

**PURE MATHEMATICS -3**

**9709**

(March, June and November series 2020 – 2023 With marking scheme)

**TRIGONOMETRY**

**EXERCISE -1**

MANJULA BALAJI

1) SP-2020\_9709\_3 Q7

(a) By first expanding  $\cos(x + 45^\circ)$ , express  $\cos(x + 45^\circ) - \sqrt{2}\sin x$  in the form  $R\cos(x + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the value of  $R$  correct to 4 significant figures and the value of  $\alpha$  correct to 2 decimal places. [5]

(b) Hence solve the equation

$$\cos(x + 45^\circ) - \sqrt{2}\sin x = 2,$$

for  $0^\circ < x < 360^\circ$ .

[4]

2) MARCH 2020\_9709\_32 Q5

(a) Show that  $\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = 2 \cot 2x$ . [4]

(b) Hence solve the equation  $\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = 4$ , for  $0 < x < \pi$ . [3]

3) MARCH 2021\_9709\_32 Q3

By first expressing the equation  $\tan(x + 45^\circ) = 2 \cot x + 1$  as a quadratic equation in  $\tan x$ , solve the equation for  $0^\circ < x < 180^\circ$ . [6]

4) MARCH 2021\_9709\_32 Q5

(a) Express  $\sqrt{7}\sin x + 2\cos x$  in the form  $R\sin(x + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . State the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

(b) Hence solve the equation  $\sqrt{7}\sin 2\theta + 2\cos 2\theta = 1$ , for  $0^\circ < \theta < 180^\circ$ . [5]

5) MARCH 2022\_9709\_32 Q5

The angles  $\alpha$  and  $\beta$  lie between  $0^\circ$  and  $180^\circ$  and are such that

$$\tan(\alpha + \beta) = 2 \quad \text{and} \quad \tan \alpha = 3 \tan \beta.$$

Find the possible values of  $\alpha$  and  $\beta$ .

[6]

6) MARCH 2023\_9709\_32 Q6

(a) Express  $5\sin \theta + 12\cos \theta$  in the form  $R\cos(\theta - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . [3]

(b) Hence solve the equation  $5\sin 2x + 12\cos 2x = 6$  for  $0 \leq x \leq \pi$ . [4]

7) JUNE 2020\_9709\_31 Q3

Express the equation  $\tan(\theta + 60^\circ) = 2 + \tan(60^\circ - \theta)$  as a quadratic equation in  $\tan \theta$ , and hence solve the equation for  $0^\circ \leq \theta \leq 180^\circ$ . [6]

8) JUNE 2020\_9709\_32 Q5

(a) Express  $\sqrt{2}\cos x - \sqrt{5}\sin x$  in the form  $R\cos(x + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the exact value of  $R$  and the value of  $\alpha$  correct to 3 decimal places. [3]

(b) Hence solve the equation  $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$ , for  $0^\circ < \theta < 180^\circ$ . [4]

9) JUNE 2020\_9709\_33 Q5

By first expressing the equation

$$\tan \theta \tan(\theta + 45^\circ) = 2 \cot 2\theta$$

as a quadratic equation in  $\tan \theta$ , solve the equation for  $0^\circ < \theta < 90^\circ$ .

[6]

10) JUNE 2021\_9709\_31 Q3

(a) Given that  $\cos(x - 30^\circ) = 2 \sin(x + 30^\circ)$ , show that  $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$ .

[4]

(b) Hence solve the equation

$$\cos(x - 30^\circ) = 2 \sin(x + 30^\circ),$$

for  $0^\circ < x < 360^\circ$ .

[2]

11) JUNE 2021\_9709\_32 Q6(a)

(a) Prove that  $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$ .

[3]

12) JUNE 2021\_9709\_33 Q5

(a) By first expanding  $\tan(2\theta + 2\theta)$ , show that the equation  $\tan 4\theta = \frac{1}{2} \tan \theta$  may be expressed as  $\tan^4 \theta + 2 \tan^2 \theta - 7 = 0$ .

[4]

(b) Hence solve the equation  $\tan 4\theta = \frac{1}{2} \tan \theta$ , for  $0^\circ < \theta < 180^\circ$ .

[3]

13) JUNE 2022\_9709\_31 Q3

Solve the equation  $2 \cot 2x + 3 \cot x = 5$ , for  $0^\circ < x < 180^\circ$ .

[6]

14) JUNE 2022\_9709\_32 Q2

Solve the equation  $3 \cos 2\theta = 3 \cos \theta + 2$ , for  $0^\circ \leq \theta \leq 360^\circ$ .

[5]

15) JUNE 2022\_9709\_33 Q2

Solve the equation  $\cos(\theta - 60^\circ) = 3 \sin \theta$ , for  $0^\circ \leq \theta \leq 360^\circ$ .

[5]

16) JUNE 2023\_9709\_31 Q4

(a) Show that the equation  $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$  can be expressed in the form

$$\cos^2 \theta + 2 \sin \theta \cos \theta - 3 \sin^2 \theta = 0.$$

[2]

(b) Hence solve the equation  $\sin 2\theta + \cos 2\theta = 2 \sin^2 \theta$  for  $0^\circ < \theta < 180^\circ$ .

[4]

17) JUNE 2023\_9709\_32 Q4

Solve the equation  $2 \cos x - \cos \frac{1}{2}x = 1$  for  $0 \leq x \leq 2\pi$ .

[5]

18) JUNE 2023\_9709\_33 Q6

(a) Express  $3 \cos x + 2 \cos(x - 60^\circ)$  in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ .

State the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places.

[4]

(b) Hence solve the equation

$$3 \cos 2\theta + 2 \cos(2\theta - 60^\circ) = 2.5$$

for  $0^\circ < \theta < 180^\circ$ .

[4]

19) OCT 2020\_9709\_31 Q6

(a) Express  $\sqrt{6} \cos \theta + 3 \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . State the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

(b) Hence solve the equation  $\sqrt{6} \cos \frac{1}{3}x + 3 \sin \frac{1}{3}x = 2.5$ , for  $0^\circ < x < 360^\circ$ . [4]

20) OCT 2020\_9709\_32 Q4

(a) Show that the equation  $\tan(\theta + 60^\circ) = 2 \cot \theta$  can be written in the form

$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0. \quad [3]$$

(b) Hence solve the equation  $\tan(\theta + 60^\circ) = 2 \cot \theta$ , for  $0^\circ < \theta < 180^\circ$ . [3]

21) OCT 2021\_9709\_31 Q2

(a) Express  $5 \sin x - 3 \cos x$  in the form  $R \sin(x - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . Give the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

(b) Hence state the greatest and least possible values of  $(5 \sin x - 3 \cos x)^2$ . [2]

22) OCT 2021\_9709\_31 Q5

(a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in  $\tan \theta$ .

[3]

(b) Hence solve the equation  $\cot 2\theta + \cot \theta = 2$ , for  $0 < \theta < \pi$ , giving your answers correct to 3 decimal places. [3]

23) OCT 2021\_9709\_32 Q8

(a) By first expanding  $(\cos^2 \theta + \sin^2 \theta)^2$ , show that

$$\cos^4 \theta + \sin^4 \theta \equiv 1 - \frac{1}{2} \sin^2 2\theta. \quad [3]$$

(b) Hence solve the equation

$$\cos^4 \theta + \sin^4 \theta = \frac{5}{9},$$

for  $0^\circ < \theta < 180^\circ$ .

[4]

24) OCT 2021\_9709\_32 Q6(a)

Using the expansions of  $\sin(3x + 2x)$  and  $\sin(3x - 2x)$ , show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x. \quad [3]$$

25) OCT 2021\_9709\_33 Q5

Solve the equation  $\sin \theta = 3 \cos 2\theta + 2$ , for  $0^\circ \leq \theta \leq 360^\circ$ .

[5]

26) OCT 2021\_9709\_33 Q6

(a) By first expanding  $\cos(x - 60^\circ)$ , show that the expression

$$2 \cos(x - 60^\circ) + \cos x$$

can be written in the form  $R \cos(x - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [5]

(b) Hence find the value of  $x$  in the interval  $0^\circ < x < 360^\circ$  for which  $2 \cos(x - 60^\circ) + \cos x$  takes its least possible value. [2]

27) OCT 2022\_9709\_31 Q4

Solve the equation  $\tan(x + 45^\circ) = 2 \cot x$  for  $0^\circ < x < 180^\circ$ . [5]

28) OCT 2022\_9709\_31 Q6

(a) Prove the identity  $\cos 4\theta + 4 \cos 2\theta + 3 \equiv 8 \cos^4 \theta$ . [4]

(b) Hence solve the equation  $\cos 4\theta + 4 \cos 2\theta = 4$  for  $0^\circ \leq \theta \leq 180^\circ$ . [3]

29) OCT 2022\_9709\_32 Q4

(a) Express  $4 \cos x - \sin x$  in the form  $R \cos(x + \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . State the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

(b) Hence solve the equation  $4 \cos 2x - \sin 2x = 3$  for  $0^\circ < x < 180^\circ$ . [5]

30) OCT 2022\_9709\_33 Q7

(a) Show that the equation  $\sqrt{5} \sec x + \tan x = 4$  can be expressed as  $R \cos(x + \alpha) = \sqrt{5}$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . Give the exact value of  $R$  and the value of  $\alpha$  correct to 2 decimal places. [4]

(b) Hence solve the equation  $\sqrt{5} \sec 2x + \tan 2x = 4$ , for  $0^\circ < x < 180^\circ$ . [4]

**MARKING SCHEME**

1) SP-2020\_9709\_3 Q7

(a)	Use $\cos(A + B)$ formula to obtain an expression in terms of $\cos x$ and $\sin x$	1	<b>M1</b>
	Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}} \sin x$ , or equivalent	1	<b>A1</b>
	Obtain $R = 2.236$	1	<b>A1</b>
	Use trig formula to find $\alpha$	1	<b>M1</b>
	Obtain $\alpha = 71.57^\circ$ with no errors seen	1	<b>A1</b>
		<b>5</b>	
(b)	Evaluate $\cos^{-1}\left(\frac{2}{2.236}\right)$ to at least 1 dp	1	<b>B1FT</b>
	Carry out an appropriate method to find a value of $x$ in the interval $0^\circ < x < 360^\circ$	1	<b>M1</b>
	Obtain answer, e.g. $x = 315^\circ$	1	<b>A1</b>
	Obtain second answer, e.g. $261.9^\circ$ and no others in the given interval	1	<b>A1</b>
		<b>4</b>	

2) MARCH 2020\_9709\_32 Q5

(a)	Express LHS correctly as a single fraction	<b>B1</b>
	Use $\cos(A \pm B)$ formula to simplify the numerator	<b>M1</b>
	Use $\sin 2A$ formula to simplify the denominator	<b>M1</b>
	Obtain the given result.	<b>A1</b>
(b)	Obtain an equation in $\tan 2x$ and use correct method to solve for $x$	<b>M1</b>
	Obtain answer, e.g. 0.232	<b>A1</b>
	Obtain second answer, e.g. 1.80	<b>A1</b>

3) MARCH 2021\_9709\_32 Q3

(a)	Sketch the graph $y = \sec x$	<b>M1</b>
	Sketch the graph $y = 2 - \frac{1}{2}x$ , and justify the given statement	<b>A1</b>
		<b>2</b>
(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	<b>M1</b>
	Complete the argument correctly with correct calculated values	<b>A1</b>
		<b>2</b>
(c)	Use the iterative formula correctly at least once	<b>M1</b>
	Obtain final answer 0.88	<b>A1</b>
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	<b>A1</b>

## 4) MARCH 2021\_9709\_32 Q5

a)	State $R = \sqrt{11}$	<b>B1</b>
	Use trig formulae to find $\alpha$	<b>M1</b>
	Obtain $\alpha = 37.09^\circ$	<b>A1</b>
		<b>3</b>
b)	Evaluate $\sin^{-1}\left(\frac{1}{\sqrt{11}}\right)$ to at least 2 dp ( $17.5484^\circ$ )	<b>B1 FT</b>
	Use correct method to find a value of $\theta$ in the interval	<b>M1</b>
	Obtain answer, e.g. $62.7^\circ$	<b>A1</b>
	Use a correct method to obtain a second answer	<b>M1</b>
	Obtain second answer, e.g. $170.2^\circ$ , and no other in the interval	<b>A1</b>

## 5) MARCH 2022\_9709\_32 Q5

	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan \alpha$ and $\tan \beta$	<b>M1</b>
	Substitute throughout for $\tan \alpha$ or for $\tan \beta$	<b>M1</b>
	Obtain $3\tan^2 \beta + 2\tan \beta - 1 = 0$ or $\tan^2 \alpha + 2\tan \alpha - 3 = 0$	<b>A1</b>
	Solve a 3-term quadratic and find an angle	<b>M1</b>
	Obtain answer $\alpha = 45^\circ, \beta = 18.4^\circ$	<b>A1</b>
	Obtain answer $\alpha = 108.4^\circ, \beta = 135^\circ$	<b>A1</b>

## 6) MARCH 2023\_9709\_32 Q6

a)	State $R = 13$	<b>B1</b>
	Use correct trig formulae to find $\alpha = \tan^{-1}(\pm 5/12) = \cos^{-1}(\pm 12/13) = \sin^{-1}(\pm 5/13)$	<b>M1</b>
	Obtain $\alpha = 0.395$	<b>A1</b>
		<b>3</b>
b)	$\cos^{-1}\left(\frac{6}{R}\right)$	<b>B1 FT</b>
	Use correct method to find a value of $2x$ in the interval	<b>M1</b>
	Obtain answer, e.g. $x = 0.743$ or $0.742$	<b>A1</b>
	Obtain second answer, e.g. $x = 2.79$ and no others in the interval	<b>A1</b>



7) JUNE 2020\_9709\_31 Q3

	Use $\tan(A \pm B)$ formula and obtain an equation in $\tan \theta$	M1
	Using $\tan 60^\circ = \sqrt{3}$ , obtain a horizontal equation in $\tan \theta$ in any correct form	A1
	Reduce the equation to $3 \tan^2 \theta + 4 \tan \theta - 1 = 0$ , or equivalent	A1
	Solve a 3-term quadratic for $\tan \theta$	M1
	Obtain a correct answer, e.g. $12.1^\circ$	A1
	Obtain a second correct answer, e.g. $122.9^\circ$ , and no others in the given interval	A1

8) JUNE 2020\_9709\_32 Q5

(a)	State $R = \sqrt{7}$	B1
	Use trig formulae to find $\alpha$	M1
	Obtain $\alpha = 57.688^\circ$	A1
		3
(b)	Evaluate $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right)$ to at least 3 d.p. ( $67.792^\circ$ ) (FT is on <i>their</i> R)	B1 FT
	Use correct method to find a value of $\theta$ in the interval	M1
	Obtain answer, e.g. $5.1^\circ$	A1
	Obtain second answer, e.g. $117.3^\circ$ , only	A1

9) JUNE 2020\_9709\_33 Q5

	Use $\tan 2A$ formula to express RHS in terms of $\tan \theta$	M1
	Use $\tan(A \pm B)$ formula to express LHS in terms of $\tan \theta$	M1
	Using $\tan 45^\circ = 1$ , obtain a correct horizontal equation in any form	A1
	Reduce equation to $2 \tan^2 \theta + \tan \theta - 1 = 0$	A1
	Solve a 3-term quadratic and find a value of $\theta$	M1
	Obtain answer $\theta = 26.6^\circ$ and no other	A1

10) JUNE 2021\_9709\_31 Q3

(a)	Use correct trig expansions and obtain an equation in $\sin x$ and $\cos x$	*M1
	Use correct exact trig ratios for $30^\circ$ in <i>their</i> expansion	B1 FT
	Obtain an equation in $\tan x$	DM1
	Obtain $\tan x = \frac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$ from correct working	A1
		4
(b)	Obtain answer in the given interval, e.g. $173.8^\circ$	B1
	Obtain a second answer and no other in the given interval, e.g. $353.8^\circ$	B1



11) JUNE 2021\_9709\_32 Q6(a)

(a)	Express the LHS in terms of $\cos 2\theta$ and $\sin 2\theta$	<b>B1</b>
	Use correct double angle formulae to express the LHS in terms of $\cos \theta$ and $\sin \theta$	<b>M1</b>
	Obtain $\tan \theta$ from correct working	<b>A1</b>
<b>Alternative method for Question 6(a)</b>		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	<b>B1</b>
	Use correct double angle formulae to express the LHS in terms of $\cos \theta$ and $\sin \theta$	<b>M1</b>
	Obtain $\tan \theta$ from correct working	<b>A1</b>
<b>Alternative method for Question 6(a)</b>		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	<b>B1</b>
	Use correct $t$ substitution or rearrangement of $\sin 2\theta$ in terms of $\sec^2 2\theta$ and $\tan \theta$ to express the LHS in terms of $\tan \theta$ .	<b>M1</b>
	Obtain $\tan \theta$ from correct working	<b>A1</b>

12) JUNE 2021\_9709\_33 Q5

(a)	Use double angle formula to express $\tan 4\theta$ in terms of $\tan 2\theta$	<b>M1</b>
	Use double angle formula to express result in terms of $\tan \theta$	<b>M1</b>
	Obtain a correct equation in $\tan \theta$ in any form	<b>A1</b>
	Obtain the given answer	<b>A1</b>
		<b>4</b>
(b)	Solve for $\tan \theta$ and obtain a value of $\theta$	<b>M1</b>
	Obtain answer, e.g. $53.5^\circ$	<b>A1</b>
	Obtain second answer, e.g. $126.5^\circ$ and no other in the interval	<b>A1</b>
		<b>3</b>

13) JUNE 2022\_9709\_31 Q3

Use correct trigonometric formulae to form an equation in $\tan x$	<b>*M1</b>
Obtain a correct linear equation in any form	<b>A1</b>
Reduce equation to a 3-term quadratic	<b>A1</b>
Solve a 3-term quadratic in $\tan x$ and obtain a value of $x$	<b>DM1</b>
Obtain answer, e.g. $x = 35.1^\circ$	<b>A1</b>
Obtain second answer, e.g. $x = 99.9^\circ$ , and no other in $(0^\circ, 180^\circ)$	<b>A1</b>
<b>Alternative method for question 3</b>	
Use correct formulae for $\sin 2x$ and $\cos 2x$ to form an equation in $\sin x$ and $\cos x$	<b>*M1</b>
Obtain $4 \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = 5$	<b>A1</b>
Reduce equation to a 3-term quadratic	<b>A1</b>
Solve a 3-term quadratic in $\tan x$ and obtain a value of $x$	<b>DM1</b>
Obtain answer, e.g. $x = 35.1^\circ$	<b>A1</b>
Obtain second answer, e.g. $x = 99.9^\circ$ , and no other in $(0^\circ, 180^\circ)$	<b>A1</b>
	<b>6</b>

14) JUNE 2022\_9709\_32 Q2

Use correct double-angle formula to obtain an equation in $\cos \theta$	<b>M1</b>
Obtain $6\cos^2\theta - 3\cos\theta - 5 = 0$ , or 3-term equivalent	<b>A1</b>
Solve a 3-term quadratic in $\cos \theta$ for $\theta$	<b>M1</b>
Obtain a correct answer, e.g. $134.1^\circ$	<b>A1</b>
Obtain a second answer, e.g. $225.9^\circ$ and no other in $[0^\circ, 360^\circ]$	<b>A1 FT</b>

15) JUNE 2022\_9709\_33 Q2

Use correct $\cos(A - B)$ formula to obtain an equation in $\cos \theta$ and $\sin \theta$	<b>B1</b>
Use trigonometric formula and substitute values for $\cos 60$ and $\sin 60$ to obtain an equation in $\tan \theta$ (or $\cos \theta$ or $\sin \theta$ )	<b>M1</b>
Obtain $\tan \theta = \frac{1}{6 - \sqrt{3}}$ or $\tan \theta = \frac{6 + \sqrt{3}}{33}$ or 0.2343, $\cos \theta = \frac{3\sqrt{3}}{\sqrt{10 - 3\sqrt{3}}}$ or 0.9736 or $\sin \theta = \frac{1}{\sqrt{10 - 3\sqrt{3}}}$ or 0.2281	<b>A1</b>
Obtain answer, e.g. $\theta = 13.2^\circ$	<b>A1</b>
Obtain second answer, e.g. $\theta = 193.2^\circ$ and no others in the given interval	<b>A1 FT</b>
<b>Alternative method for question 2 – using <math>R\cos(\theta \pm \alpha)</math> or equivalent</b>	
Use correct $\cos(A - B)$ formula to obtain an equation in $\cos \theta$ and $\sin \theta$	<b>B1</b>
Correct method for finding $\tan \alpha$ from $p\cos \theta + q\sin \theta = 0$	<b>M1</b>
Correct value of $\alpha$	<b>A1</b>
Obtain answer, e.g. $\theta = 13.2^\circ$	<b>A1</b>
Obtain second answer, e.g. $\theta = 193.2^\circ$ and no others in the given interval	<b>A1 FT</b>

16) JUNE 2023\_9709\_31 Q4

a)	Use correct double angle formulae	<b>M1</b>
	Obtain $\cos^2 \theta + 2\sin \theta \cos \theta - 3\sin^2 \theta = 0$ from <b>full and correct</b> working	<b>A1</b>
		<b>2</b>

(b)	Factorise to obtain $(\cos \theta - \sin \theta)(\cos \theta + 3 \sin \theta) = 0$	<b>B1</b>
	Solve a quadratic in $\sin \theta$ and $\cos \theta$ to obtain a value for $\theta$ .	<b>M1</b>
	Obtain one correct value e.g. $45^\circ$	<b>A1</b>
	Obtain a second correct value e.g. $161.6^\circ$ and no others in the interval	<b>A1</b>
<b>Alternative Method 1</b>		
	Obtain $3 \tan^2 \theta - 2 \tan \theta - 1 = 0$	<b>B1</b>
	Solve a 3 term quadratic in $\tan \theta$ to obtain a value for $\theta$ .	<b>M1</b>
	Obtain one correct value e.g. $45^\circ$	<b>A1</b>
	Obtain a second correct value e.g. $161.6^\circ$ and no others in the interval	<b>A1</b>
<b>Alternative Method 2</b>		
	Obtain $(\cos \theta + \sin \theta)^2 = (2 \sin \theta)^2$	<b>B1</b>
	Solve to obtain a value for $\theta$ .	<b>M1</b>
	Obtain one correct value e.g. $45^\circ$	<b>A1</b>
	Obtain a second correct value e.g. $161.6^\circ$ and no others in the interval	<b>A1</b>
		<b>4</b>

17) JUNE 2023\_9709\_32 Q4

Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only	<b>*M1</b>
Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$ ,	<b>A1</b>
Obtain $\cos\left(\frac{x}{2}\right) = -\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right) = 1$	<b>A1</b>
Solve for the <b>original</b> $x$	<b>DM1</b>
Obtain $x = 0$ and $4.84$ and no others in the interval	<b>A1</b>
<b>Alternative Method for Question 4</b>	
Use correct double angle formula to obtain an equation in $\cos x$ only	<b>*M1</b>
Obtain a 3 term quadratic in $\cos x$ ,	<b>A1</b>
Obtain $\cos x = \frac{1}{8}$ and $\cos x = 1$	<b>A1</b>
Solve for $x$	<b>DM1</b>
Obtain answers $x = 0$ and $4.84$ and no others in the interval	<b>A1</b>
	<b>5</b>

18) JUNE 2023\_9709\_33 Q6

a)	Expand $\cos(x - 60^\circ)$ correctly and evaluate $3 \cos x + 2 \cos(x - 60^\circ)$ to obtain $4 \cos x + \sqrt{3} \sin x$ or unsimplified coefficients	<b>B1</b>
	State $R = \sqrt{19}$ [ $R \cos \alpha = 4$ $R \sin \alpha = \sqrt{3}$ ]	<b>B1 FT</b>
	Use correct trig formulae for their expansion to find $\alpha$ e.g. $\alpha = \tan^{-1} \frac{\sqrt{3}}{4}$ or $\cos^{-1} \frac{4}{\sqrt{19}}$ or $\sin^{-1} \frac{\sqrt{3}}{\sqrt{19}}$	<b>M1</b>
	Obtain $\alpha = 23.41^\circ$	<b>A1</b>
		<b>4</b>
b)	$\cos^{-1}\left(\frac{2.5}{R}\right)$	<b>B1 FT</b>
	Use a correct method to find a value of $2\theta$ (not $x$ ) in the interval. Allow sign error in moving $\alpha$ to right side	<b>M1</b>
	Obtain one correct answer e.g. $39.2^\circ$	<b>A1</b>
	Obtain second correct answer e.g. $164.2^\circ$ and no others in the interval	<b>A1</b>
		<b>4</b>

19) OCT 2020\_9709\_31 Q6

a)	State $R = \sqrt{15}$	<b>B1</b>
	Use trig formulae to find $\alpha$	<b>M1</b>
	Obtain $\alpha = 50.77$	<b>A1</b>
		<b>3</b>
b)	Evaluate $\beta = \cos^{-1} \frac{2.5}{\sqrt{15}}$ ( $49.797^\circ$ to 4 d.p.)	<b>B1 FT</b>
	Use correct method to find a value of $\frac{x}{3}$ in the interval	<b>M1</b>
	Obtain answer rounding to $x = 301.6^\circ$ to $301.8^\circ$	<b>A1</b>
	Obtain second answer rounding to $x = 2.9(0)^\circ$ to $2.9(2)^\circ$ and no others in the interval	<b>A1</b>
		<b>4</b>

20) OCT 2020\_9709\_32 Q4

(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan \theta$	<b>M1</b>
	Use $\tan 60^\circ = \sqrt{3}$ and obtain a correct horizontal equation in any form	<b>A1</b>
	Reduce to $\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0$ correctly	<b>A1</b>
		<b>3</b>
(b)	Solve the given quadratic to obtain a value for $\theta$	<b>M1</b>
	Obtain one correct answer e.g. $\theta = 19.8^\circ$	<b>A1</b>
	Obtain second correct answer $\theta = 100.2^\circ$ and no others in the given interval	<b>A1</b>
		<b>3</b>

21) OCT 2021\_9709\_31 Q2

(a)	State $R = \sqrt{34}$	<b>B1</b>
	Use trig formulae to find $\alpha$	<b>M1</b>
	Obtain $\alpha = 0.54$	<b>A1</b>
		<b>3</b>
(b)	State greatest value 34	<b>B1 FT</b>
	State least value 0	<b>B1</b>
		<b>2</b>

22) OCT 2021\_9709\_31 Q5

(a)	Use correct trig formulae and express equation in terms of $\tan \theta$	<b>M1</b>
	Obtain a correct equation in $\tan \theta$ in any form	<b>A1</b>
	Reduce to $\tan^2 \theta + 4 \tan \theta - 3 = 0$ , or 3-term equivalent	<b>A1</b>
		<b>3</b>
(b)	Solve a 3-term quadratic for $\tan \theta$ and calculate $\theta$	<b>M1</b>
	Obtain answer, e.g. 0.573	<b>A1</b>
	Obtain second answer, e.g. 1.783 and no other	<b>A1</b>
		<b>3</b>

23) OCT 2021\_9709\_32 Q8

(a)	Expand the square and equate to 1	<b>B1</b>
	Use correct double angle formula	<b>M1</b>
	Obtain $\cos^4 \theta + \sin^4 \theta = 1 - \frac{1}{2} \sin^2 2\theta$	<b>A1</b>
		<b>3</b>
(b)	Use the identity and carry out a method for finding a root	<b>M1</b>
	Obtain answer $35.3^\circ$	<b>A1</b>
	Obtain a second answer, e.g. $54.7^\circ$	<b>A1 FT</b>
	Obtain the remaining answers, e.g. $144.7^\circ$ and $125.3^\circ$ and no others in the given interval	<b>A1 FT</b>

24) OCT 2021\_9709\_32 Q6(a)

	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	<b>B1</b>
	Substitute expansions in $\frac{1}{2}(\sin 5x + \sin x)$ , or equivalent	<b>M1</b>
	Simplify and obtain $\frac{1}{2}(\sin 5x + \sin x) = \sin 3x \cos 2x$	<b>A1</b>
		<b>3</b>

25) OCT 2021\_9709\_33 Q5

	Use double angle formula and obtain an equation in $\sin \theta$	<b>M1</b>
	Reduce to $6\sin^2 \theta + \sin \theta - 5 = 0$ , or 3-term equivalent	<b>A1</b>
	Solve a 3-term quadratic in $\sin \theta$ and calculate $\theta$	<b>M1</b>
	Obtain answer, e.g. $56.4^\circ$	<b>A1</b>
	Obtain second and third answers, e.g. $123.6^\circ$ and $270^\circ$ and no others in the given interval	<b>A1</b>
		<b>5</b>

26) OCT 2021\_9709\_33 Q6

(a)	Use $\cos(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	<b>M1</b>
	Collect terms and reach $2 \cos x + \sqrt{3} \sin x$	<b>A1</b>
	State $R = \sqrt{7}$	<b>A1</b>
	Use trig formula to find $\alpha$	<b>M1</b>
	Obtain $\alpha = 40.89^\circ$	<b>A1</b>
		<b>5</b>
(b)	Use correct method to find $x$	<b>M1</b>
	Obtain answer $x = 220.9^\circ$	<b>A1</b>
		<b>2</b>



## 27) OCT 2022\_9709\_31 Q4

Use correct $\tan(A+B)$ formula and obtain an equation in $\tan x$ or an equation in $\cos x$ and $\sin x$	<b>M1</b>
Obtain correct 3 term equation $\tan^2 x + 3 \tan x - 2 = 0$ , or equivalent	<b>A1</b>
Solve a 3-term quadratic in $\tan x$ and obtain a value for $x$	<b>M1</b>
Obtain answer, e.g. $29.3^\circ$	<b>A1</b>
Obtain second answer, e.g. $105.7^\circ$ and no other	<b>A1</b>
	<b>5</b>

## 28) OCT 2022\_9709\_31 Q6

(a)	Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	<b>B1</b>
	Express $\cos 2\theta$ in terms of $\cos \theta$ and/or $\sin \theta$	<b>B1</b>
	Expand to obtain a correct expression in terms of $\cos \theta$	<b>B1</b>
	Reduce correctly to $\cos 4\theta + 4 \cos 2\theta + 3 = 8 \cos^4 \theta$	<b>B1</b>
		<b>4</b>
(b)	Use the identity and carry out method to calculate a root	<b>M1</b>
	Obtain answer, e.g. $14.7^\circ$	<b>A1</b>
	Obtain second answer, e.g. $165.3^\circ$ , and no other in the given interval	<b>A1 FT</b>

## 29) OCT 2022\_9709\_32 Q4

(a)	State $R = \sqrt{17}$	<b>B1</b>
	Use correct trig formulae to find $\alpha$ (Correct expansion and correct expression for trig ratio for $\alpha$ )	<b>M1</b>
	Obtain $\alpha = 14.04^\circ$	<b>A1</b>
		<b>3</b>
(b)	Evaluate $\cos^{-1} \left( \frac{3}{\sqrt{17}} \right)$ to at least 1 d.p. ( $43.3138\dots^\circ$ )	<b>B1 FT</b>
	Use correct method to find a value of $x$ in the interval	<b>M1</b>
	Obtain answer, e.g. $14.6^\circ$	<b>A1</b>
	Use a correct method to find a second answer in the interval	<b>M1</b>
	Obtain second answer in the interval, e.g. $151.3^\circ$ , and no other in the interval	<b>A1</b>
		<b>5</b>

30) OCT 2022\_9709\_33 Q7

(a)	Rearrange and obtain $4\cos x - \sin x = \sqrt{5}$	<b>B1</b>
	State $R = \sqrt{17}$	<b>B1</b>
	Use trig formulae to find $\alpha$	<b>M1</b>
	Obtain $\alpha = 14.04^\circ$	<b>A1</b>
		<b>4</b>
(b)	Evaluate $\cos^{-1}\left(\frac{\sqrt{5}}{\sqrt{17}}\right)$	<b>B1 FT</b>
	Carry out a correct method to find a value of $x$ in the given interval	<b>M1</b>
	Obtain answer, e.g. $21.6^\circ$	<b>A1</b>
	Obtain a second answer, e.g. $144.4^\circ$ and no other in the interval	<b>A1</b>
		<b>4</b>