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P_3

Pure Maths_3.

Contents Pages.

Polynomials and Modulus functions Notes

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Polynomials.

§ Polynomial:

It is an expression of the form:

$$a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-2} x^2 + a_{n-1} x + a_n$$

where 'n' is a non-negative integer and $a_0, a_1, a_2 \ldots a_{n-1}$ are real coefficients; a_n is constant term.
'n' is the degree of the polynomial.

Example (i) $2x+3$ is a polynomial of degree 1. (Linear polynomial)
(ii) $3x^2-5x+8$ is a polynomial of degree 2. (Quadratic polynomial).
(iii) $4x^3-2x^2+x-7$ is a poly. of degree 3 (Cubic polynomial)
(iv) $\frac{7}{3}$ ($\frac{7}{3}x^0$) is constant of degree 0.// (v) Number 0 does not have a degree.

§ Divisional of Polynomials: $P(x) \div g(x)$

Given two polynomial $P(x)$ and $g(x)$: degree of $g(x) \leq$ degree of $P(x)$

"$P(x) = G(x) \cdot Q(x) + R(x)$" where $Q(x)$ is the quotient and $R(x)$ remainder.
Dividend; divisor; quotient; remainder

In Numbers, $a, b \in R$
Division Algorithm: $(a \div b)$
$a = bq + r$; $b \leq a$; $r < b$
Consider $37 \div 8$
or $= 4\frac{5}{8}$
$37 = 8 \times 4 + 5$
$a = b\,q + r$ $\left[\frac{a}{b} = q + \frac{r}{b}\right]$

Example 1: Find the quotient and remainder:
$(8x^3+x^2-2x+1) \div (2x-1)$

$$\begin{array}{r l}
2x-1 \,\big)\, 8x^3+x^2-2x+1 \,\big(\, 4x^2+\frac{5}{2}x+\frac{1}{4} & \\
-8x^3 \mp 4x^2 & \\
\hline
5x^2-2x+1 & \\
-5x^2 \mp \frac{5}{2}x & \\
\hline
\frac{1}{2}x+1 & \\
-\frac{1}{2}x \mp \frac{1}{4} & \\
\hline
\frac{5}{4} &
\end{array}$$

here Quotient $= 4x^2+\frac{5}{2}x+\frac{1}{4}$ ✓

Remainder $= \frac{5}{4}$ ✓

Division Algorithm:

$$P(x) = G(x) \cdot Q(x) + R(x) \; : \; \deg R(x) < \deg G(x)$$

$$8x^3+x^2-2x+1 = (2x-1)\left(4x^2+\frac{5}{2}x+\frac{1}{4}\right) + \frac{5}{4}$$

Example 2: The polynomial $4x^4 + ax^2 + 11x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 2$

(i) Find the values of a and b. --[5]

(ii) When a and b have these values, find the real roots of the equation $p(x) = 0$ [W-16/33/Q] --[2]

Solution: Since $(x^2 - x + 2)$ is a factor of $(4x^4 + ax^2 + 11x + b)$; $p(x)$

(i) can be written as:

$$4x^4 + ax^2 + 11x + b = (x^2 - x + 2)(px^2 + qx + r) \quad \text{---(i)}$$
$$= (px^4 + (q-p)x^3 + (r - q + 2p)x^2 + (2q - r)x + 2r)$$

Equating the coefficients: coeff of $x^4 \Rightarrow p = 4$ ---(ii)

Coeff of x^3: $q - p = 0 \Rightarrow q = p = 4$ from (ii) $\Rightarrow q = 4$ ---(iii)

Coeff of x^2: $r - q + 2p = a \Rightarrow r - 4 + 8 = a$ $[\because q = 4, p = 4]$

$\Rightarrow a = r + 4$ ---(iv)

Coeff of x: $2q - r = 11 \Rightarrow 8 - r = 11$ $[\because q = 4]$

$\Rightarrow r = -3$ ✓ ---(v)

Constant term $b = 2r = 2 \times (-3) = -6$ ✓ $[\because r = -3]$

from (iv) and (v) $a = -3 + 4 = 1$ ✓

$\therefore$ $a = 1$ and $b = -6$ and the factor is $(4x^2 + 4x - 3)$

(ii) $\therefore$ from (i) $4x^4 + x^2 + 11x - 6 = (x^2 - x + 2)(4x^2 + 4x - 3)$ ----(vi)

$\therefore$ Equation $p(x) = 0 \Rightarrow (x^2 - x + 2)(4x^2 + 4x - 3) = 0$

or $(x^2 - x + 2)(2x + 3)(2x - 1) = 0$

$x = -3/2$; $x = \frac{1}{2}$ ✓

$\left\{ \begin{array}{l} \because x^2 - x + 2 = 0 \\ B^2 - 4AC = 1 - 4 \times 1 \times 2 = -7 < 0 \\ \text{No real roots} \end{array} \right.$

(*) Alternate method: (Using Long division)

$$\begin{array}{r|l} x^2 - x + 2 & 4x^4 + ax^2 + 11x + b \quad (4x^2 + 4x + (a-4) \\ & \underline{-4x^4 + 8x^2 \quad \mp 4x^3} \\ & 4x^3 + (a-8)x^2 + 11x + b \\ & \underline{-4x^3 \mp 4x^2 \pm 8x} \\ & (a-4)x^2 + 3x + b \\ & \underline{-(a-4)x^2 \mp (a-4)x \pm (2a-8)} \\ & (a-1)x + (b - 2a + 8) \end{array}$$

$\because (x^2 - x + 2)$ is a factor of $p(x)$

Remainder $R(x) = 0$

$\Rightarrow (a-1)x + (b - 2a + 8) = 0$

$\Rightarrow a - 1 = 0$ & $b - 2a + 8 = 0$

$\Rightarrow a = 1$ ✓ & $b - 2 + 8 = 0$

$b = -6$ ✓

Example 3: Find the quotient and remainder when, $6x^4+x^3-x^2+5x-6$ is divided by $2x^2-x+1$. --- [3]

[S-20/32/Q1]

Solution:

$$2x^2-x+1 \,\big)\, 6x^4+x^3-x^2+5x-6 \,\big(\, 3x^2+2x-1$$
$$-6x^4 \mp 3x^3 \pm 3x^2$$
$$4x^3-4x^2+5x$$
$$-4x^3 \mp 2x^2 \pm 2x$$
$$-2x^2+3x-6$$
$$\mp 2x^2 \pm x \mp 1$$
$$(2x-5)$$

Quotient $= 3x^2+2x-1$ ✓
and
Remainder $= 2x-5$ ✓

Example 4 (i) Use algebric division to show that the polynomial, $p(x) = 2x^3-5x^2+7x-10$ has a factor $(x-2)$.
(ii) Verify $p(2)=0$

Solution:
(i)

$$x-2 \,\big)\, 2x^3-5x^2+7x-10 \,\big(\, 2x^2-x+5$$
$$-2x^3 \mp 4x^2$$
$$-x^2+7x$$
$$\mp x^2 \pm 2x$$
$$5x-10$$
$$5x-10$$
$$- \quad +$$
$$\times$$

Quotient $= 2x^2-x+5$
Remainder $= 0$
$p(x) = (x-2)(2x^2-x+5)$ --- (1)

$\therefore (x-2)$ is a factor of $p(x)$.

(ii) $p(2) = 2\times 2^3 - 5\times 2^2 + 7\times 2 - 10 = 16-20+14-10 = 0$ ✓
(also from (1) $p(2) = (2-2)(2\times 2^2-2+5) = 0$ ✓)

§ **The Factor theorem:**
If for a polynomimel $P(x)$; $P(a)=0$,
then $(x-a)$ is a factor of $P(x)$. (and conversely)
(Refer: Example 4(ii))

§ A linear polynomial $(ax+b)$ is a factor of $P(x)$ if $P\left(-\frac{b}{a}\right)=0$
(Note: $ax+b=0 \Rightarrow x=-\frac{b}{a}$)

Example 5: The polynomial $6x^3 - 23x^2 - 38x + 15$ is denoted by $p(x)$
(a) Show that $(x-5)$ is a factor of $p(x)$
(b) Factorise $f(x)$ completely and write down the roots of $p(x)=0$

Solution: $p(x) = 6x^3 - 23x^2 - 38x + 15$ {for $(x-5)$ be a factor verify $p(5)=0$

(a) $p(5) = 6\times5^3 - 23\times5^2 - 38\times5 + 15$
$= 750 - 575 - 190 + 15$
$= 765 - 765 = 0$
$\therefore$ Using factor theorem it mean $(x-5)$ is a factor of $p(x)$.

(b) Now $p(x) = (x-5)(6x^2+7x-3)$
$= (x-5)[6x^2+9x-2x-3]$
$= (x-5)[3x(2x+3)-1(2x+3)]$
$= (x-5)(3x-1)(2x+3)$

$\because$

$$\begin{array}{r} 6x^2+7x-3 \\ x-5\overline{)6x^3-23x^2-38x+15} \\ -6x^3 \mp 30x^2 \\ \hline 7x^2-38x \\ -7x^2 \mp 35x \\ \hline -3x+15 \\ \mp 3x \pm 15 \\ \hline \times \end{array}$$

Now $p(x) = 0$
$\Rightarrow (x-5)(3x-1)(2x+3) = 0$
roots of $p(x)$ are $5, \frac{1}{3}$ and $-\frac{3}{2}$

Example 6: The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$. (i) Find the value of a. ---[2]
(ii) When a has this value (a) Factorise $p(x)$ --[2]
(b) Solve the inequality $p(x) > 0$ $\boxed{M-16/32/Q4}$ ---[3]

Solution: $p(x) = 4x^3 + ax + 2$ --(i)

(i) for $(2x+1)$ a factor of $p(x)$
Check: $p(-\frac{1}{2}) = 0$ $[\because 2x+1=0 \Rightarrow x = -\frac{1}{2}]$
$p(-\frac{1}{2}) = 4(-\frac{1}{2})^3 + a(-\frac{1}{2}) + 2 = 0$
$\Rightarrow -\frac{1}{2} - \frac{1}{2}a + 2 = 0 \Rightarrow -\frac{1}{2}a = -\frac{3}{2}$
$\Rightarrow a = 3$

from (i)
$p(x) = 4x^3 + 3x + 2$ ---(ii)

(ii)(a) $= (2x+1)(2x^2 - x + 2)$
$\because (2x^2-x+2)$ Can't be factorised
(as $b^2 - 4ac = 1 - 16 = -15 < 0$)

$(\because$

$$\begin{array}{r} 2x+1\overline{)4x^3+3x+2}\;(2x^2-x+2 \\ -4x^3 \underset{-}{+} 2x^2 \\ \hline -2x^2+3x \\ \mp 2x^2 \mp x \\ \hline 4x+2 \\ 4x+2 \\ \hline \times \end{array}$$

(ii)(b) To Solve $p(x) > 0$
$(2x+1)(2x^2-x+2) > 0$
$\Rightarrow (2x+1) > 0$
$x > -\frac{1}{2}$

$\because 2x^2 - x + 2$
$= 2[x - \frac{1}{4}]^2 + \frac{15}{8} > 0$
for all reals.

Example 7. The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x+1)$ and $(x+2)$ are factors of $p(x)$.

(i) Find the values of a and of b. --[4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by (x^2+1). --[3]

[W-14/33/Q3]

Solution: $p(x) = 4x^3 + ax^2 + bx - 2$ ---- (i)

(i) $(x+1)$ is a factor of $p(x) \Rightarrow p(-1) = 0$ (using factor theorem)

$p(-1) = 4(-1)^3 + a(-1)^2 + b(-1) - 2 = 0$

$\Rightarrow -4 + a - b - 2 = 0 \Rightarrow a - b = 6$ --- (ii)

Again $(x+2)$ is a factor of $p(x) \Rightarrow p(-2) = 0$ (factor theorem)

$\Rightarrow p(-2) = 4(-2)^3 + a(-2)^2 + b(-2) - 2 = 0$

$\Rightarrow -32 + 4a - 2b - 2 = 0 \Rightarrow 4a - 2b = 34$

or $2a - b = 17$ --- (iii)

Solving (ii) and (iii)

$a = 11$ and $b = 5$ ✓

from (i) $p(x) = 4x^3 + 11x^2 + 5x - 2$ ✓

(ii) Now $p(x) \div (x^2+1)$

$$
\begin{array}{r|l}
x^2+1 & 4x^3 + 11x^2 + 5x - 2 \;(4x+11 \\
 & \underline{-4x^3 \qquad \pm 4x} \\
 & 11x^2 + x - 2 \\
 & \underline{-11x^2 \qquad \pm 11} \\
 & x - 13 \checkmark
\end{array}
$$

$\therefore$ The remainder $= \underline{(x-13)}$ ✓

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Example 8: The polynomial $2x^3+5x^2-7x+11$ is denoted by $p(x)$, find the remainder when $p(x)$ is divided by $(x-2)$.

Solution: $p(x) = 2x^3+5x^2-7x+11$ ....(i)

$$
\begin{array}{r|l}
x-2 & 2x^3+5x^2-7x+11 \;\big(2x^2+9x+11 \\
 & \underline{-2x^3 \mp 4x^2} \\
 & \quad 9x^2-7x \\
 & \quad \underline{-9x^2 \mp 18x} \\
 & \qquad 11x+11 \\
 & \qquad \underline{-11x \mp 22} \\
 & \qquad\quad 33
\end{array}
$$

Remainder $= 33$ ✓

Now Note: $\left(\begin{array}{l} p(x) = (x-a)\cdot q(x) + r \\ \Rightarrow p(a) = \quad 0 \quad + r \end{array}\right)$

$p(x) = 2x^3+5x^2-7x+11 = (x-2)(2x^2+9x+11) + 33 \Rightarrow P(2) = 33$

$r = p(2) = 2\times2^3+5\times2^2-7\times2+11 = 16+20-14+11 = 33$ ✓

§ **Remainder Theorem:**

If a polynomial $p(x)$ is divided by $(x-a)$ {a linear polynomial} the remainder $r = p(a)$ ✓

$\left[\begin{array}{l} p(x) = (x-a)q(x) + r \\ p(a) = 0 + r \Rightarrow r = p(a) \end{array}\right]$

§ **Note:** If $p(x)$ is divided by $(ax+b)$, then remainder $r = p\left(-\frac{b}{a}\right)$ ✓ $\left[\because ax+b=0 \Rightarrow x = -\frac{b}{a}\right]$

Example 9: Find the remainder when $p(x) = 8x^3+x^2-2x+1$ is divided by $(2x-1)$.

Solution: $p(x) = 8x^3+x^2-2x+1$ is divided by $(2x-1)$

$\left[\begin{array}{l} 2x-1=0, \\ x = \frac{1}{2} \end{array}\right.$

$r = \left(\frac{1}{2}\right) = 8\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 2\times\frac{1}{2} + 1$

$= 1 + \frac{1}{4} - 1 + 1 = \frac{5}{4}$ ✓ ⊛

$\therefore$ remainder $= \frac{5}{4}$ ✓

[Refer Example 1/Page 1 using long division]

Example 10: The polynomial $6x^3+ax^2+bx-2$, where a and b are constants is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$ and when $p(x)$ is divided by $(x+2)$ the remainder is -24. Find the values of a and b. ---[5]

[W-19/33/Q2]

Solution: $p(x) = 6x^3+ax^2+bx-2$ ---(1)

Given $(2x+1)$ is a factor of $p(x) \Rightarrow p(-\frac{1}{2})=0$ [$\because$ critical value of $2x+1=0$, $x=-\frac{1}{2}$]

$p(-\frac{1}{2}) = 6(-\frac{1}{2})^3 + a(-\frac{1}{2})^2 + b(-\frac{1}{2}) - 2 = 0$

$\Rightarrow -\frac{6}{8} + \frac{1}{4}a - \frac{1}{2}b - 2 = 0 \Rightarrow \frac{1}{4}a - \frac{1}{2}b = \frac{11}{4}$

$\Rightarrow a - 2b = 11$ ---- (2)

Also $p(x)$ is divided by $(x+2)$ then remainder $= -24$

$R = p(-2) = 6(-2)^3 + a(-2)^2 + b(-2) - 2$ ($\Rightarrow p(-2) = -24$, $\{x+2=0, x=-2\}$

$p(-2) = -48 + 4a - 2b - 2 = -24$ given

$\Rightarrow 4a - 2b = 26$ ---- (3)

Solving (2) and (3) we get $a=5$ ✓ and $b=-3$ ✓

Example 11: Given a polynomial $p(x) = 16x^3 - 24x^2 - 15x - 2$

(i) Using factor theorem factorise $p(x)$ completely. --- [4]

(ii) Find the roots of $p(x)=0$ ---[1]

Solution: Given $p(x) = 16x^3 - 24x^2 - 15x - 2$ --- (1)

[Consider the factors of constant term, $2 = 1 \times 2$. Zeros of $p(x)$ may be ± 1 or ± 2]

(i) $p(1) = 16 \times 1^3 - 24 \times 1^2 - 15 \times 1 - 2 = -25 \neq 0$

Now $p(2) = 16 \times 2^3 - 24 \times 2^2 - 15 \times 2 - 2 = 0$ ✓

$\therefore$ $(x-2)$ is factor of $p(x)$ using factor theorem.

$$
\begin{array}{r|l}
x-2 & 16x^3 - 24x^2 - 15x - 2 \quad (16x^2+8x+1 \\
 & \underline{16x^3 \mp 32x^2} \\
 & 8x^2 - 15x \\
 & \underline{8x^2 \mp 16x} \\
 & x - 2 \\
 & \underline{x \mp 2} \\
 & \times
\end{array}
$$

$\therefore p(x) = (x-2)(16x^2+8x+1)$

$= (x-2) \cdot (4x+1)^2$ are the required factors.

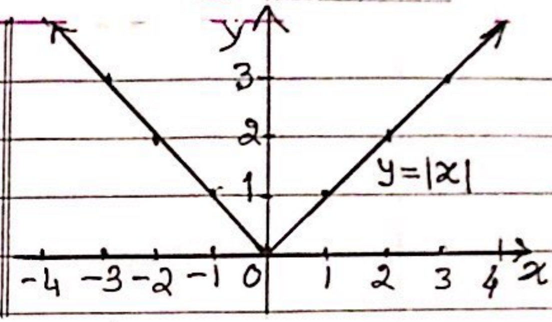
(ii) $(x-2)(4x+1)^2 = 0$

$x = 2$ ✓, $x = -\frac{1}{4}$ ✓

Modulus Function.

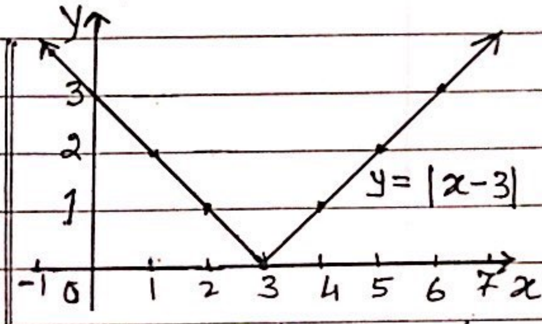
§ Modulus function:

$$|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$$

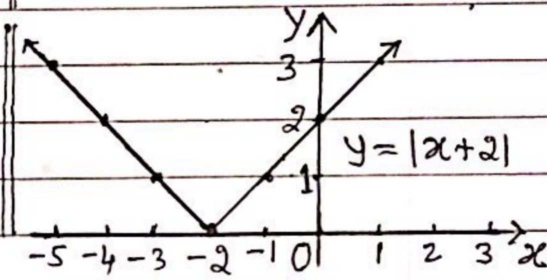


Example.

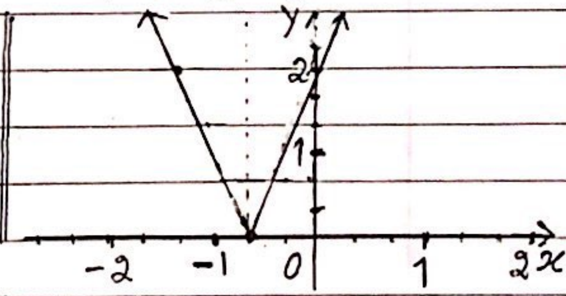
(i) $y = |x-3| = \begin{cases} x-3, & \text{for } x \geq 3 \\ -(x-3) \\ \text{or } 3-x & \text{for } x < 3 \end{cases}$



(ii) $y = |x+2| = \begin{cases} x+2, & \text{for } x \geq -2 \\ -(x+2) & \text{for } x < -2 \end{cases}$



(iii) $y = |3x+2| = \begin{cases} 3x+2, & \text{for } x \geq -\frac{2}{3} \\ -(3x+2), & \text{for } x < -\frac{2}{3} \end{cases}$

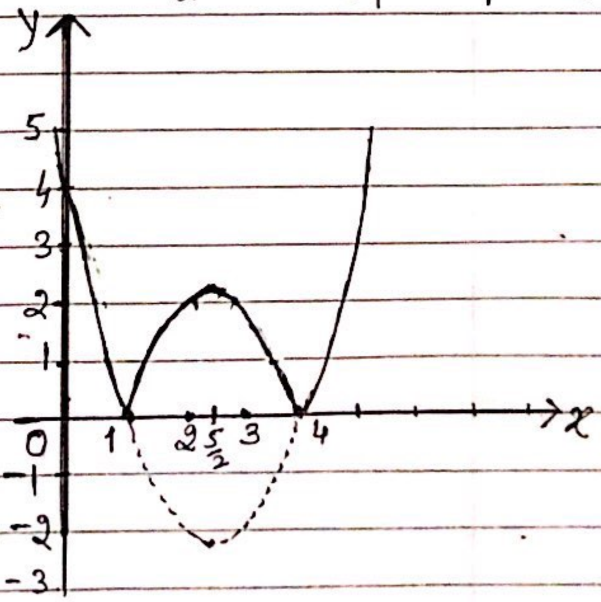


(iv) $y = |x^2 - 5x + 4|$

$= |(x-1)(x-4)|$

$= \begin{cases} (x-1)(x-4); & \text{for } (x-1)(x-4) \geq 0 \\ -(x-1)(x-4); & \text{for } (x-1)(x-4) < 0 \end{cases}$

$= \begin{cases} (x-1)(x-4); & \text{for } x \leq 1 \text{ or } x \geq 4 \\ -(x-1)(x-4); & \text{for } 1 \leq x \leq 4 \end{cases}$



§ Some Properties of modulus functions:

(i) $|a \cdot b| = |a||b|$

(ii) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

(iii) $|a-x| = |x-a|$

(iv) $|x| = |y| \Rightarrow x = y$ or $x = -y$

$|x| = |y| \Rightarrow x^2 = y^2$

(v) $|x| = a \Rightarrow x = \pm a$; $a>0$

(vi) $|x| < a \Rightarrow -a < x < a$; $a>0$

(vii) $|x| > a \Rightarrow x < -a$ or $x > a$; $a>0$

(viii) $|x| > |a| \Leftrightarrow x^2 > a^2$

(ix) $|x| < |a| \Leftrightarrow x^2 < a^2$

(x) $|x^2| = x^2 = |x|^2$; $x \in R$

(xi) $\sqrt{x^2} = |x|$; $x \in R$

(xii) $|f(x)| = \begin{cases} f(x); & \text{for } f(x) \geq 0 \\ -f(x); & \text{for } f(x) < 0 \end{cases}$

(xiii) $|x-b| < a \Rightarrow b-a < x < b+a$ for $a>0$

Example 12:

for graphical solution see example 24/Page 13.

Solve: $|4x-3| = 7$

$\Rightarrow 4x-3 = 7$ or $4x-3 = -7$

$\Rightarrow 4x = 10$; $4x = -4$

$x = \frac{5}{2}$ ✓ or $x = -1$ ✓

Example 13: Solve;

$|2x+7| = 3x$ ---(i)

$\Rightarrow (2x+7) = 3x$ or $2x+7 = -3x$

$\Rightarrow 7 = 3x - x$ or $2x + 3x = -7$

$x = 7$ ✓ or $5x = -7$ ✗

$x = -\frac{7}{5}$

$\therefore$ $x = 7$ ✓

Verify the answer

$\because$ from (i) for $x = -\frac{7}{5}$

$|2 \cdot -\frac{7}{5} + 7| = 3 \cdot -\frac{7}{5}$

$\left|\frac{21}{5}\right| = -\frac{21}{5}$

$\frac{21}{5} = -\frac{21}{5}$ ✗

false.

Example 14: Solve: $2|x-1| = 3|x|$ [S-16/31/Q1(i)] --- [3]

Solution: $2|x-1| = 3|x|$

$\Rightarrow 2(x-1) = 3x$ or $2(x-1) = -3x$

$2x-2 = 3x$ or $2x-2 = -3x$

$\Rightarrow x = -2$; $5x = 2$

$\underline{x = -2}$ ✓; $\underline{x = \frac{2}{5}}$ ✓

Alternate method:

$2|x-1| = 3|x|$

$\Rightarrow (2|x-1|)^2 = (3|x|)^2$

$\Rightarrow 4(x-1)^2 = 9x^2$

$4(x^2-2x+1) = 9x^2$

$\Rightarrow 4x^2 - 8x + 4 = 9x^2$

$\Rightarrow 9x^2 - 4x^2 + 8x - 4 = 0$

$5x^2 + 8x - 4 = 0$

$\Rightarrow (x+2)(5x-2) = 0$

$\Rightarrow \underline{x = -2}$ ✓ or $\underline{x = \frac{2}{5}}$ ✓

Example 15: Solve $|4x-1| = |x-3|$ [S-13/31/Q4(i)] --- [3]

Solution: $|4x-1| = |x-3|$

$\Rightarrow 4x-1 = x-3$ or $4x-1 = -(x-3)$

$\Rightarrow 3x = -2$; $4x-1 = -x+3$

$x = -\frac{2}{3}$; $5x = 4$

$x = 4/5$

$\therefore \underline{x = -\frac{2}{3}}$ ✓ or $\underline{\frac{4}{5}}$ ✓

Alternate Method:

$|4x-1| = |x-3|$ sq. both sides

$\Rightarrow (4x-1)^2 = (x-3)^2$

$\Rightarrow 16x^2 - 8x + 1 = x^2 - 6x + 9$

$\Rightarrow 15x^2 - 2x - 8 = 0$

$\Rightarrow (3x+2)(5x-4) = 0$

$\underline{x = -2/3}$ ✓; $\underline{x = \frac{4}{5}}$ ✓

Example 16: Solve: $|x+3| + |x-1| = 6$

Solution: $|x+3| + |x-1| = 6$

$\Rightarrow |x+3| = 6 - |x-1|$

$\Rightarrow x+3 = 6 - |x-1|$

$x+3 = 6-(x-1)$ or $x+3 = 6-(1-x)$

$x+3 = 7-x$ or $x+3 = 5+x$

$2x = 4$ or $3 = 5$ ✗ false

$x = 2$ ✓

or

$x+3 = -(6-|x-1|)$

$\Rightarrow x+3 = |x-1| - 6$

$\Rightarrow x+3 = (x-1)-6$ or $x+3 = 1-x-6$

$\Rightarrow 3 = -7$ ✗ false or $2x = -8$

$x = -4$ ✓

$\therefore \underline{x = 2 \text{ or } x = -4}$ ✓

(Find graphical solution: Example 26/Page 14)

Example 17: Solve $|3x-2| \geq 7$

Solution: $|3x-2| \geq 7$ $\quad \left(\because |x| \geq a \;;\; a>0 \Rightarrow x \leq -a \text{ or } x \geq a \right)$

$\Rightarrow 3x-2 \leq -7$ or $3x-2 \geq 7$

$\Rightarrow 3x \leq -5$ or $3x \geq 9$

$\Rightarrow x \leq -\frac{5}{3}$ or $x \geq 3$ ✓

Example 18: Solve; $|2-x| < 4$

Solution: $|2-x| < 4$ $\quad \left\{ |x| < a \Rightarrow -a < x < a \;;\; a>0 \right.$

$\Rightarrow -4 < 2-x < 4$

$\Rightarrow -6 < -x < 2$

$\Rightarrow 6 > x > -2 \Rightarrow -2 < x < 6$ ✓

Example 19: Solve $|x-2| > 2x-3$

Solution: $|x-2| > 2x-3$ $\quad |x-2| = \begin{cases} x-2 \;;\; \text{for } x \geq 2 \\ 2-x \;;\; \text{for } x < 2 \end{cases}$

$\Rightarrow \{x-2 > 2x-3 \text{ if } x \geq 2\}$

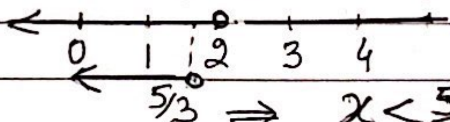
or $\{2-x > 2x-3 \text{ if } x < 2\}$

$\Rightarrow \{x < 1 \text{ if } x \geq 2\}$ or $\{3x < 5 \text{ if } x < 2\}$

or $\{x < \frac{5}{3} \text{ if } x < 2\}$

Not possible

or



$\Rightarrow x < \frac{5}{3}$ ✓

Example 20: Solve the inequality $2-5x > 2|x-3|$ --- [4]

[W-20/31/Q1]

Solution: $2-5x > 2|x-3|$ ---- ①

$\Rightarrow (2-5x)^2 > (2(x-3))^2$

$\Rightarrow 4 + 25x^2 - 20x > 4(x^2 - 6x + 9)$

$\Rightarrow 25x^2 - 20x + 4 - 4x^2 + 24x - 36 > 0$

$\Rightarrow 21x^2 + 4x - 32 > 0$

$\Rightarrow (3x+4)(7x-8) > 0$ (critical values $-\frac{4}{3}; \frac{8}{7}$)

$\Rightarrow x < -\frac{4}{3} \;;\; x > \frac{8}{7}$ ✗ (does not satisfy ①)

$\therefore x < -\frac{4}{3}$ ✓

Example 21: Solve the inequality: $|2x-3| > 4|x+1|$ --- [4]

[W-19/31/Q2]

Solution: $|2x-3| > 4|x+1|$

$\Rightarrow (2x-3)^2 > (4(x+1))^2$ $\quad (\because |x|>|y| \Rightarrow x^2>y^2)$

$4x^2-12x+9 > 16(x^2+2x+1)$

$4x^2-12x+9 > 16x^2+32x+16$

$\rightarrow 12x^2+44x+7 < 0$ $\quad$ critical values are

$(2x+7)(6x+1) < 0$ $\quad \left(x=-\frac{7}{2},\ x=-\frac{1}{6}\right)$

$\Rightarrow -\frac{7}{2} < x < -\frac{1}{6}$ ✓

Example 22: (a) Sketch the graph of $y=|2x-3|$ --- [1]

(b) Solve the inequality $3x-1 > |2x-3|$ --- [3]

[SP-20/03/Q3]

Solution (a) $y=|2x-3| = \begin{cases} 2x-3; & \text{for } x \geq 3/2 \quad \text{---} ① \\ 3-2x; & \text{for } x < \frac{3}{2} \end{cases}$

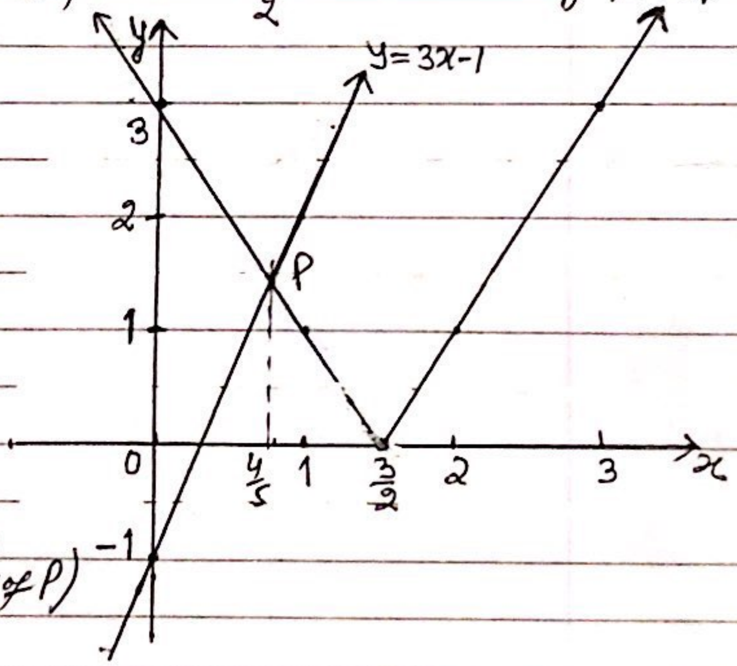
(b) consider $y = 3x-1$ ---- ②

for the intersection of ① & ② on the left of $x=\frac{3}{2}$

$3x-1 = 3-2x$

$5x = 4$

$x=\frac{4}{5}$ at P



from graph we find:

$3x-1 > |2x-3|$

for $x > \frac{4}{5}$ (On the right of P)

$\therefore$ Required solution: $x > \frac{4}{5}$ ✓

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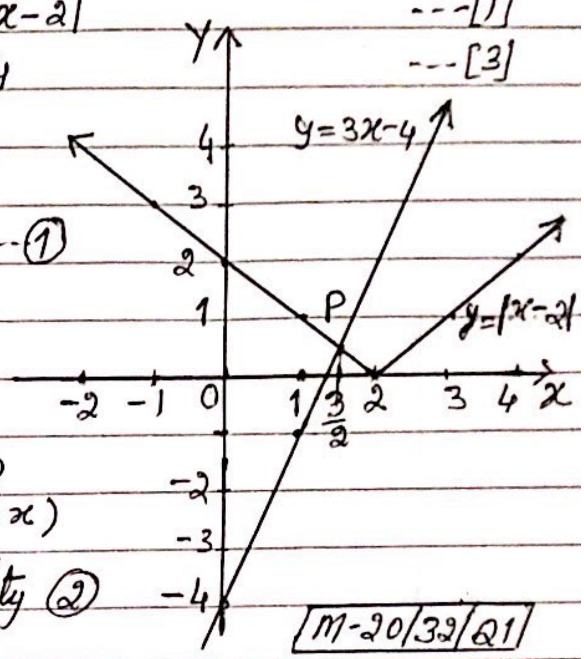
Example 23 (a) Sketch the graph of $y = |x-2|$ ---[1]

(b) Solve the inequality. $|x-2| < 3x-4$ ---[3]



Solution: $y = |x-2| = \begin{cases} x-2; \text{ for } x \geq 2 \\ 2-x; \text{ for } x < 2 \end{cases}$ ---①

(b) To solve $|x-2| < 3x-4$ ------②

Consider $y = 3x-4$ ---③

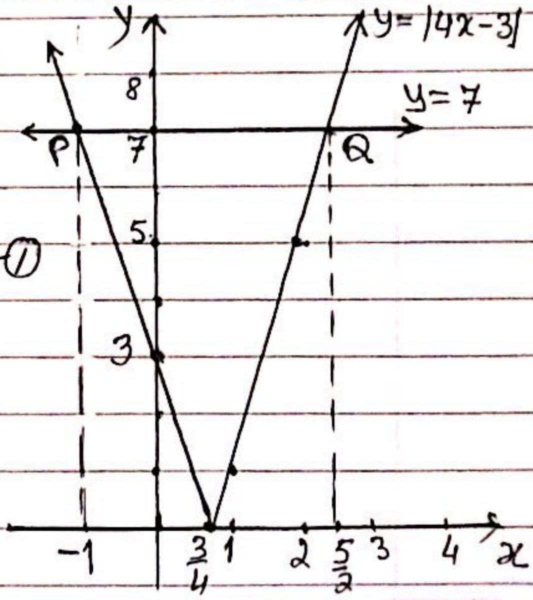
① and ③ (Graphs) intersect at P.

at P, $x = \frac{3}{2}$, $(3x-4 = 2-x \Rightarrow x = 3/2)$

from graph we find the inequality ② is true for $x > \frac{3}{2}$ ✓

[M-20/32/Q1]

Example 24: Sketch the graph of $y = |4x-3|$ and hence solve $|4x-3| = 7$.



Solution: $y = |4x-3| = \begin{cases} 4x-3; \text{ for } x \geq \frac{3}{4} \\ 3-4x, \text{ for } x < \frac{3}{4} \end{cases}$ ---①

To Solve $|4x-3| = 7$ ---②

Consider $y = 7$ ----③

Curve ① and ③ intersect at P and Q

at $x = -1$ and $x = \frac{5}{2}$ ✓

are the required solutions.

See. example 12/Page 9

Example 25: Solve graphically,

$|2x+7| = 3x$ ----(1)

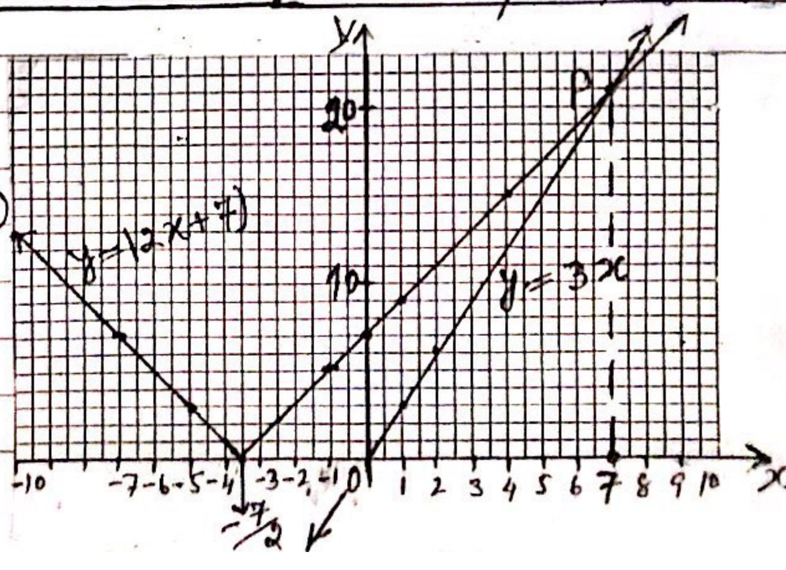
Consider $y = |2x+7| = \begin{cases} 2x+7; \text{ for } x \geq -\frac{7}{2} \\ -(2x+7); \text{ for } x < -\frac{7}{2} \end{cases}$ -②

and $y = 3x$ ---③

Graphs ② and ③ intersect at P.

$x = 7$ ✓

(For algebraic solution see: Example 13/Page 9)



Example 26(a) Sketch the graph: $y = |x+3| + |x-1|$

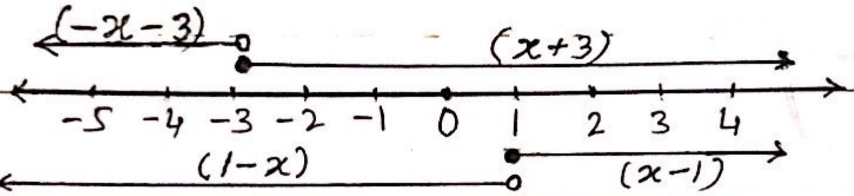
(b) Hence solve $|x+3| + |x-1| = 6$

Solution: (a)

$$|x+3| = \begin{cases} x+3 & \text{if } x \geq -3 \\ -(x+3) & \text{if } x < -3 \end{cases} \quad \text{---(i)}$$

and

$$|x-1| = \begin{cases} x-1 & \text{if } x \geq 1 \\ 1-x & \text{if } x < 1 \end{cases} \quad \text{---(ii)}$$



add (i) and (ii) using the number line (above)

$$y = |x+3| + |x-1| = \begin{cases} 2x+2, & \text{for } x \geq 1 \\ 4, & \text{for } -3 \leq x < 1 \\ -(2x+2), & \text{for } x < -3 \end{cases} \quad \text{---(iii)}$$

(b) Consider $y = 6$ ---(iv)

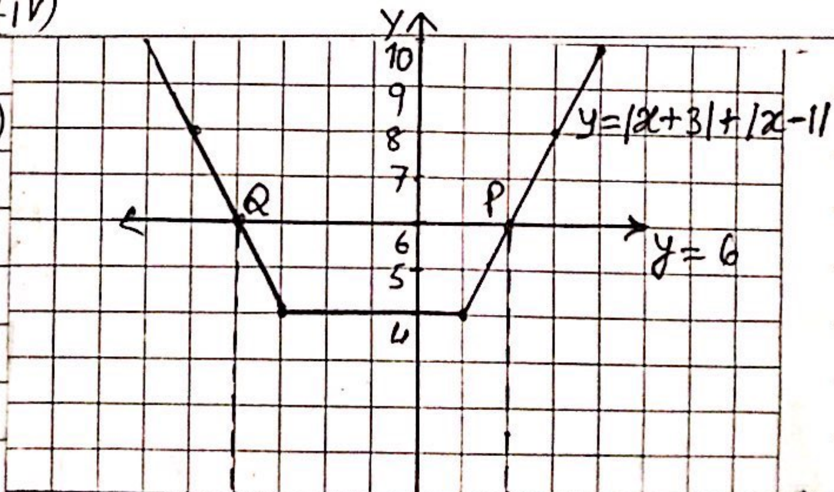
Now to solve

$|x+3| + |x-1| = 6$ ---(V)

find the points of intersection of (iii) & (iv)

point P at $x = 2$

point Q at $x = -4$



∴ The solution of the equation (V) are:

$x = 2$; $x = -4$ ✓

Note:
For the algebraic solution of this question see:
Example: 16/Page-10