classmate

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Pure Maths - 3

Vectors.

Exercise 1 - Solution (Revision)

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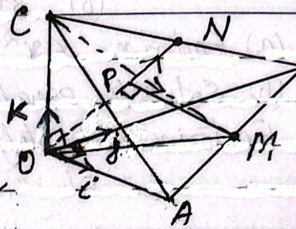
Vectors

Example 1: In the diagram, OABC is a pyramid in which OA = 2 units, OB = 4 units and OC = 2 units, the edge OC is vertical, the base OAB is horizontal and angle AOB = 90°. Unit vectors i, j, k are parallel to OA, OB and OC respectively. The mid points of AB and BC are M and N resp.

(a) Express the vectors $\vec{ON}$ and $\vec{CM}$ in terms of i, j and k. --[3]

(b) Calculate the angle between the directions of $\vec{ON}$ and $\vec{CM}$. --[3]

(c) Show that the length of perpendicular from M to ON is $\frac{3}{5}\sqrt{5}$. --[4]



[SP-20/03/Q8]

Solution: $\vec{OA} = 2i$, $\vec{OB} = 4j$, $\vec{OC} = 2k$

(a) $\vec{ON} = \frac{\vec{OC}+\vec{OB}}{2} = \frac{1}{2}(2k+4j) = (2j+k)$ ✓

$\vec{OM} = \frac{\vec{OA}+\vec{OB}}{2} = \frac{1}{2}(2i+4j) = (i+2j)$

$\vec{CM} = \vec{OM} - \vec{OC} = (i+2j-2k)$ ✓

(b) Angle between $\vec{ON}$ and $\vec{CM}$ is θ.

$\cos\theta = \frac{\vec{ON}\cdot\vec{CM}}{|\vec{ON}||\vec{CM}|} = \frac{(2j+k)\cdot(i+2j-2k)}{\sqrt{5}\sqrt{9}}$

$= \frac{4-2}{3\sqrt{5}} = \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{15}$

$\theta = \cos^{-1}\left(\frac{2\sqrt{5}}{15}\right) = \underline{72.7°}$ ✓

(C) Let MP is perp. to ON.

$\Rightarrow MP = \sqrt{OM^2 - OP^2}$ —— ①

Projection of $\vec{OM}$ on $\vec{ON}$ = OP

$= \frac{\vec{OM}\cdot\vec{ON}}{|\vec{ON}|} = \frac{(i+2j)\cdot(2j+k)}{\sqrt{5}}$

$OP = \frac{4}{\sqrt{5}}$ ✓

In rt triangle OPM

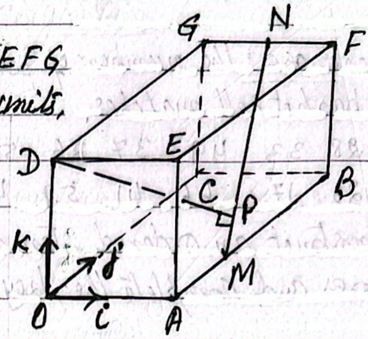
$MP = \sqrt{OM^2 - OP^2}$

$= \sqrt{5 - \frac{16}{5}} = \frac{3}{\sqrt{5}}$

Req perp. distance $= \frac{3\sqrt{5}}{5}$ ✓

Example 2:

In the diagram, OABCDEFG is a cuboid in which OA = 2 units, OC = 3 units and OD = 2 units. Unit vectors i, j and k are parallel to OA, OC and OD, respectively. The point M on AB is such that MB = 2AM. The mid point of FG is N.



(a) Express the vectors $\vec{OM}$ and $\vec{MN}$ in terms of i, j and k. --[3]

(b) Find a vector equation for the line through M and N. --[2]

(c) Find the position vector of P, the foot of perpendicular from D the line through M and N. [M-20/32/Q8] --[4]

Solution:

(a) $\vec{AM} = \frac{1}{3}\vec{AB} = \frac{1}{3}\times 3j = j$

$\vec{OM} = \vec{OA} + \vec{AM} = (2i + j)$ ✓ —— ①

$\vec{OF} = 2i + 3j + 2k$; $\vec{OG} = 3j + 2k$

$\Rightarrow \vec{ON} = \frac{1}{2}(\vec{OF} + \vec{OG})$

$= \frac{1}{2}(2i + 6j + 4k)$

$= (i + 3j + 2k)$ —— ②

fr ① and ②

$\vec{MN} = \vec{ON} - \vec{OM} = (-i + 2j + 2k)$ ✓

(b) Vector Equ^n of line MN.

$\vec{r} = \vec{OM} + \lambda(\vec{MN})$

$\vec{r} = (2i + j) + \lambda(-i + 2j + 2k)$ ✓ —— ③

(c) Any point P on MN fr ③

$\vec{OP} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2-\lambda \\ 1+2\lambda \\ 2\lambda \end{pmatrix}$ —— ④

$\vec{OD} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$ —— ⑤

fr ④ & ⑤ $\vec{DP} = \vec{OP} - \vec{OD}$

$= \begin{pmatrix} 2-\lambda \\ 1+2\lambda \\ -2+2\lambda \end{pmatrix}$ —— ⑥

Now $\vec{DP} \perp \vec{MN}$

$\Rightarrow \vec{DP} \cdot \vec{MN} = 0$

$\begin{pmatrix} 2-\lambda \\ 1+2\lambda \\ -2+2\lambda \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$

$\Rightarrow -2 + \lambda + 2 + 4\lambda - 4 + 4\lambda = 0$

$-4 + 9\lambda = 0$

$\Rightarrow \lambda = 4/9$

$\therefore$ from ④ Put $= 4/9$

$\vec{OP} = \begin{pmatrix} 2-\frac{4}{9} \\ 1+2\times\frac{4}{9} \\ 2\times\frac{4}{9} \end{pmatrix} = \begin{pmatrix} \frac{14}{9} \\ \frac{17}{9} \\ 8/9 \end{pmatrix}$

$\vec{OP} = \frac{14}{9}i + \frac{17}{9}j + \frac{8}{9}k$ ✓

3. Two lines have equations; $r = \begin{pmatrix}1\\3\\2\end{pmatrix} + s\begin{pmatrix}2\\-1\\3\end{pmatrix}$ and $r = \begin{pmatrix}2\\1\\4\end{pmatrix} + t\begin{pmatrix}-1\\1\\4\end{pmatrix}$

(a) Show that the lines are skew. ---[5]

(b) Find the acute angle between the directions of the two lines. ---[3]

[M-21/32/Q7]

Solution(a). To show lines are skew $\rightarrow$ Non-parallel, Non-intersecting

The lines are not parallel as their directions are not proportional. $\begin{pmatrix}2\\-1\\3\end{pmatrix} \neq k\begin{pmatrix}1\\-1\\4\end{pmatrix}$; $k \neq 0$

Now for the intersection of L_1 and L_2.

$$\Rightarrow \begin{pmatrix}1\\3\\2\end{pmatrix} + s\begin{pmatrix}2\\-1\\3\end{pmatrix} = \begin{pmatrix}2\\1\\4\end{pmatrix} + t\begin{pmatrix}1\\-1\\4\end{pmatrix} \Rightarrow \begin{pmatrix}1+2s\\3-s\\2+3s\end{pmatrix} = \begin{pmatrix}2+t\\1-t\\4+4t\end{pmatrix}$$

$$\Rightarrow \begin{cases} 1+2s = 2+t \\ 3-s = 1-t \\ 2+3s = 4+4t \end{cases} \Rightarrow \begin{cases} t = 2s-1 & ---(1) \\ t = s-2 & ---(2) \\ 4t = 3s-2 & ---(3) \end{cases}$$

Solving equations (1) & (2) $\Rightarrow s = -1,\ t = -3$ ✓

Put $s = -1$ and $t = -3$ in (3) $\Rightarrow 4\times(-3) = 3(-1)-2 \Rightarrow -12 = -5$ false ✓

$\therefore$ Two lines have no point in common.

$\therefore$ Two lines are **skew lines**.

(b) To find the acute angle between the two lines: $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$$\vec{a} = \begin{pmatrix}2\\-1\\3\end{pmatrix};\ \vec{b} = \begin{pmatrix}1\\-1\\4\end{pmatrix}$$

$$\vec{a}\cdot\vec{b} = \begin{pmatrix}2\\-1\\3\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\4\end{pmatrix} = 2\times1 + (-1)(-1) + 3\times4 = 2+1+12 = 15 ✓$$

$|\vec{a}| = \sqrt{4+1+9} = \sqrt{14}$ and $\sqrt{b} = 1+1+16 = \sqrt{18}$

$$\therefore \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{15}{\sqrt{14}\cdot\sqrt{18}} = \frac{15}{\sqrt{252}} = 0.9449$$

$$\therefore \theta = \cos^{-1}0.9449 = \underline{19.1^\circ} ✓$$

4. The points A and B have position vectors $2i+j+k$ and $i-2j+2k$ respectively. The line l has vector equation,
$$r = (i+2j-3k)+\mu(i-3j-2k)$$
(a) Find the vector equation for the line through A and B --- [3]
(b) Find the acute angle between the directions of AB and l, giving your answer in degrees. --- [3]
(c) Show that the line through A and B does not intersect the line l. --- [4]

[M-22 / 32 / Q 10]

Solution: $\vec{OA}=(2i+j+k)$, $\vec{OB}=(i-2j+2k) \Rightarrow \vec{AB}=(-i-3j+k)$ — ①

$\therefore$ Equation of line AB: $r = \vec{OA}+\lambda\vec{AB}$

$\Rightarrow \underline{r=(2i+j+k)+\lambda(-i-3j+k)}$ — ②

(b) $l: r=(i+2j-3k)+\mu(i-3j-2k)$ — ③

Angle 'θ' between $\vec{AB}$ and l

$\left\{\begin{array}{l} \text{direction of } \vec{AB};\ \vec{V_1}=(-i-3j+k) \\ \text{direction of } l;\ \vec{V_2}=(i-3j-2k) \\ |\vec{V_1}|=\sqrt{1^2+3^2+1^2}=\sqrt{11} \\ |\vec{V_2}|=\sqrt{1^2+3^2+2^2}=\sqrt{14} \end{array}\right.$

$$\cos\theta = \frac{\vec{V_1}\cdot\vec{V_2}}{|\vec{V_1}||\vec{V_2}|}$$

$$= \frac{(-i-3j+k)\cdot(i-3j-2k)}{\sqrt{11}\sqrt{14}} = \frac{-1+9-2}{\sqrt{11}\sqrt{14}} = \frac{6}{\sqrt{154}}$$

$$\therefore \theta = \cos^{-1}\left(\frac{6}{\sqrt{154}}\right) = \cos^{-1}(0.4835) = 61.086$$

$$\Rightarrow \underline{\theta = 61.1^\circ}$$

(c) Any point of line AB $=\begin{pmatrix}2\\1\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\-3\\1\end{pmatrix}=\begin{pmatrix}2-\lambda\\1-3\lambda\\1+\lambda\end{pmatrix}$ — ④

Any point of line l $=\begin{pmatrix}1\\2\\-3\end{pmatrix}+\mu\begin{pmatrix}1\\-3\\-2\end{pmatrix}=\begin{pmatrix}1+\mu\\2-3\mu\\-3-2\mu\end{pmatrix}$ — ⑤

from ④ and ⑤

for $\vec{AB}$ and l to intersect $\begin{pmatrix}2-\lambda\\1-3\lambda\\1+\lambda\end{pmatrix}=\begin{pmatrix}1+\mu\\2-3\mu\\-3-2\mu\end{pmatrix} \Rightarrow$
$\lambda+\mu=1$ — ⑥
$-3\lambda+3\mu=1$ — ⑦
$\lambda+2\mu=-4$ — ⑧

Solving ⑥ & ⑦, $\lambda=\frac{1}{3}$, $\mu=2/3$

Put $\lambda=1/3$, $\mu=2/3$ in ⑧ $\Rightarrow$ $5/3=-4$ false.

$\therefore$ Equation ⑥, ⑦, ⑧ are not consistent $\Rightarrow$ lines AB and l do not intersect.

5. With respect to the origin O, the points A, B, C and D have position vectors:
$\vec{OA}=\begin{pmatrix}3\\-1\\2\end{pmatrix}$, $\vec{OB}=\begin{pmatrix}1\\2\\-3\end{pmatrix}$, $\vec{OC}=\begin{pmatrix}1\\-2\\5\end{pmatrix}$ and $\vec{OD}=\begin{pmatrix}5\\-6\\11\end{pmatrix}$.

(a) Find the obtuse angle between the vectors $\vec{OA}$ and $\vec{OB}$. ---[3]

The line l passes through the points A and B.

(b) Find the vector equation for the line l. ---[2]

(c) Find the position vector of the point of intersection of the line l and the line passing through C and D. ---[4]

M-23/32/Q10

Solution: $\vec{OA}\cdot\vec{OB}=\begin{pmatrix}3\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}1\\2\\-3\end{pmatrix}=3-2-6$
$=-5$

(a) $|\vec{OA}|=\sqrt{3^2+(-1)^2+2^2}=\sqrt{14}$

$|\vec{OB}|=\sqrt{1^2+2^2+(-3)^2}=\sqrt{14}$

angle 'θ' between $\vec{OA}$ and $\vec{OB}$:

$\cos\theta=\dfrac{\vec{OA}\cdot\vec{OB}}{|\vec{OA}|\cdot|\vec{OB}|}=\dfrac{-5}{\sqrt{14}\sqrt{14}}=\dfrac{-5}{14}$
$=-\cos 1.205$

$\theta=\pi-1.205$

$\theta=1.94$ radians.

(b) $\vec{AB}=\vec{OB}-\vec{OA}=\begin{pmatrix}1\\2\\-3\end{pmatrix}-\begin{pmatrix}3\\-1\\2\end{pmatrix}=\begin{pmatrix}-2\\3\\-5\end{pmatrix}$

Equation of line l ($\vec{AB}$)

$\vec{r}=\vec{OA}+\lambda\vec{AB}$

$\vec{r}=(3i-j+2k)+\lambda(-2i+3j-5k)$

(c) Any point on line $\vec{AB}=\begin{pmatrix}3-2\lambda\\-1+3\lambda\\2-5\lambda\end{pmatrix}$ — (1)

Equation of line CD

$\vec{r}=\vec{OC}+\mu\vec{CD}$

$=\begin{pmatrix}1\\-2\\5\end{pmatrix}+\mu\begin{pmatrix}4\\-4\\6\end{pmatrix}$

$\left(\vec{CD}=\vec{OD}-\vec{OC}=\begin{pmatrix}5\\-6\\11\end{pmatrix}-\begin{pmatrix}1\\-2\\5\end{pmatrix}=\begin{pmatrix}4\\-4\\6\end{pmatrix}\right.$

any point on $\vec{CD}=\begin{pmatrix}1+4\mu\\-2-4\mu\\5+6\mu\end{pmatrix}$ — (2)

for the point of intersection of lines l and $\vec{CD}$; from (1) & (2)

$\begin{pmatrix}3-2\lambda\\-1+3\lambda\\2-5\lambda\end{pmatrix}=\begin{pmatrix}1+4\mu\\-2-4\mu\\5+6\mu\end{pmatrix}$

$\Rightarrow 3-2\lambda=1+4\mu\Rightarrow 2\lambda+4\mu=2$ --- (3)

$-1+3\lambda=-2-4\mu\Rightarrow 3\lambda+4\mu=-1$ — (4)

$2-5\lambda=5+6\mu\Rightarrow 5\lambda+6\mu=-3$ — (5)

Solving (3) & (4) $\lambda=-3$ and $\mu=2$ — (6)

for $\lambda=-3$ and $\mu=2$ put in (5)

$5(-3)+6\times 2=-3$ True

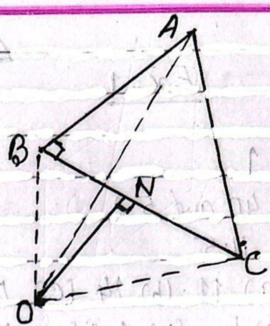
Hence the lines intersect

Put $\lambda=-3$ in (1) (or $\mu=2$ in (2))

$\vec{r}=\begin{pmatrix}9\\-10\\17\end{pmatrix}$

$\therefore$ Required Point of Intersection $=9i-10j+17k$

Vectors



Example 6: With respect to the origin O, the vertices of a triangle ABC have position vectors, $\vec{OA} = 2i + 5k$, $\vec{OB} = 3i + 2j + 3k$, $\vec{OC} = i + j + k$.

(a) Using a scalar product, show that angle ABC is a right angle. --[3]

(b) Show that triangle ABC is isosceles. --[2]

(c) Find the exact length of perpendicular from O to the line through B and C. --[4]

[S-20/31/Q9]

Solution: $\vec{OA} = 2i+5k$, $\vec{OB} = 3i+2j+3k$, $\vec{OC} = i+j+k$

(a) $\vec{BA} = \vec{OA} - \vec{OB} = (2i+5k) - (3i+2j+3k) = (-i-2j+2k)$

$\vec{BC} = \vec{OC} - \vec{OB} = (i+j+k) - (3i+2j+3k) = (-2i-j-2k)$

Now $\vec{BA} \cdot \vec{BC} = (-i-2j+2k)\cdot(-2i-j-2k)$

$= (-1)\cdot(-2) + (-2)(-1) + (2)\cdot(-2)$

$= 2+2-4 = 0 \Rightarrow$ angle ABC is a right angle.

(b) $|\vec{BA}| = |-i-2j+2k| = \sqrt{1^2+2^2+2^2} = 3$ —— ①

$|\vec{BC}| = |-2i-j-2k| = \sqrt{2^2+1^2+2^2} = 3$ —— ②

from ① & ② Sides BA = BC

Hence triangle ABC is isosceles.

(c) Let $\vec{ON} \perp \vec{BC}$ Req. is the length ON = ?

Projection of $\vec{OC}$ on $\vec{BC}$ or $|\vec{NC}| = \left|\frac{\vec{OC}\cdot\vec{BC}}{|\vec{BC}|}\right| = \left|\frac{(i+j+k)\cdot(-2i-j-2k)}{\sqrt{2^2+1^2+2^2}}\right|$

$= \left|\frac{-2-1-2}{3}\right| = 5/3$

and $|\vec{OC}| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

In rt. triangle ONC.

$ON = \sqrt{OC^2 - NC^2}$

$= \sqrt{(\sqrt{3})^2 - \left(\frac{5}{3}\right)^2}$

$= \sqrt{3 - \frac{25}{9}} = \sqrt{\frac{2}{9}} = \frac{1}{3}\sqrt{2}$

Example 7: With respect to the origin O, the points A and B have position vectors given by $\vec{OA} = 6i+2j$ and $\vec{OB} = 2i+2j+3k$. The mid point of OA is M. The point N lying on AB, between A and B, is such that AN = 2NB.

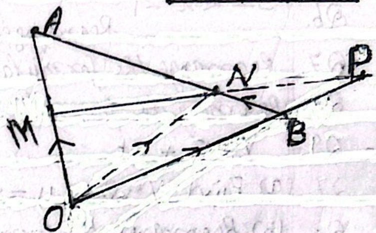
(a) Find a vector equation for the line through M and N. ---[5]

The line through M and N intersects the line through O and B at the point P.

(b) Find the position vector of P. ---[3]

(c) Calculate angle OPM, giving your answer in degrees. --[3]

[S-20/32/Q10]



Solution (a) $\vec{OA} = 6i+2j$, $\vec{OB} = 2i+2j+3k$.

M is the mid point of OA.

$\vec{OM} = (3i+j)$ ✓ —— ①

$\vec{AB} = \vec{OB} - \vec{OA} = (-4i+3k)$

$\vec{BA} = (4i-3k)$

$\vec{BN} = \frac{1}{3}\vec{BA} = (\frac{4}{3}i - k)$

$\vec{ON} = \vec{OB} + \vec{BN}$

$= (2i+2j+3k)+(\frac{4}{3}i-k)$

$\vec{ON} = (\frac{10}{3}i+2j+2k)$ ✓ —— ②

$\vec{MN} = \vec{ON} - \vec{OM}$

$= (\frac{10}{3}i+2j+2k)-(3j+j)$

$= (\frac{1}{3}i+j+2k)$ ✓

∴ Equation of Line MN

$\vec{r} = \vec{OM} + \lambda\vec{MN}$

$= (3i+j)+\lambda(\frac{1}{3}i+j+2k)$ ✓ —— ③

(b) Equation of OB, $\vec{r} = \mu(2i+2j+3k)$ —— ④

for the point of Intersection of MN and OB

from ③ & ④,

$$\begin{pmatrix} 3+\frac{1}{3}\lambda \\ 1+\lambda \\ 2\lambda \end{pmatrix} = \begin{pmatrix} 2\mu \\ 2\mu \\ 3\mu \end{pmatrix} \quad —— ⑤$$

from ⑤

$1+\lambda = 2\mu \Rightarrow 2\mu - \lambda = 1$ —— ⑥

and $2\lambda = 3\mu \Rightarrow \mu = \frac{2}{3}\lambda$ —— ⑦

Solve ⑥ and ⑦ $\lambda = 3, \mu = 2$

from ⑤

$\vec{OP} = (4i+4j+6k)$ ✓

(c) angle OPM = θ

$$\cos\theta = \vec{PO}\cdot\vec{PN} = \frac{\vec{PO}\cdot\vec{NM}}{|\vec{PO}||\vec{NM}|}$$

$$= \frac{\frac{4}{3}+4+12}{\sqrt{\frac{1}{9}+1+4}\sqrt{16+16+36}}$$

$$= \frac{52/3}{\sqrt{\frac{46}{9}\times 68}} = \frac{17.33}{18.64}$$

$\cos\theta = 0.9297$

$\theta = 21.6°$ ✓

Example 8 Relative to the origin O, the points A, B and D have position vectors given by:

$\vec{OA} = i + 2j + k$, $\vec{OB} = 2i + 5j + 3k$ and $\vec{OD} = 3i + 2k$

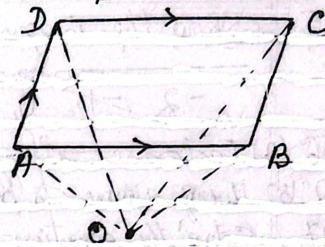
A fourth point C is such that ABCD is a parallelogram,

(a) Find the position vector of C and verify that the parallelogram is not a rhombus. --- [5]

(b) Find the angle BAD, giving your answer in degrees. --- [3]

(c) Find the area of the parallelogram correct to 3 s.f. --- [2]

[S-20/33/Q 8]



Solution (a) $\vec{OA} = i + 2j + k$, $\vec{OB} = 2i + 5j + 3k$

$\vec{OD} = 3i + 2k$, Let $\vec{OC} = (ai + bj + ck)$

$\vec{AB} = \vec{OB} - \vec{OA} = (i + 3j + 2k)$

$\vec{DC} = \vec{OC} - \vec{OD} = ((a-3)i + bj + (c-2)k)$

for given ABCD is a parallelogram.

$\vec{DC} = \vec{AB}$

$$\begin{pmatrix} a-3 \\ b \\ c-2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \Rightarrow \left.\begin{matrix} a-3=1 \\ b=3 \\ c-2=2 \end{matrix}\right\} \Rightarrow \vec{OC}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \\ 4 \end{pmatrix}$$

$\therefore \vec{OC} = 4i + 3j + 4k$ ✓

$AB = \sqrt{14}$

$AD = 3$

$AB \neq AD$

$\therefore$ ABCD is not a rhombus

(b) Let angle $BAD = \theta$

$$\cos\theta = \frac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}||\vec{AD}|} = \frac{(i+3j+2k)\cdot(2i-2j+k)}{\sqrt{1^2+3^2+2^2}\sqrt{2^2+2^2+1^2}}$$

$$\cos\theta = \frac{2-6+2}{\sqrt{14}\sqrt{9}} = \frac{-2}{\sqrt{126}}$$

$$\cos\theta = \frac{-2}{11.23} = -0.1781$$

$\theta = \cos^{-1}(-0.1781)$

$= 180 - (79.74)$

$\theta = \underline{100.3^\circ}$

(c) Area of Parallelogram ABCD

$= AB \times AD \sin\theta$

$= \sqrt{14} \times 3 \times \sin 100.3^\circ$

$= 11$ Sq unit ✓

$= \underline{11}$ ✓

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9. With respect to the origin O, the points A and B have position vectors given by: $\vec{OA} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\vec{OB} = \begin{pmatrix}3\\1\\-2\end{pmatrix}$ and the line l has equation: $\vec{r} = \begin{pmatrix}2\\3\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\1\end{pmatrix}$

(a) Find the acute angle between the lines AB and l. --- [4]

(b) Find the position vector of the point P on l such that AP = BP --- [5]

S-21/31/Q 8

Solution: (a) $\vec{OA} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\vec{OB} = \begin{pmatrix}3\\1\\-2\end{pmatrix} \Rightarrow \vec{AB} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}$ — ①

Line l: $\vec{r} = \begin{pmatrix}2\\3\\1\end{pmatrix} + \lambda\begin{pmatrix}1\\-2\\1\end{pmatrix}$ — ②

Angle between the lines AB & l, 'θ'

$$\cos\theta = \frac{\vec{V_1}\cdot\vec{V_2}}{|\vec{V_1}||\vec{V_2}|} =$$

$\vec{V_1} = \sqrt{2^2+1^2+3^2} = \sqrt{14}$

$|\vec{V_2}| = \sqrt{1^2+2^2+1^2} = \sqrt{6}$

$$= \frac{1}{\sqrt{14}\cdot\sqrt{6}}\begin{pmatrix}2\\-1\\-3\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix}$$

$$= \frac{(2+2-3)}{\sqrt{84}} = \frac{1}{\sqrt{84}} =$$

$\cos\theta = 0.1091$

$\theta = \cos^{-1}(0.1091) = \underline{83.7^\circ}$

(b) Any point P on l;

$\vec{OP} = \begin{pmatrix}2+\lambda\\3-2\lambda\\1+\lambda\end{pmatrix}$ — ③

$\vec{OA} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\vec{OB} = \begin{pmatrix}3\\1\\-2\end{pmatrix}$

$\vec{AP} = \begin{pmatrix}1+\lambda\\1-2\lambda\\\lambda\end{pmatrix}$ and $\vec{BP} = \begin{pmatrix}-1+\lambda\\2-2\lambda\\3+\lambda\end{pmatrix}$

Given $|\vec{AP}| = \vec{BP}$

$\Rightarrow AP^2 = BP^2$

$(1+\lambda)^2 + (1-2\lambda)^2 + \lambda^2 = (-1+\lambda)^2 + (2-2\lambda)^2 + (3+\lambda)^2$

$\Rightarrow 2+6\lambda^2-2\lambda = 14+6\lambda^2-4\lambda$

$\Rightarrow 2\lambda = 12 \Rightarrow \lambda = 6$

from $\vec{OP} = (8i - 9j + 7k)$

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10 With respect to the origin O, the points A and B have position vectors given by $\vec{OA} = 2i - j$ and $\vec{OB} = j - 2k$.

(a) Show that OA = OB and use a scalar product to calculate angle AOB in degrees. --[4]

The mid point of AB is M. The point P on the line through O and M is such that PA : OA = $\sqrt{7}$: 1.

(b) Find the position position vectors of P. --[6]

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Solution (a) $\vec{OA} = 2i - j$ —① $\Rightarrow OA = \sqrt{2^2+1^2} = \sqrt{5}$

$\vec{OB} = j - 2k$ —② $\Rightarrow OB = \sqrt{1^2+2^2} = \sqrt{5}$

$\therefore OA = OB = \sqrt{5}$ ✓

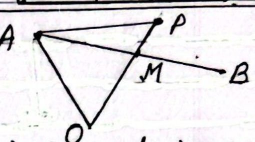
$\cos AOB = \dfrac{\vec{OA}\cdot\vec{OB}}{|\vec{OA}||\vec{OB}|} = \dfrac{(2i-j)\cdot(j-2k)}{\sqrt{5}\cdot\sqrt{5}}$

$= \dfrac{0-1+0}{5} = -0.2$

$= -\cos 78.46$

$\therefore$ angle AOB $= 180 - 78.46 = 101.5^\circ$

(b)



$\vec{OM} = \frac{1}{2}(\vec{OA} + \vec{OB}) = \frac{1}{2}(2i - 2k)$

$\Rightarrow \vec{OM} = (i - k)$ —③

P lies on $\vec{OM}$;

$\Rightarrow \vec{OP} = \lambda(i - k)$ —④

$\vec{PA} = (2-\lambda)i - j + \lambda k$

$|\vec{PA}| = \sqrt{(2-\lambda)^2 + 1^2 + \lambda^2}$

$PA^2 = (2\lambda^2 - 4\lambda + 5)$ —⑤

and $OA^2 = 5$ —⑥

Given PA : OA = $\sqrt{7}$: 1

$\Rightarrow \dfrac{PA^2}{OA^2} = 7$

$\Rightarrow \dfrac{2\lambda^2 - 4\lambda + 5}{5} = 7 \Rightarrow 2\lambda^2 - 4\lambda + 5 = 35$

$2\lambda^2 - 4\lambda - 30 = 0$

$\lambda^2 - 2\lambda - 15 = 0$

$(\lambda - 5)(\lambda + 3) = 0$

$\lambda = 5, \ \lambda = -3$

from ④

(i) for $\lambda = 5$

$\vec{PA} = (5i - 5k)$ ✓ and for $\lambda = -3$

$\vec{PA} = (-3i + 3k)$ ✓

11 The quadrilateral ABCD is a trapezium in which AB and DC are parallel. With respect to the origin O, the position vectors of A, B and C are given by $\vec{OA} = -i + 2j + 3k$, $\vec{OB} = i + 3j + k$ and $\vec{OC} = 2i + 2j - 3k$

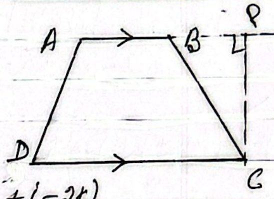
(a) Given that $\vec{DC} = 3\vec{AB}$, find the position vector of D. ---[3]

(b) State a vector equation for the line through A and B. ---[1]

(c) Find the distance between the parallel sides and hence find the area of the trapezium. ---[5]

S-21/33/Q9

Solution: $\vec{OA} = -i + 2j + 3k$; $\vec{OB} = i + 3j + k$, $\vec{OC} = 2i + 2j - 3k$



(a) $\vec{AB} = \vec{OB} - \vec{OA} = 2i + j - 2k$ — ① and $\vec{DC} = 3\vec{AB}$

Let the Position vector of D, $\vec{OD} = (ai + bj + ck)$ — ②

$\Rightarrow \vec{DC} = (2-a)i + (2-b)j + (-3-c)k$ — ③

Given $\vec{DC} = 3\vec{AB} \Rightarrow (2-a)i + (2-b)j + (-3-c)k = 3(2i + j - 2k)$

$\Rightarrow \begin{cases} 2-a = 6 \\ 2-b = 3 \\ -3-c = -6 \end{cases} \Rightarrow \begin{array}{l} a = -4 \\ b = -1 \\ c = 3 \end{array}$

∴ from ② Position Vector of D.

$\vec{OD} = -4i - j + 3k$ — ④

(b) Vector equation of line AB, $\vec{r} = \vec{OA} + \lambda(\vec{AB})$

or $\vec{r} = (-i + 2j + 3k) + \lambda(2i + j - 2k)$ — ⑤

(c) Let 'P' is any point of AB $\rightarrow \vec{OP} = (-1+2\lambda, 2+\lambda, 3-2\lambda)$

$\vec{OC} = (2i + 2j - 3k)$

$\Rightarrow \vec{PC} = (3-2\lambda, -\lambda, -6+2\lambda)$ — ⑥

for $\vec{PC} \perp \vec{AB} \Rightarrow \begin{pmatrix} 3-2\lambda \\ -\lambda \\ -6+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix} = 0 \Rightarrow 6 - 4\lambda - \lambda + 12 - 4\lambda = 0$

$\vec{PC} \cdot \vec{AB} = 0$ $\qquad -9\lambda = -18 \Rightarrow \lambda = 2$

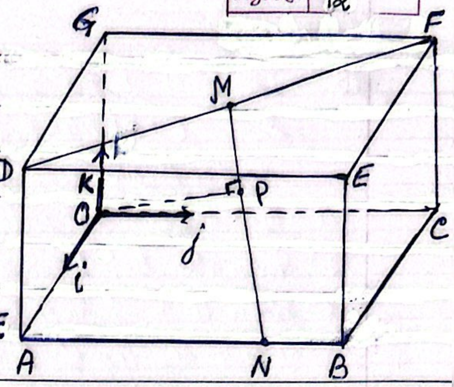
for $\lambda = 2$, $\vec{PC} = (-i - 2j - 2k)$

∴ $|\vec{PC}| = \sqrt{1^2 + 2^2 + 2^2} = 3$ (distance between the parallel lines AB & DC).

$|\vec{AB}| = \sqrt{2^2 + 1^2 + 2^2} = 3$; $|\vec{DC}| = 3|\vec{AB}| = 3 \times 3 = 9$

∴ Area of Trapezium $= \frac{1}{2} \times 3 \times (3 + 9) = $ **18 sq units.**

12 The diagram OABCDEFG is a Cuboid in which OA = 2 units, OC = 4 and OG = 2 units. Unit vectors i, j, k are parallel to OA, OC and OG respectively. The point M is the midpoint of DF. The point N is on AB such that AN = 3NB.



(a) Express the vectors $\vec{OM}$ and $\vec{MN}$ in terms of i, j and k. ---[3]

(b) Find a vector equation for the line through M and N. ---[2]

(C) Show that the length of perpendicular from O to the line through M and N is $\sqrt{\frac{53}{6}}$. ---[4]

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Solution:

(a) $\vec{OM} = \vec{OD} + \vec{OF}$

$= \frac{(2i + 2k) + (4j + 2k)}{2}$

$\vec{OM} = i + 2j + 2k$ ---①

Now:

$\vec{MN} = \vec{ON} - \vec{OM}$

$(\vec{AN} = \frac{3}{4}\vec{AB})$ $= (2i + 3j) - (i + 2j + 2k)$

$= i + j - 2k$ ---②

(b) Vector equation of line $\vec{r} = \vec{a} + \lambda\vec{v}$

Vector equation of MN

$\vec{r} = \vec{ON} + \lambda\,\vec{MN}$ ---③

$\Rightarrow \vec{r} = (2i + 3j) + \lambda(i + j - 2k)$

(c) Let $\vec{OP}$ is perpendicular to $\vec{MN}$

P is the foot of perp. on MN

$\vec{OP} = \begin{pmatrix} 2+\lambda \\ 3+\lambda \\ -2\lambda \end{pmatrix}$ ---④ [P lies on $\vec{MN}$ from ③

$\vec{OP} \perp \vec{MN} = \vec{OP} \cdot \vec{v} = 0$

$\vec{v} = (i + j - 2k)$

$\Rightarrow (2+\lambda)\times 1 + (3+\lambda)\times 1 - 2\times(-2\lambda) = 0$

$6\lambda + 5 = 0$

$\lambda = -\frac{5}{6}$

from ④

$\vec{OP} = \begin{pmatrix} 2 - 5/6 \\ 3 - 5/6 \\ -2\times(-\frac{5}{6}) \end{pmatrix} = \begin{pmatrix} 7/6 \\ 13/6 \\ \frac{10}{6} \end{pmatrix}$

$\therefore$ length OP $= |\vec{OP}|$

$= \sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{13}{6}\right)^2 + \left(\frac{10}{6}\right)^2}$

$= \sqrt{\frac{318}{36}}$

$= \sqrt{\frac{53}{6}}$

13 The lines l and m have vector equations:
$r = -i+3j+4k+\lambda(2i-j-k)$ and $r = 5i+4j+3k+\mu(ai+bj+k)$
respectively, where a and b are constants.

(a) Given l and m intersect, show that $2b-a=4$ ---[4]

(b) Given also that l and m are perpendicular, find the values of a and b. ---[4]

(c) When a and b have those values, find the position vector of the point of intersection of l and m. ---[2]

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Solution: $l: \vec{r} = -i+3j+4k+\lambda(2-j-k)$ --- ① any point on l: $\begin{pmatrix}-1+2\lambda\\3-\lambda\\4-\lambda\end{pmatrix}$ --- ③

$m: \vec{r} = 5i+4j+3k+\mu(ai+bj+k)$ --- ②

any point of m: $\begin{pmatrix}5+a\mu\\4+b\mu\\3+\mu\end{pmatrix}$ --- ④

Now for l and m to intersect,

from ③ & ④ $\begin{pmatrix}-1+2\lambda\\3-\lambda\\4-\lambda\end{pmatrix} = \begin{pmatrix}5+a\mu\\4+b\mu\\3+\mu\end{pmatrix} \Rightarrow \left.\begin{array}{l}-1+2\lambda = 5+a\mu\\3-\lambda = 4+b\mu\\4-\lambda = 3+\mu\end{array}\right\} \Rightarrow \begin{array}{ll}2\lambda - a\mu = 6 & \text{--- ⑤}\\-\lambda - b\mu = 1 & \text{--- ⑥}\\\lambda+\mu = 1 & \text{--- ⑦}\end{array}$

Equation ⑤ + 2×⑥ $\Rightarrow -\mu(a+2b) = 8 \Rightarrow \mu(a+2b) = -4$ --- ⑧

⑥ + ⑦ $\Rightarrow \mu(1-b) = 2 \Rightarrow \mu = \frac{2}{1-b}$ --- ⑨

fr ⑧ & ⑨ $\frac{2}{(1-b)}(a+2b) = -4 \Rightarrow a+2b = -4(1-b)$

$\Rightarrow a+2b = -4+4b$

$\Rightarrow \underline{2b-a=4}$ ✓ --- ⑩

(b) $l \perp m \Rightarrow (2i-j-k)\cdot(ai+bj+k) = 0 \Rightarrow 2a-b-1 = 0 \Rightarrow b = 2a-1$ --- ⑪

for ⑩ and ⑪ $2(2a-1) - a = 4 \Rightarrow 3a = 6 \Rightarrow \underline{a=2}$ ✓

fr ⑪ $b = 2\times2-1 = \underline{3}$ ✓

(c) from ⑨ $\mu = \frac{2}{1-b} = \frac{2}{1-3} = -1$ (for $b=3$)

from ⑦ $\lambda+\mu = 1 \Rightarrow \lambda - 1 = 1 \Rightarrow \lambda = 2$ ✓ $(\mu = -1)$

Put $\lambda = 2$ in ③ point of intersection $= \begin{pmatrix}-1+4\\3-2\\4-2\end{pmatrix} = \begin{pmatrix}3\\1\\2\end{pmatrix}$

The position vector of the point of intersection of l and m is, $\underline{(3i+j+2k)}$

14 With respect to the origin O, the point A has position vector given by $\overrightarrow{OA} = i + 5j + 6k$. The line l has vector equation:

$$r = 4i + k + \lambda(-i + 2j + 3k)$$

(a) Find in degrees the acute angle between the directions of OA and l, ---[3]

(b) Find the position vector of the foot of perpendicular from A to l. ---[4]

(c) Hence find the position vector of the reflection of A in l. ---[2]

[S-22/33/Q9]

Solution: $\overrightarrow{OA} = i + 5j + 6k$ ---①

(a) Line l: $r = 4i + k + \lambda(-i + 2j + 3k)$ --② $[\vec{r} = a + \lambda\vec{v}]$

angle θ, between $\overrightarrow{OA}$ & l: $\cos\theta = \dfrac{\overrightarrow{OA}\cdot\vec{v}}{|\overrightarrow{OA}||\vec{v}|} = \dfrac{(i+5j+6k)\cdot(-i+2j+3k)}{\sqrt{1^2+5^2+6^2}\cdot\sqrt{1^2+2^2+3^2}}$

$\Rightarrow \cos\theta = \dfrac{-1+10+18}{\sqrt{62}\sqrt{14}} = \dfrac{27}{\sqrt{868}} = 0.9164$

$\Rightarrow \theta = \cos^{-1} 0.9164 = \underline{23.6^\circ}$ ✓

(b) Any point P on line 'l'

$P(4-\lambda, +2\lambda, 1+3\lambda)$ and $A(1,5,6)$ --- ②

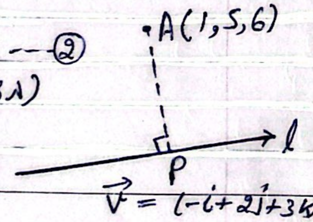
$\Rightarrow \overrightarrow{AP}$ has components $(3-\lambda, -5+2\lambda, -5+3\lambda)$

$\overrightarrow{AP} \perp$ line $l \Rightarrow \overrightarrow{AP}\cdot\vec{v} = 0$

$\Rightarrow (3-\lambda, -5+2\lambda, -5+3\lambda)\cdot(-1, 2, 3) = 0$

$\Rightarrow -3+\lambda - 10 + 4\lambda - 15 + 9\lambda = 0$

$\Rightarrow 14\lambda - 28 = 0 \Rightarrow \lambda = 2$



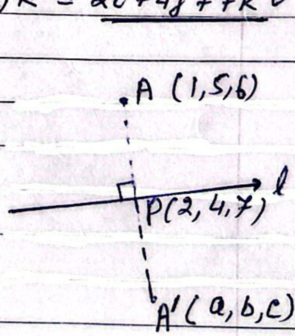
for $\lambda = 2 \Rightarrow$ from ② $\overrightarrow{OP} = (4-2)i + 2\times2j + (1+3\times2)k = \underline{2i + 4j + 7k}$ ✓

(c) $A(1,5,6)$, $P(2,4,7)$

Let $A'(a,b,c)$ be reflection A in l.

Then P is the mid point of AA'.

$$\Rightarrow \begin{cases} \frac{a+1}{2} = 1 \\ \frac{b+5}{2} = 4 \\ \frac{c+6}{2} = 7 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = 3 \\ c = 8 \end{cases}$$



$\therefore$ Position vector of A' is $\underline{i + 3j + 8k}$ ✓

15 Relative to the origin O, the point A, B and C have position vectors given by $\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix}$ and $\vec{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}$.

The quadrileteral ABCD is a parallelogram.

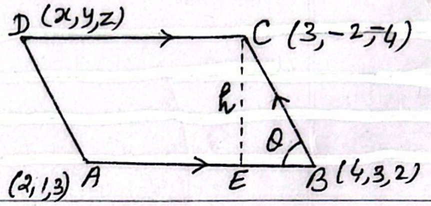
(a) Find the position vector of D. ---[3]

(b) The angle between BA and BC is θ; Find the exact value of $\cos\theta$. ---[3]

(c) Hence find the area of ABCD, answer in the form $p\sqrt{q}$; p, q are integers. ---[4]

S-23|31|Q6

Solution: Let $\vec{OD} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$



(a) ABCD is a parallelogram $\Rightarrow \vec{AB} = \vec{DC}$ --(1)

$\vec{AB} = \begin{pmatrix} 4-2 \\ 3-1 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ & $\vec{DC} = \begin{pmatrix} 3-x \\ -2-y \\ -4-z \end{pmatrix}$

from (1) $\vec{DC} = \vec{AB} \Rightarrow \begin{pmatrix} 3-x \\ -2-y \\ 4-z \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$

$\Rightarrow 3-x=2 \Rightarrow x=1$, $-2-y=2 \Rightarrow y=-4$

and $-4-z=-1 \Rightarrow z=-3$

$\therefore \vec{OD} = \begin{pmatrix} 1 \\ -4 \\ -3 \end{pmatrix} = (i-4j-3k)$

(b) $\vec{BA} = \begin{pmatrix} -2 \\ -2 \\ +1 \end{pmatrix}$, $\vec{BC} = \begin{pmatrix} -1 \\ -5 \\ -6 \end{pmatrix}$

$\Rightarrow \vec{BA} \cdot \vec{BC} = 2+10-6 = 6$

$|\vec{BA}| = \sqrt{4+4+1} = 3$ & $|\vec{BC}| = \sqrt{1+25+36}$ ---(2)

$= \sqrt{62}$

$\cos\theta = \dfrac{\vec{BA} \cdot \vec{BC}}{|\vec{BA}||\vec{BC}|} = \dfrac{6}{3 \times \sqrt{62}} = \dfrac{2}{\sqrt{62}}$ ---(3)

(c) Area of parallelogram $= BA \times h$

from (2) $= |\vec{BA}| \times |\vec{BC}| \sin\theta$

$= 3 \times \sqrt{62} \times \dfrac{\sqrt{58}}{\sqrt{62}}$

$= 3\sqrt{58}$

from (3) $\cos\theta = \dfrac{2}{\sqrt{62}}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{1-\dfrac{4}{62}} = \sqrt{\dfrac{58}{62}}$

16 The points A and B have position vectors $i+2j-2k$ and $2i-j+k$ respectively. The line l has equation $r = i-j+3k+\mu(2i-3j+4k)$.

(a) Show that l does not intersect the line passing through A and B. ---[5]

(b) Find the position vector of the foot of perpendicular from A to l. ---[4]

S-23/32/Q11

Solution: Line l; $r = i-j+3k+\mu(2i-3j+4k)$ -- ①

$\overrightarrow{OA} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$; $\overrightarrow{OB} = \begin{pmatrix}2\\-1\\1\end{pmatrix}$; $\overrightarrow{[illegible]} = \begin{pmatrix}-1\\3\\-3\end{pmatrix}$

Equation of line $\overrightarrow{AB} \Rightarrow r = \overrightarrow{OA} + \lambda\overrightarrow{AB}$

Equ^n of $\overrightarrow{AB}$: $\vec{r} = (i+2j-2k)+\lambda(i-3j+3k)$ -- ②

Any point on line $\overrightarrow{AB} = \begin{pmatrix}1+\lambda\\2-3\lambda\\-2+3\lambda\end{pmatrix}$ -- ③

Any point of line l. $= \begin{pmatrix}1+2\mu\\-1-3\mu\\3+4\mu\end{pmatrix}$ --- ④

In case line $\overrightarrow{AB}$ and line l intersect:

from ③ & ④ $\begin{pmatrix}1+\lambda\\2-3\lambda\\-2+3\lambda\end{pmatrix} = \begin{pmatrix}1+2\mu\\-1-3\mu\\3+4\mu\end{pmatrix}$

$\Rightarrow 1+\lambda = 1+2\mu \Rightarrow \lambda - 2\mu = 0$ — ⑤

$2-3\lambda = -1-3\mu \Rightarrow -3\lambda+3\mu = -3$ -- ⑥

$-2+3\lambda = 3+4\mu \Rightarrow 3\lambda - 4\mu = 5$ -- ⑦

Solving ⑥ & ⑦ $\Rightarrow \lambda = -1$ & $\mu = -2$

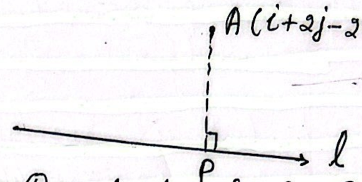
Put the values of λ & μ in ⑤

$-1-2(-2) = 0 \Rightarrow \underline{3=0 \text{ false}}$

$\therefore$ There is no common point between the lines 'l' and line 'AB'

Hence the two lines don't intersect.

ⓑ



from ④ any point of line l, say $\overrightarrow{OP} = \begin{pmatrix}1+2\mu\\-1-3\mu\\3+4\mu\end{pmatrix}$ ⑧ & $\overrightarrow{OA} = \begin{pmatrix}1\\2\\-2\end{pmatrix}$

$\Rightarrow \overrightarrow{PA} = \overrightarrow{OA} - \overrightarrow{OP} = \begin{pmatrix}-2\mu\\3+3\mu\\-5-4\mu\end{pmatrix}$ — ⑨

Direction vector of line 'l'

$\vec{v} = \begin{pmatrix}2\\-3\\4\end{pmatrix}$ — ⑩

for 'P' is the foot of perpendicular from point 'A' to line 'l':

$\overrightarrow{PA} \cdot \vec{v} = 0$ from ⑨ & ⑩

$\begin{pmatrix}-2\mu\\3+3\mu\\-5-4\mu\end{pmatrix} \cdot \begin{pmatrix}2\\-3\\4\end{pmatrix} = 0$

$\Rightarrow -4\mu - 9 - 9\mu - 20 - 16\mu = 0$

$-29\mu - 29 = 0 \Rightarrow \underline{\mu = -1}$ ✓

Put $\mu = -1$ in ⑧ $\overrightarrow{OP} = \begin{pmatrix}1-2\\-1+3\\3-4\end{pmatrix} = \begin{pmatrix}-1\\2\\-1\end{pmatrix}$

$\therefore$ Position vector of the required foot of perpendicular is:

$\underline{(-i+2j-k)}$ ✓

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17 The lines l and m have equations, $l: r = ai + 3j + bk + \lambda(ci - 2j + 4k)$
and $m: r = i + 2j + 3k + \mu(2i - 3j + k)$

Relative to the origin O, the position vector of the point P is $4i + 7j - 2k$.

(a) Given that l is perpendicular to m and P lies on l, find the values of the constants a, b and c. ---[4]

(b) The perpendicular from P meets the line m at Q. The R lies on PQ extended, with PQ : QR = 2 : 3. Find the position vector of R. ---[6]

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Solution: Line l: $r = ai + 3j + bk + \lambda(ci - 2j + 4k)$ ---①

(a) Line m: $r = i + 2j + 3k + \mu(2i - 3j + k)$ --②

Point P: $\vec{OP} = (4i + 7j - 2k)$ --③

$l \perp m \Rightarrow u \cdot v = 0$ {Direct. of $l = u$, Direction of $m = v$

$\Rightarrow (ci - 2j + 4k)\cdot(2i - 3j + k) = 0$

$\Rightarrow 2c + 6 + 4 = 0 \Rightarrow \underline{c = -5}$ ✓ ---④

Any point of line l: $\vec{r} = \begin{pmatrix} a + c\lambda \\ 3 - 2\lambda \\ b + 4\lambda \end{pmatrix}$ ---⑤

$\vec{OP} = \begin{pmatrix} 4 \\ 7 \\ -2 \end{pmatrix}$ ---⑥

for ⑤ & ⑥

P lies on line $l \Rightarrow \begin{cases} a + c\lambda = 4 \\ 3 - 2\lambda = 7 \Rightarrow \lambda = -2 \\ b + 4\lambda = -2 \end{cases}$

for $\lambda = -2$

Now $a + c\lambda = 4 \Rightarrow a - 2c = 4$

from ④ $c = -5 \Rightarrow a - 2(-5) = 4$

$\Rightarrow a = -6$ ✓

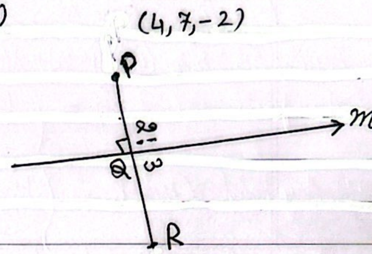
$b + 4\lambda = -2$

$b + 4\times(-2) = -2$

$b = 6$ ✓

$\therefore \underline{a = -6,\ b = 6 \text{ and } c = -5}$ ✓

(b)



Any point Q on line 'm' from ②

$\vec{OQ} = r = \begin{pmatrix} 1 + 2\mu \\ 2 - 3\mu \\ 3 + \mu \end{pmatrix}$ ---⑦

$\vec{PQ} = \vec{OQ} - \vec{OP} = \begin{pmatrix} 1 + 2\mu - 4 \\ 2 - 3\mu - 7 \\ 3 + \mu + 2 \end{pmatrix} = \begin{pmatrix} 2\mu - 3 \\ -3\mu - 5 \\ \mu + 5 \end{pmatrix}$ ---⑧

as $\vec{PQ}$ perpendicular to line 'm'

$\vec{V} \cdot \vec{PQ} = 0$ [Direction of m $\vec{V} = (2i - 3j + k)$

$\Rightarrow \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2\mu - 3 \\ -3\mu - 5 \\ \mu + 5 \end{pmatrix} = 0$

$2(2\mu - 3) - 3(-3\mu - 5) + 1(\mu + 5) = 0$

$\Rightarrow 14\mu + 14 = 0 \Rightarrow \mu = -1$ ✓

from ⑦ $\vec{OQ} = (-i + 5j + 2k)$ --

and $\vec{PQ} = (-5i - 2j + 4k)$ --⑨

Now PQ : QR = 2 : 3 $\Rightarrow$ PR $= \frac{5}{2}$ PQ

$\therefore \vec{OR} = \vec{OP} + \frac{5}{2}\vec{PQ}$

from ③ & ⑨

$= (4i + 7j - 2k) + \frac{5}{2}(-5i - 2j + 4k)$

$= \underline{\left(-\frac{17}{2}i + 2j + 8k\right)}$ ✓

18. Two lines have equations $r = i+2j+k+\lambda(ai+2j-k)$ and $r = 2i+j-k+\mu(2i-j+k)$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. ---[5]

(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a. ---[6]

W-20/31/Q11

Solution (a)

$L_1: \vec{r} = i+2j+k+\lambda(a+2j-k) = \begin{pmatrix} 1+a\lambda \\ 2+2\lambda \\ 1-\lambda \end{pmatrix}$ —— ①

$L_2: \vec{r} = (2i+j-k)+\mu(2i-j+k) = \begin{pmatrix} 2+2\mu \\ 1-\mu \\ -1+\mu \end{pmatrix}$ —— ②

for L_1 and L_2 to intersect $\Rightarrow \begin{pmatrix} 1+a\lambda \\ 2+2\lambda \\ 1-\lambda \end{pmatrix} = \begin{pmatrix} 2+2\mu \\ 1-\mu \\ -1+\mu \end{pmatrix}$

$\Rightarrow 1+a\lambda = 2+2\mu$ —— ③

$2+2\lambda = 1-\mu \Rightarrow 2\lambda+\mu = -1$ —— ④

$1-\lambda = -1+\mu \Rightarrow +\lambda+\mu = 2$ —— ⑤

Solving ④ & ⑤ $\Rightarrow \lambda = -3$ and $\mu = 5$

$\therefore$ from ③ $1+(-3)a = 2+2\times5 \Rightarrow a = -\frac{11}{3}$ ✓

$\therefore$ from ② the point of intersection for $\mu = 5 \Rightarrow \begin{pmatrix} 12 \\ -4 \\ 4 \end{pmatrix}$

$\therefore$ Position Vector of the point of intersection $= 12i-4j+4k$ ✓

(b) Angle between the two lines with directions $(a+2j-k)$ & $(2i-j+k)$

$\cos\alpha = \dfrac{2a-2-1}{\sqrt{a^2+5}\cdot\sqrt{6}} = \pm\dfrac{1}{6}$ $\left[\cos\alpha = \dfrac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\cdot\sqrt{a_2^2+b_2^2+c_2^2}}\right]$

Square:

$\Rightarrow \dfrac{(2a-3)^2}{6(a^2+5)} = \dfrac{1}{36} \Rightarrow 6(2a-3)^2 = a^2+5$

$\Rightarrow 23a^2-72a+49=0$

$23a^2-23a-49a+49=0$

$23a(a-1)-49(a-1)=0$

$(a-1)(23a-49)=0$

$\Rightarrow a=1$; $a = \frac{49}{23}$ ✓

19 With respect to the origin O, the position vectors of the points A, B, C and D are:
$\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \vec{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \vec{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \vec{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}$

(a) Show that $AB = 2CD$ ---[3]

(b) Find the angle between the directions of $\vec{AB}$ and $\vec{CD}$ ---[3]

(c) Show that the line through A and B does not intersect the line through C and D $\boxed{\text{W-20/32/Q8}}$ ---[4]

Solution:

(a) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix} \Rightarrow AB = \sqrt{2^2+2^2+4^2} = \sqrt{24}$ —(1)

$\vec{CD} = \vec{OD} - \vec{OC} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \Rightarrow CD = \sqrt{2^2+1^2+1^2} = \sqrt{6}$ —(2)

from (1) $AB = \sqrt{24} = 2\sqrt{6} = 2CD$ from (2) $\Rightarrow \underline{AB = 2CD}$ ✓

(b) angle α, between $\vec{AB}$ and $\vec{CD}$,

$$\cos\alpha = \frac{2\times2 + (-2)\times1 + (-4)\times1}{\sqrt{24}\,\sqrt{6}} = \frac{-2}{12} = -\frac{1}{6}$$

$\cos\alpha = -\frac{1}{6} = -\cos 80.4$

$\Rightarrow \alpha = 180 - 80.4$

$\therefore$ Angle between $\vec{AB}$ and $\vec{CD}$ $= \underline{99.6^\circ}$ ✓

(c) Equation of line AB; $\vec{r} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 2 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2+2\lambda \\ 1-2\lambda \\ 5+4\lambda \end{pmatrix}$ —(3)

Equation of line CD, $\vec{r} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ 1+\mu \\ 2+\mu \end{pmatrix}$ —(4)

for Intersection of $\vec{AB}$ and $\vec{CD}$ from (3) & (4)

$$\left.\begin{array}{l} 2+2\lambda = 1+2\mu \Rightarrow 2\lambda - 2\mu = -1 \quad \text{—(5)} \\ 1-2\lambda = 1+\mu \Rightarrow 2\lambda + \mu = 0 \quad \text{—(6)} \\ 5+4\lambda = 2+\mu \Rightarrow 4\lambda - \mu = -3 \quad \text{—(7)} \end{array}\right\}$$

Solving (5) & (6) $\lambda = -\frac{1}{6}$ and $\mu = \frac{1}{3}$

Put λ & μ in (7) $\Rightarrow$ $4\times\left(-\frac{1}{6}\right) - \frac{1}{3} = -3 \Rightarrow -1 = -3$ false ✓

$\therefore$ Lines AB does not intersect line CD. ✓

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20. Two lines l and m have equations: $r = 3i+2j+5k+s(4i-j+3k)$ and $r = i-j-2k+t(-i+2j+2k)$ respectively.

(a) Show that l and m are perpendicular. ---[2]

(b) Show that l and m intersect and state the position vector of the point of intersection. ---[5]

(c) Show that the length of perpendicular from the origin to the line m is $\frac{1}{3}\sqrt{5}$. ---[4]

[N-21/31/Q9]

Solution: l: $r = 3i+2j+5k+s(4i-j+3k)$ — ① $\qquad \vec{r} = a + s\vec{V_1}$

Line → m: $r = i-j-2k+t(-i+2j+2k)$ — ② $\qquad \vec{r} = b + s\vec{V_2}$

(a) Consider $\vec{V_1}\cdot\vec{V_2} = (4i-j+3k)\cdot(-i+2j+2k)$ (Check $\vec{V_1}\cdot\vec{V_2} = 0$)

$= -4-2+6 = 0 \Rightarrow$ l and m are perpendicular.

(b) Any point on l: $\vec{r} = \begin{pmatrix} 3+4s \\ 2-s \\ 5+3s \end{pmatrix}$ — ③ ; Any point on m: $\vec{r} = \begin{pmatrix} 1-t \\ -1+2t \\ -2+2t \end{pmatrix}$ — ④

for the intersection of l and m, equating ③, ④ respective components.

$$\left.\begin{aligned} 3+4s &= 1-t \\ 2-s &= -1+2t \\ 5+3s &= -2+2t \end{aligned}\right\} \Rightarrow \begin{aligned} t+4s &= -2 \quad ⑤ \\ 2t+s &= 3 \quad ⑥ \\ 2t-3s &= 7 \quad ⑦ \end{aligned}$$

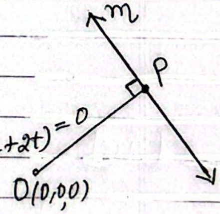
Solving ⑤ & ⑥ $\Rightarrow s = -1$ and $t = 2$, Put these values in ⑦ $2\times2-3(-1) = 7$ True

∴ All three equations are consistent ⇒ l & m intersect.

for the point of intersection of l and m → Put $s=-1$ in ③ (or $t=2$ in ④)

∴ Req. point of intersection is $(-i+3j+2k)$ ✓

(c) Any point P on m: $\vec{OP} = \begin{pmatrix} 1-t \\ -1+2t \\ -2+2t \end{pmatrix}$ — ④



Direction of m: $V_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$

Now $\vec{OP} \perp m \Rightarrow \vec{OP}\cdot\vec{V_2} = 0 \Rightarrow -1(1-t)+2(-1+2t)+2(-2+2t) = 0$

$\Rightarrow -7+9t = 0 \Rightarrow t = 7/9$

from ④ from $t = 7/9$

$\therefore \vec{OP} = \begin{pmatrix} 2/9 \\ \frac{5}{9} \\ -\frac{4}{9} \end{pmatrix} \Rightarrow |\vec{OP}| = \sqrt{\left(\frac{2}{9}\right)^2+\left(\frac{5}{9}\right)^2+\left(\frac{4}{9}\right)^2} = \sqrt{\frac{5}{9}} = \frac{1}{3}\sqrt{5}$ ✓

21. With respect to the origin O, the position vectors of the points A and B are given by: $\vec{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$

(a) Find the vector equation for the line l through A and B ---[3]

(b) The point C lies on l and is such that $\vec{AC} = 3\vec{AB}$
Find the position vector of C. ---[2]

(c) Find the possible position vectors of the point P on l such that $OP = \sqrt{14}$. ---[5]

[W-21 / 32 / Q 10]

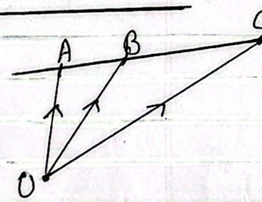
Solution:
(a) $\vec{AB} = \vec{OB} - \vec{OA} = \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix} = (-i + j + 2k)$; $\vec{OA} = (i + 2j - k)$

$\vec{r} = \vec{OA} + \lambda \vec{AB}$

$\therefore$ Equation of line through A & B; l: $r = (i + 2j - k) + \lambda(-i + j + 2k)$ — ①

(b) $\vec{OC} = \vec{OA} + \vec{AC}$ — ②

$\vec{AC} = 3\vec{AB} = 3(-i + j + 2k) = (-3i + 3j + 6k)$



for ②
$\vec{OC} = (i + 2j - k) + (-3i + 3j + 6k)$
$\therefore \vec{OC} = (-2i + 5j + 5k)$ ✓

(c) from ① let P is any point on l; $\vec{OP} = (1-\lambda)i + (2+\lambda)j + (-1+2\lambda)k$ — ②

$|\vec{OP}| = \sqrt{(1-\lambda)^2 + (2+\lambda)^2 + (-1+2\lambda)^2} = \sqrt{14}$

$\Rightarrow 1 + \lambda^2 - 2\lambda + 4 + \lambda^2 + 4\lambda + 1 + 4\lambda^2 - 4\lambda = 14$

$\Rightarrow 6\lambda^2 - 2\lambda + 6 = 14$

$\Rightarrow 3\lambda^2 - \lambda - 4 = 0$

$\Rightarrow (3\lambda - 4)(\lambda + 1) = 0 \Rightarrow \lambda = \frac{4}{3}$; $\lambda = -1$

from ② for $\lambda = 4/3$
$\vec{OP} = -\frac{1}{3}i + \frac{10}{3}j + \frac{5}{3}k$ ✓

and for $\lambda = -1$ in ②
$\vec{OP} = 2i + j - 3k$ ✓

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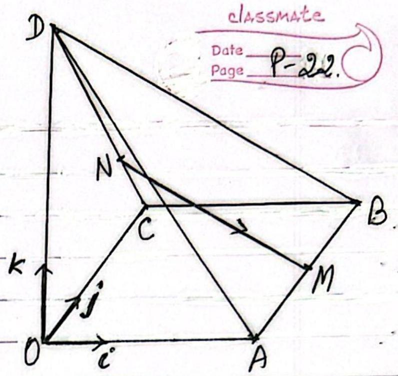


22. In the diagram, OABCD is a pyramid with vertex D. The horizontal base OABC is a square of side 4 units. The edge OD is vertical and OD = 4 units.
The unit vectors i, j, k are parallel to OA, OC, and OD respectively.
The mid point of AB is M and the point N on CD is such that DN = 3NC.

(a) Find a vector equation for the line through M and N. --- [5]
(b) Show that the length of the perpendicular from O to MN is $\frac{1}{3}\sqrt{82}$ --- [4]

[W-21/33/Q8]

Solution: $\vec{OA} = 4i$; $\vec{OB} = 4i+4j$; $\vec{OC} = 4j$; $\vec{OD} = 4k$

$\vec{OM} = \vec{OA} + \vec{AM} = 4i + 2j$ —— (1)

$\vec{CD} = (4k - 4j)$; $\vec{CN} = \frac{1}{4}\vec{CB}$ ($\because$ DN = 3NC Given)

$\vec{CN} = \frac{1}{4}(4k - 4j) = (-j + k)$ —— (2)

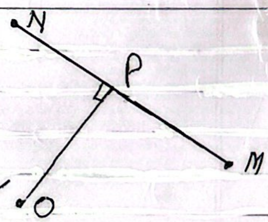
$\vec{ON} = \vec{OC} + \vec{CN} = 4j + (-j+k) = 3j + k$ —— (3)

$\therefore$ from (1) & (3) $\vec{NM} = \vec{OM} - \vec{ON} = -(3j+k) + (4i+2j) = (4i - j - k)$ —— (4)

$\therefore$ Equation of line through M and N:

$r = (3j + k) + \lambda(4i - j - k) \rightarrow [r = \vec{ON} + \lambda(\vec{NM})]$ —— (5)

(b) Let P is any point on $\vec{NM}$ line



$\vec{OP} = (4\lambda i + (3-\lambda)j + (1-\lambda)k)$ —— (6)

$\vec{OP} \perp NM \Rightarrow \vec{OP} \cdot \vec{NM} = 0$ ((4) & (6))

$(4\lambda i + (3-\lambda)j + (1-\lambda)k) \cdot (4i - j - k) = 0$

$\Rightarrow 16\lambda - 3 + \lambda - 1 + \lambda = 0 \Rightarrow 18\lambda = 4 \Rightarrow \lambda = \frac{2}{9}$ ✓

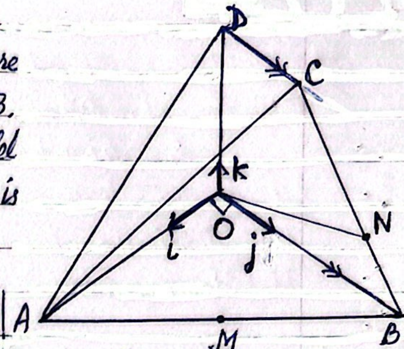
from (6) Now for $\lambda = \frac{2}{9} \Rightarrow \vec{OP} = \frac{8}{9}i + \frac{25}{9}j + \frac{7}{9}k$

$\therefore$ Length of perpendicular from O to line NM $= |\vec{OP}|$

$= \sqrt{\left(\frac{8}{9}\right)^2 + \left(\frac{25}{9}\right)^2 + \left(\frac{7}{9}\right)^2} = \sqrt{\frac{738}{81}}$

$= \sqrt{\frac{82}{9}} = \frac{1}{3}\sqrt{82}$ ✓

23 In the diagram, OABCD is a solid figure in which $OA = OB = 4$ units and $OD = 3$. The edge OD is vertical, DC is parallel to OB and $DC = 1$ unit. The base, OAB, is horizontal and angle $AOB = 90°$. Unit vectors i, j and k are parallel to OA, OB and OD respectively. The mid point of AB is M and the point N on BC is such that $CN = 2NB$.



(a) Express vectors vectors $\vec{MD}$ and $\vec{ON}$ in terms of i, j and k. ---[4]

(b) Calculate the angle in degrees between the directions of $\vec{MD}$ and $\vec{ON}$. ---[3]

(c) Show that the length of perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. ---[4]

W-22/31/Q11

Solution:

(a) $\vec{MD} = \vec{OD} - \vec{OM}$

$= 3k - \left(\frac{4i+4j}{2}\right)$

$= (-2i - 2j + 3k)$

$\vec{ON} = \vec{OC} + \frac{2}{3}\vec{CB}$

$= (\vec{OD} + \vec{DC}) + \frac{2}{3}(\vec{OB} - \vec{OC})$

$= (3k + j) + \frac{2}{3}(4j - (3k+j))$

$= (3k+j) + \frac{2}{3}(4j - 3k - j)$

$= (3j + k)$

(b) Angle 'θ' between $\vec{MD}$ and $\vec{ON}$

$\cos\theta = \frac{\vec{MD}\cdot\vec{ON}}{|\vec{MD}||\vec{ON}|}$

$= \frac{(-2i - 2j + 3k)\cdot(3j + k)}{\sqrt{2^2+2^2+3^2}\cdot\sqrt{3^2+1^2}}$

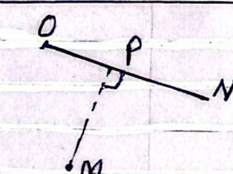
$= \frac{-6+3}{\sqrt{17}\sqrt{10}} = \frac{-3}{\sqrt{170}}$

$\cos\theta = -\cos\alpha$; $\left(\alpha = \cos^{-1}\frac{3}{\sqrt{170}} = \cos^{-1}0.23 = 76.7°\right)$

$\theta = 180 - \alpha = 180° - 76.7°$

$= 103.3°$

(c)



Let P is any point on $\vec{ON}$

$\vec{OP} = \lambda(3j + k)$ --- ①

$\vec{OM} = (2i + 2j)$

$\Rightarrow \vec{MP} = \lambda(3j + k) - (2i + 2j)$ --- ②

$= (-2i + (-2+3\lambda)j + \lambda k)$

$\vec{MP} \perp \vec{OP}$

$\vec{MP}\cdot\vec{OP} = 0$

$\Rightarrow \begin{pmatrix} -2 \\ -2+3\lambda \\ \lambda \end{pmatrix}\cdot\begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = 0$

$\Rightarrow -6 + 9\lambda + \lambda = 0$

$\Rightarrow \lambda = 3/5$

from ②

$\vec{MP} = (-2i - \frac{1}{5}j + \frac{3}{5}k)$

$MP = \sqrt{2^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{3}{5}\right)^2}$

$= \sqrt{\frac{110}{25}} = \sqrt{\frac{22}{5}}$

24 Relative to the origin O, the points A, B and C have position vectors given by: $\vec{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\vec{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$

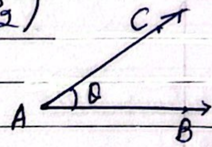
(a) Using scalar product, find the cosine of angle BAC. --- [4]

(b) Hence find the area of triangle ABC. Give your answer in a simplified exact form. --- [4]

W-22/32/Q6

Solution: Given $\vec{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, $\vec{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ and $\vec{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}$

(a)



$\vec{AB} = \vec{OB} - \vec{OA} = 2i - 2j + k$

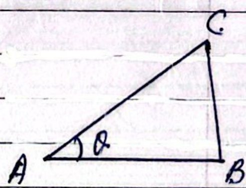
$\vec{AC} = \vec{OC} - \vec{OA} = 4i - 3k$

$\cos BAC = \dfrac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}||\vec{AC}|} = \dfrac{(2i - 2j + k) \cdot (4i - 3k)}{\sqrt{2^2 + 2^2 + 1^2}\,\sqrt{4^2 + 3^2}}$

$= \dfrac{8 - 3}{\sqrt{9}\sqrt{25}} = \dfrac{5}{3 \times 5} = \dfrac{1}{3}$

$\therefore \cos BAC = 1/3$. ✓

(b)



Area of $\Delta ABC = \frac{1}{2} AB \times AC \sin\theta$ — (1)

$AB = |\vec{AB}| = \sqrt{2^2 + 2^2 + 1^2} = 3$

$AC = |\vec{AC}| = \sqrt{4^2 + 3^2} = 5$

$\cos\theta = \frac{1}{3} \Rightarrow \sin\theta = \sqrt{1 - \cos^2\theta}$

$= \sqrt{1 - \left(\frac{1}{3}\right)^2} = \sqrt{8/9} = \frac{2\sqrt{2}}{3}$

Hence from (1) Area of $\Delta ABC = \frac{1}{2} \times 3 \times 5 \times \frac{2\sqrt{2}}{3} = 5\sqrt{2}$ ✓

25 With respect to the origin O, the position vector of the points A, B and C are given by: $\vec{OA}=\begin{pmatrix}0\\5\\2\end{pmatrix}$, $\vec{OB}=\begin{pmatrix}1\\0\\1\end{pmatrix}$ and $\vec{OC}=\begin{pmatrix}4\\-3\\-2\end{pmatrix}$.

The mid point of AC is M and the point N lies on BC, and is such that $BN = 2NC$.

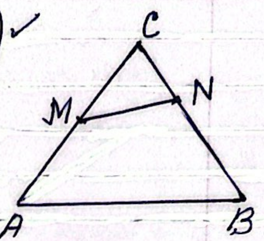
(a) Find the position vectors of M and N. ---[3]

(b) Find the vector equation for the line through M and N. ---[2]

(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. --[4]

W-22/33/Q9

Solution:
(a) $\vec{OM} = \frac{\vec{OA}+\vec{OC}}{2} = \frac{1}{2}\begin{pmatrix}0+4\\5-3\\2-2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}4\\2\\0\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix}$ ✓



$\vec{ON} = \vec{OC} + \frac{1}{3}\vec{CB}$

$= \begin{pmatrix}4\\-3\\-2\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1-4\\0+3\\1+2\end{pmatrix} = \begin{pmatrix}4\\-3\\-2\end{pmatrix} + \frac{1}{3}\begin{pmatrix}-3\\3\\3\end{pmatrix}$

$\vec{ON} = \begin{pmatrix}3\\-2\\-1\end{pmatrix}$ ✓

(b) Equation of line through M & N: $\vec{r} = \vec{OM} + \lambda\vec{MN}$

$\vec{r} = \begin{pmatrix}2\\1\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\-3\\-1\end{pmatrix}$ --- (1) $\quad \left[\vec{MN} = \begin{pmatrix}1\\-3\\-1\end{pmatrix}\right.$

(c) Equation of line AB is $\vec{r} = \vec{OB} + \mu\vec{AB}$ $\quad \left\{\vec{AB} = \begin{pmatrix}1\\-5\\-1\end{pmatrix}\right.$

$\vec{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + \mu\begin{pmatrix}1\\-5\\-1\end{pmatrix}$ --- (2)

for (1) any point on $\vec{MN} = \begin{pmatrix}2+\lambda\\1-3\lambda\\-\lambda\end{pmatrix}$ --- (3)

and any point on $\vec{AB} = \begin{pmatrix}1+\mu\\-5\mu\\1-\mu\end{pmatrix}$ --- (4)

for the intersection of lines $\vec{MN}$ and $\vec{AB}$

$\begin{pmatrix}2+\lambda\\1-3\lambda\\-\lambda\end{pmatrix} = \begin{pmatrix}1+\mu\\-5\mu\\1-\mu\end{pmatrix} \Rightarrow \begin{cases}\lambda-\mu=-1 & \text{--- (5)}\\ -3\lambda+5\mu=-1 & \text{--- (6)}\\ -\lambda+\mu=1 & \text{--- (7)}\end{cases}$ Solving (5) & (6) $\lambda=-3$, $\mu=-2$

Put $\lambda=-3$ in (3), Point of intersection $Q\begin{pmatrix}-1\\10\\3\end{pmatrix}$ ✓