

for ① & ② $3x+5 < -(2x+1)$

$\Rightarrow 5x < -6$

$\Rightarrow x < -\frac{6}{5}$ ✓

2.3. The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x+3)$ is a factor of $p(x)$.

(a) Find the value of a. --- [2]

(b) When a has this value, solve the inequality $p(x) < 0$ --- [4]

W-22 / 32 / Q2

Solution $p(x) = 2x^3 - x^2 + a$ --- ①

(a) $(2x+3)$ is a factor of $(2x+3)$

hence $x = -3/2$ (for $2x+3=0$) is a root of $p(x) \Rightarrow p(-\frac{3}{2}) = 0$

from ① $\Rightarrow 2(-\frac{3}{2})^3 - (-\frac{3}{2})^2 + a = 0$

$-\frac{27}{4} - \frac{9}{4} + a = 0 \Rightarrow a = 9$ ✓

for $a = 9$ in ①

$p(x) = 2x^3 - x^2 + 9$

$p(x) = (2x+3)(x^2 - 2x + 3)$ {$(2x+3)$ is a factor of $p(x)$}

To Solve

$(2x+3)(x^2-2x+3) < 0$ {for $x^2-2x+3 = (x-1)^2 + 2 > 0$ for all $x \in R$}

$\Rightarrow 2x+3 < 0$

$\Rightarrow x < -3/2$ ✓