# PURE MATHEMATICS -3

# 9709

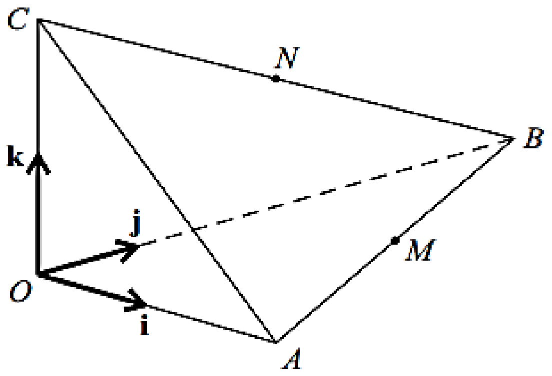
## (March, June and November series 2020 – 2023 With marking scheme)

# VECTORS

## EXERCISE -1

MANJULA BALAJI

**1)** SP-2020 _9709_3 Q8



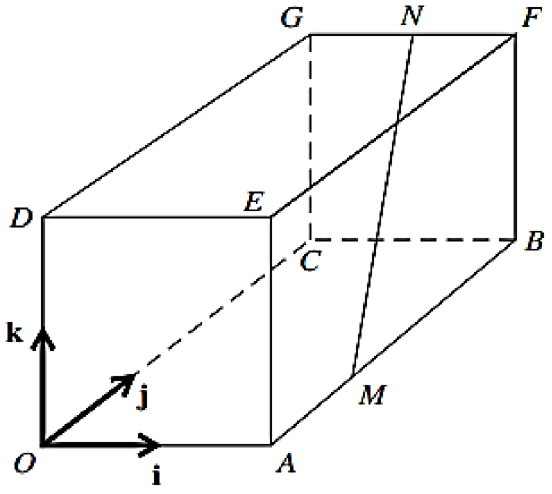
In the diagram, $OABC$ is a pyramid in which $OA = 2$ units, $OB = 4$ units and $OC = 2$ units. The edge $OC$ is vertical, the base $OAB$ is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to $OA$, $OB$ and $OC$ respectively. The midpoints of $AB$ and $BC$ are $M$ and $N$ respectively.

**(a)** Express the vectors $\overrightarrow{ON}$ and $\overrightarrow{CM}$ in terms of **i**, **j** and **k**. [3]

**(b)** Calculate the angle between the directions of $\overrightarrow{ON}$ and $\overrightarrow{CM}$. [3]

**(c)** Show that the length of the perpendicular from $M$ to $ON$ is $\frac{3}{5}\sqrt{5}$. [4]

**2)** MARCH-2020 _9709_32 Q8



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 3$ units and $OD = 2$ units. Unit vectors **i**, **j** and **k** are parallel to $OA$, $OC$ and $OD$ respectively. The point $M$ on $AB$ is such that $MB = 2AM$. The midpoint of $FG$ is $N$.

**(a)** Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

**(b)** Find a vector equation for the line through $M$ and $N$. [2]

**(c)** Find the position vector of $P$, the foot of the perpendicular from $D$ to the line through $M$ and $N$. [4]

**3)** MARCH-2021 _9709_32 Q7

Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

**(a)** Show that the lines are skew. [5]

**(b)** Find the acute angle between the directions of the two lines. [3]

**4)** MARCH-2022 _9709_32 Q10

The points $A$ and $B$ have position vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ respectively. The line $l$ has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$.

**(a)** Find a vector equation for the line through $A$ and $B$. [3]

**(b)** Find the acute angle between the directions of $AB$ and $l$, giving your answer in degrees. [3]

**(c)** Show that the line through $A$ and $B$ does not intersect the line $l$. [4]

**5)** MARCH-2023 _9709_32 Q10

With respect to the origin $O$, the points $A$, $B$, $C$ and $D$ have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

**(a)** Find the obtuse angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [3]

The line $l$ passes through the points $A$ and $B$.

**(b)** Find a vector equation for the line $l$. [2]

**(c)** Find the position vector of the point of intersection of the line $l$ and the line passing through $C$ and $D$. [4]

**6)** JUNE-2020 _9709_31 Q9

With respect to the origin $O$, the vertices of a triangle $ABC$ have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

**(a)** Using a scalar product, show that angle $ABC$ is a right angle. [3]

**(b)** Show that triangle $ABC$ is isosceles. [2]

**(c)** Find the exact length of the perpendicular from $O$ to the line through $B$ and $C$. [4]

**7)** JUNE-2020 _9709_32 Q10

With respect to the origin $O$, the points $A$ and $B$ have position vectors given by $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of $OA$ is $M$. The point $N$ lying on $AB$, between $A$ and $B$, is such that $AN = 2NB$.

**(a)** Find a vector equation for the line through $M$ and $N$. [5]

The line through $M$ and $N$ intersects the line through $O$ and $B$ at the point $P$.

**(b)** Find the position vector of $P$. [3]

**(c)** Calculate angle $OPM$, giving your answer in degrees. [3]

**8)** JUNE-2020 _9709_33 Q8

Relative to the origin $O$, the points $A$, $B$ and $D$ have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{k}.$$

A fourth point $C$ is such that $ABCD$ is a parallelogram.

**(a)** Find the position vector of $C$ and verify that the parallelogram is not a rhombus. [5]

**(b)** Find angle $BAD$, giving your answer in degrees. [3]

**(c)** Find the area of the parallelogram correct to 3 significant figures. [2]

**9)** JUNE-2021 _9709_31 Q8

With respect to the origin $O$, the points $A$ and $B$ have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line $l$ has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

**(a)** Find the acute angle between the directions of $AB$ and $l$. [4]

**(b)** Find the position vector of the point $P$ on $l$ such that $AP = BP$. [5]

**10)** JUNE-2021 _9709_32 Q11

With respect to the origin $O$, the points $A$ and $B$ have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{j} - 2\mathbf{k}$.

**(a)** Show that $OA = OB$ and use a scalar product to calculate angle $AOB$ in degrees. [4]

The midpoint of $AB$ is $M$. The point $P$ on the line through $O$ and $M$ is such that $PA : OA = \sqrt{7} : 1$.

**(b)** Find the possible position vectors of $P$. [6]

**11)** JUNE-2021 _9709_33 Q9

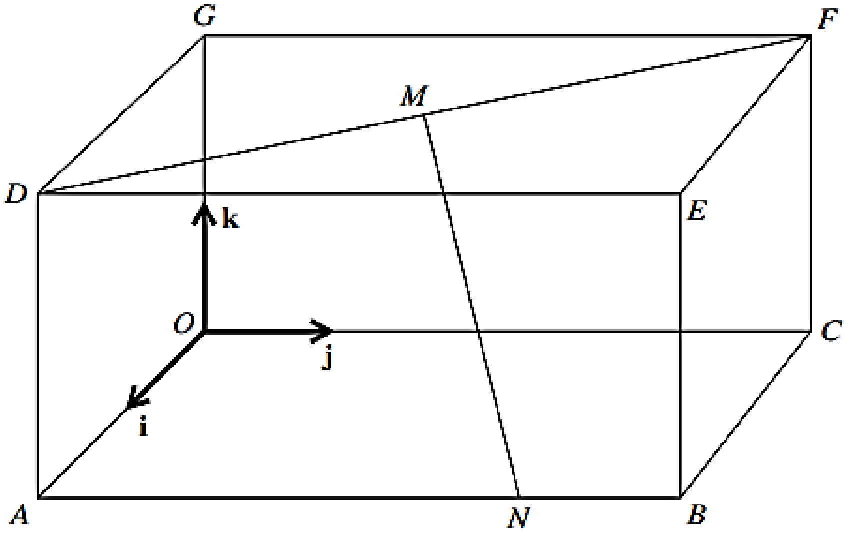
The quadrilateral $ABCD$ is a trapezium in which $AB$ and $DC$ are parallel. With respect to the origin $O$, the position vectors of $A$, $B$ and $C$ are given by $\overrightarrow{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

**(a)** Given that $\overrightarrow{DC} = 3\overrightarrow{AB}$, find the position vector of $D$. [3]

**(b)** State a vector equation for the line through $A$ and $B$. [1]

**(c)** Find the distance between the parallel sides and hence find the area of the trapezium. [5]

**12)** JUNE-2022 _9709_31 Q9



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 4$ units and $OG = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $OA$, $OC$ and $OG$ respectively. The point $M$ is the midpoint of $DF$. The point $N$ on $AB$ is such that $AN = 3NB$.

**(a)** Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

**(b)** Find a vector equation for the line through $M$ and $N$. [2]

**(c)** Show that the length of the perpendicular from $O$ to the line through $M$ and $N$ is $\sqrt{\frac{53}{6}}$. [4]

**13)** JUNE-2022 _9709_32 Q9

The lines $l$ and $m$ have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where $a$ and $b$ are constants.

**(a)** Given that $l$ and $m$ intersect, show that $2b - a = 4$. [4]

**(b)** Given also that $l$ and $m$ are perpendicular, find the values of $a$ and $b$. [4]

**(c)** When $a$ and $b$ have these values, find the position vector of the point of intersection of $l$ and $m$. [2]

**14)** JUNE-2022 _9709_33 Q9

With respect to the origin $O$, the point $A$ has position vector given by $\overrightarrow{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The line $l$ has vector equation $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

**(a)** Find in degrees the acute angle between the directions of $OA$ and $l$. [3]

**(b)** Find the position vector of the foot of the perpendicular from $A$ to $l$. [4]

**(c)** Hence find the position vector of the reflection of $A$ in $l$. [2]

**15)** JUNE-2023 _9709_31 Q6

Relative to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral $ABCD$ is a parallelogram.

**(a)** Find the position vector of $D$. [3]

**(b)** The angle between $BA$ and $BC$ is $\theta$.

Find the exact value of $\cos\theta$. [3]

**(c)** Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where $p$ and $q$ are integers. [4]

**16)** JUNE-2023 _9709_32 Q11

The points $A$ and $B$ have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line $l$ has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

**(a)** Show that $l$ does not intersect the line passing through $A$ and $B$. [5]

**(b)** Find the position vector of the foot of the perpendicular from $A$ to $l$. [4]

**17)** JUNE-2023 _9709_33 Q9

The lines $l$ and $m$ have equations

$$l: \quad \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$
$$m: \quad \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin $O$, the position vector of the point $P$ is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

**(a)** Given that $l$ is perpendicular to $m$ and that $P$ lies on $l$, find the values of the constants $a$, $b$ and $c$. [4]

**(b)** The perpendicular from $P$ meets line $m$ at $Q$. The point $R$ lies on $PQ$ extended, with $PQ : QR = 2 : 3$.

Find the position vector of $R$. [6]

**18)** OCT 2020_9709_31 Q11

Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where $a$ is a constant.

**(a)** Given that the two lines intersect, find the value of $a$ and the position vector of the point of intersection. [5]

**(b)** Given instead that the acute angle between the directions of the two lines is $\cos^{-1}(\frac{1}{6})$, find the two possible values of $a$. [6]

**19)** OCT 2020_9709_32 Q8

With respect to the origin $O$, the position vectors of the points $A$, $B$, $C$ and $D$ are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

**(a)** Show that $AB = 2CD$. [3]

**(b)** Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$. [3]

**(c)** Show that the line through $A$ and $B$ does not intersect the line through $C$ and $D$. [4]

**20)** OCT 2021_9709_31 Q9

Two lines $l$ and $m$ have equations $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + s(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ respectively.

**(a)** Show that $l$ and $m$ are perpendicular. [2]

**(b)** Show that $l$ and $m$ intersect and state the position vector of the point of intersection. [5]

**(c)** Show that the length of the perpendicular from the origin to the line $m$ is $\frac{1}{3}\sqrt{5}$. [4]

**21)** OCT 2021_9709_32 Q10

With respect to the origin $O$, the position vectors of the points $A$ and $B$ are given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$.

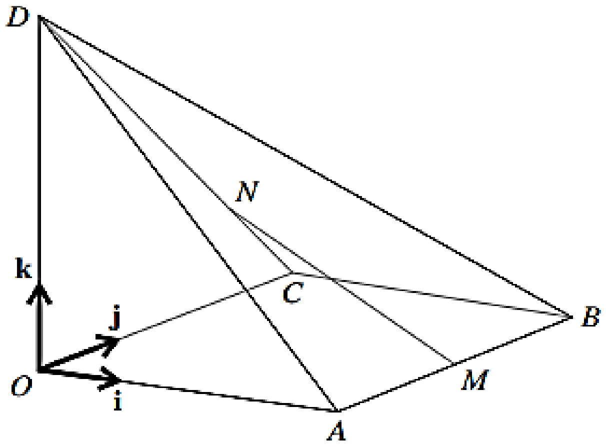
**(a)** Find a vector equation for the line $l$ through $A$ and $B$. [3]

**(b)** The point $C$ lies on $l$ and is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$.

Find the position vector of $C$. [2]

**(c)** Find the possible position vectors of the point $P$ on $l$ such that $OP = \sqrt{14}$. [5]

**22)** OCT 2021_9709_33 Q8



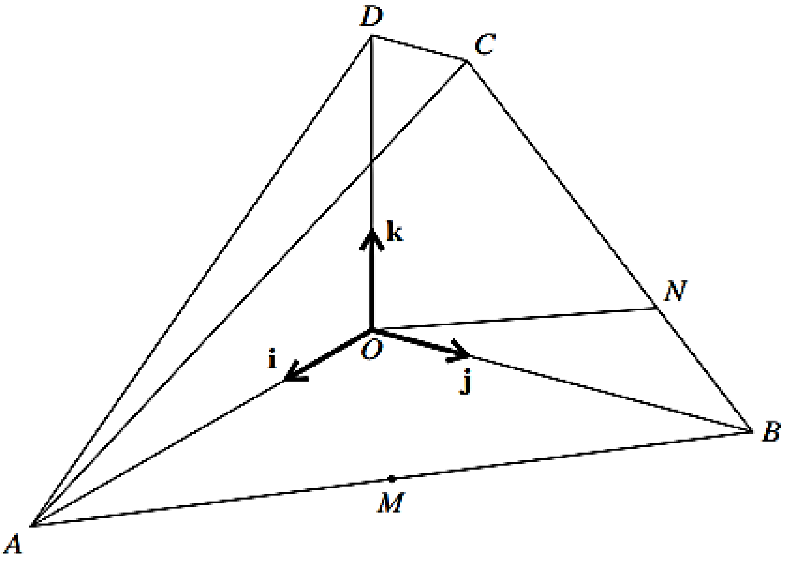
In the diagram, $OABCD$ is a pyramid with vertex $D$. The horizontal base $OABC$ is a square of side 4 units. The edge $OD$ is vertical and $OD = 4$ units. The unit vectors **i**, **j** and **k** are parallel to $OA$, $OC$ and $OD$ respectively.

The midpoint of $AB$ is $M$ and the point $N$ on $CD$ is such that $DN = 3NC$.

**(a)** Find a vector equation for the line through $M$ and $N$. [5]

**(b)** Show that the length of the perpendicular from $O$ to $MN$ is $\frac{1}{3}\sqrt{82}$. [4]

**23)** OCT 2022-9709_31 Q11



In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge $OD$ is vertical, $DC$ is parallel to $OB$ and $DC = 1$ unit. The base, $OAB$, is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to $OA$, $OB$ and $OD$ respectively. The midpoint of $AB$ is $M$ and the point $N$ on $BC$ is such that $CN = 2NB$.

**(a)** Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of **i**, **j** and **k**. [4]

**(b)** Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

**(c)** Show that the length of the perpendicular from $M$ to $ON$ is $\sqrt{\frac{22}{5}}$. [4]

**24)** OCT 2022_9709_32 Q6

Relative to the origin $O$, the points $A$, $B$ and $C$ have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

**(a)** Using a scalar product, find the cosine of angle $BAC$. [4]

**(b)** Hence find the area of triangle $ABC$. Give your answer in a simplified exact form. [4]

**25)** OCT 2022_9709_33 Q9

With respect to the origin $O$, the position vectors of the points $A$, $B$ and $C$ are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of $AC$ is $M$ and the point $N$ lies on $BC$, between $B$ and $C$, and is such that $BN = 2NC$.

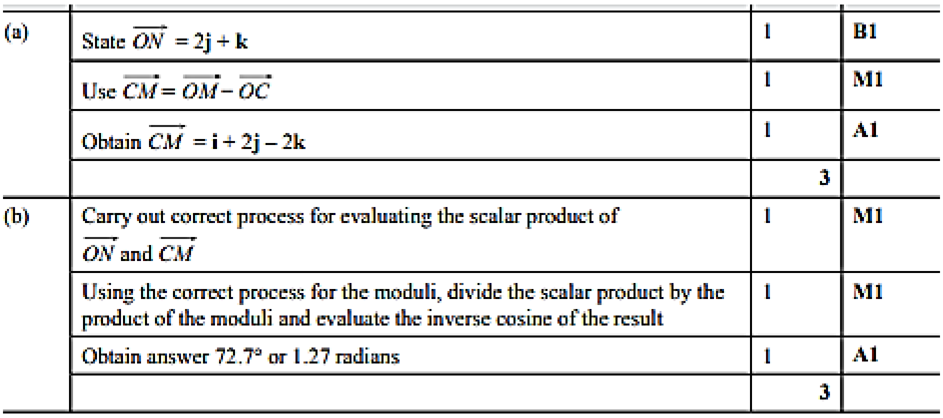
**(a)** Find the position vectors of $M$ and $N$. [3]

**(b)** Find a vector equation for the line through $M$ and $N$. [2]

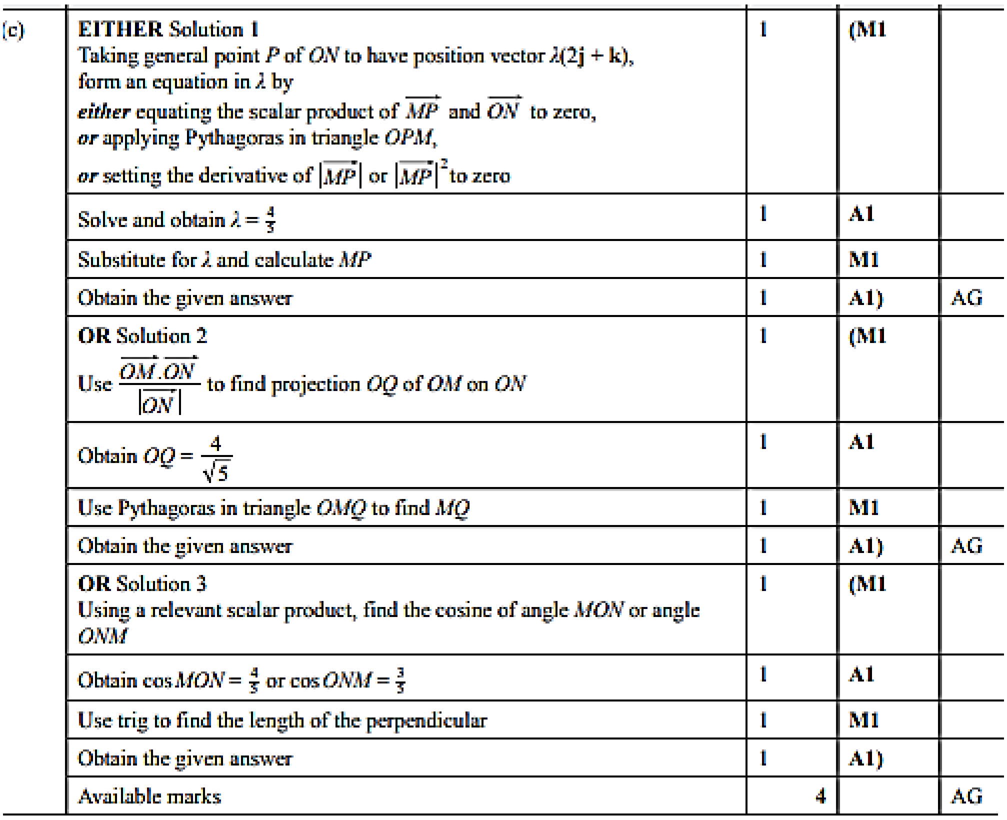
**(c)** Find the position vector of the point $Q$ where the line through $M$ and $N$ intersects the line through $A$ and $B$. [4]

## MARKING SCHEME

**1)** SP-2020 _9709_3 Q8



| | | | |
|---|---|---|---|
| (a) | State $\overrightarrow{ON} = 2\mathbf{j} + \mathbf{k}$ | 1 | B1 |
| | Use $\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC}$ | 1 | M1 |
| | Obtain $\overrightarrow{CM} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ | 1 | A1 |
| | | 3 | |
| (b) | Carry out correct process for evaluating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{CM}$ | 1 | M1 |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result | 1 | M1 |
| | Obtain answer 72.7° or 1.27 radians | 1 | A1 |
| | | 3 | |



| | | | | |
|---|---|---|---|---|
| (c) | **EITHER** Solution 1<br>Taking general point *P* of *ON* to have position vector $\lambda(2\mathbf{j} + \mathbf{k})$, form an equation in $\lambda$ by<br>***either*** equating the scalar product of $\overrightarrow{MP}$ and $\overrightarrow{ON}$ to zero,<br>***or*** applying Pythagoras in triangle *OPM*,<br>***or*** setting the derivative of $\lvert\overrightarrow{MP}\rvert$ or $\lvert\overrightarrow{MP}\rvert^2$ to zero | 1 | (M1 | |
| | Solve and obtain $\lambda = \frac{4}{5}$ | 1 | A1 | |
| | Substitute for $\lambda$ and calculate *MP* | 1 | M1 | |
| | Obtain the given answer | 1 | A1) | AG |
| | **OR** Solution 2<br>Use $\dfrac{\overrightarrow{OM}.\overrightarrow{ON}}{\lvert\overrightarrow{ON}\rvert}$ to find projection *OQ* of *OM* on *ON* | 1 | (M1 | |
| | Obtain $OQ = \dfrac{4}{\sqrt{5}}$ | 1 | A1 | |
| | Use Pythagoras in triangle *OMQ* to find *MQ* | 1 | M1 | |
| | Obtain the given answer | 1 | A1) | AG |
| | **OR** Solution 3<br>Using a relevant scalar product, find the cosine of angle *MON* or angle *ONM* | 1 | (M1 | |
| | Obtain $\cos MON = \frac{4}{5}$ or $\cos ONM = \frac{3}{5}$ | 1 | A1 | |
| | Use trig to find the length of the perpendicular | 1 | M1 | |
| | Obtain the given answer | 1 | A1) | |
| | Available marks | 4 | | AG |

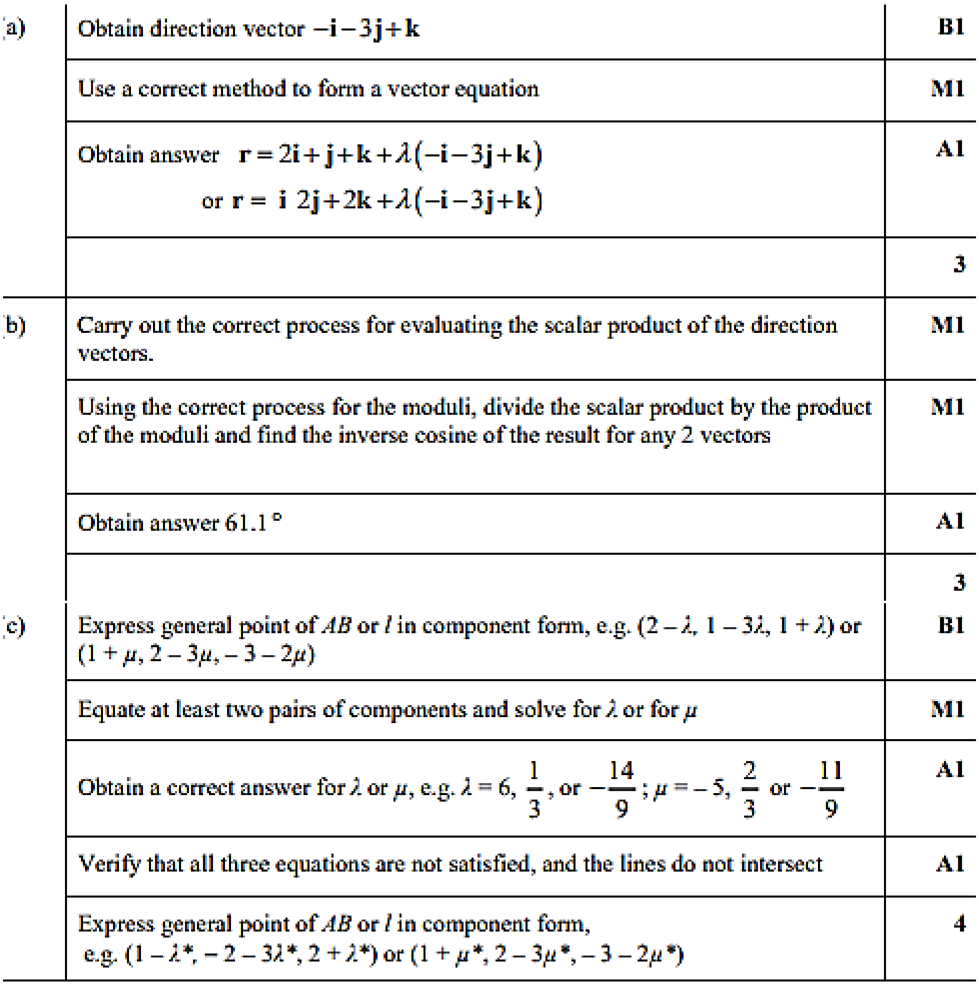
**2)** MARCH-2020 _9709_32 Q8

| | | |
|---|---|---|
| a) | Obtain $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j}$ | B1 |
| | Use a correct method to find $\overrightarrow{MN}$ | M1 |
| | Obtain $\overrightarrow{MN} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ | A1 |
| | | 3 |
| b) | Use a correct method to form an equation for *MN* | M1 |
| | Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent | A1 |
| | | 2 |
| c) | Find $\overrightarrow{DP}$ for a point *P* on *MN* with parameter $\lambda$, e.g. $(2-\lambda, 1+2\lambda, -2+2\lambda)$ | B1 |
| | Equate scalar product of $\overrightarrow{DP}$ and a direction vector for *MN* to zero and solve for $\lambda$ | M1 |
| | Obtain $\lambda = \frac{4}{9}$ | A1 |
| | State that the position vector of *P* is $\frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$ | A1 |
| | | 4 |

**3)** MARCH-2021 _9709_32 Q7

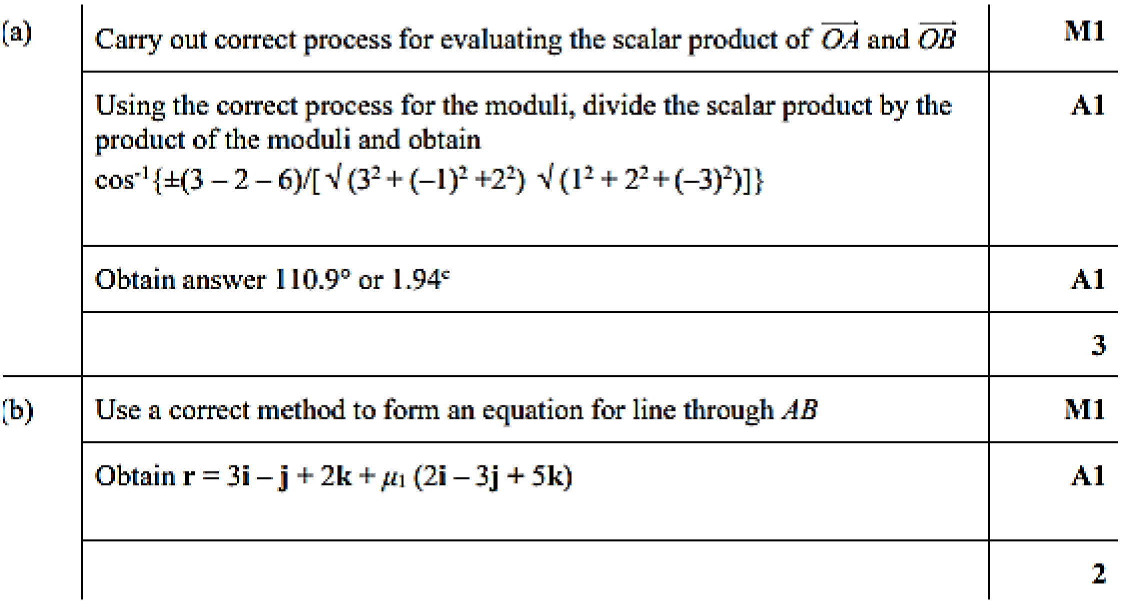
| | | |
|---|---|---|
| (a) | Express general point of a line in component form, e.g.<br>$(1 + 2s, 3 - s, 2 + 3s)$ or $(2 + t, 1 - t, 4 + 4t)$ | B1 |
| | Equate at least two pairs of components and solve for *s* or for *t* | M1 |
| | Obtain correct answer for *s* or for *t* (possible answers are $-1, 6, \frac{2}{5}$ for *s* and $-3, 4, -\frac{1}{5}$ for *t* ) | A1 |
| | Verify that all three component equations are not satisfied | A1 |
| | Show that the lines are not parallel and are thus skew | A1 |
| | | 5 |
| (b) | Carry out correct process for evaluating the scalar product of the direction vectors | M1 |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result | M1 |
| | Obtain answer 19.1° or 0.333 radians | A1 |
| | | 3 |

## 4) MARCH-2022 _9709_32 Q10



| | | |
|---|---|---|
| a) | Obtain direction vector $-\mathbf{i}-3\mathbf{j}+\mathbf{k}$ | B1 |
| | Use a correct method to form a vector equation | M1 |
| | Obtain answer $\mathbf{r}=2\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$<br>or $\mathbf{r}=\ \mathbf{i}\ 2\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$ | A1 |
| | | 3 |
| b) | Carry out the correct process for evaluating the scalar product of the direction vectors. | M1 |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result for any 2 vectors | M1 |
| | Obtain answer 61.1 ° | A1 |
| | | 3 |
| c) | Express general point of $AB$ or $l$ in component form, e.g. $(2-\lambda,\ 1-3\lambda,\ 1+\lambda)$ or $(1+\mu, 2-3\mu, -3-2\mu)$ | B1 |
| | Equate at least two pairs of components and solve for $\lambda$ or for $\mu$ | M1 |
| | Obtain a correct answer for $\lambda$ or $\mu$, e.g. $\lambda = 6,\ \frac{1}{3}$, or $-\frac{14}{9}$; $\mu = -5,\ \frac{2}{3}$ or $-\frac{11}{9}$ | A1 |
| | Verify that all three equations are not satisfied, and the lines do not intersect | A1 |
| | Express general point of $AB$ or $l$ in component form,<br>e.g. $(1-\lambda^*, -2-3\lambda^*, 2+\lambda^*)$ or $(1+\mu^*, 2-3\mu^*, -3-2\mu^*)$ | 4 |

## 5) MARCH-2023 _9709_32 Q10



| | | |
|---|---|---|
| (a) | Carry out correct process for evaluating the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ | M1 |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain<br>$\cos^{-1}\{\pm(3-2-6)/[\surd(3^2+(-1)^2+2^2)\ \surd(1^2+2^2+(-3)^2)]\}$ | A1 |
| | Obtain answer 110.9° or $1.94^c$ | A1 |
| | | 3 |
| (b) | Use a correct method to form an equation for line through $AB$ | M1 |
| | Obtain $\mathbf{r} = 3\mathbf{i}-\mathbf{j}+2\mathbf{k}+\mu_1(2\mathbf{i}-3\mathbf{j}+5\mathbf{k})$ | A1 |
| | | 2 |

| | | |
|---|---|---|
| (c) | Obtain a correct equation for line through *CD*<br>e.g. [**r** = ] **i** – 2**j** + 5**k** + $\lambda_1$(–4**i** + 4**j** – 6**k**) | **B1** |
| | Equate two pairs of components of general points on *their l* and *their CD* and solve for $\lambda$ or for $\mu$ | **M1** |
| | Obtain e.g. $\lambda_1 = -2$ or $\mu_1 = 3$ or $\lambda_2 = -1$ or $\mu_2 = -4$ | **A1** |
| | Obtain position vector 9**i** – 10**j** + 17**k** | **A1** |
| | | **4** |

**6)** JUNE-2020 _9709_31 Q9

| | | |
|---|---|---|
| (a) | State $\overrightarrow{AB}\left(\text{or } \overrightarrow{BA}\right)$ and $\overrightarrow{BC}\left(\text{or } \overrightarrow{CB}\right)$ in vector form | **B1** |
| | Calculate their scalar product | **M1** |
| | Show product is zero and confirm angle *ABC* is a right angle | **A1** |
| | | **3** |
| (b) | Use correct method to calculate the lengths of *AB* and *BC* | **M1** |
| | Show that *AB* = *BC* and the triangle is isosceles | **A1** |
| | | **2** |
| (c) | State a correct equation for the line through *B* and *C*,<br>e.g. $\mathbf{r} = \mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\mu(-2\mathbf{i}-\mathbf{j}-2\mathbf{k})$ | **B1** |
| | Taking a general point of *BC* to be *P*, form an equation in $\lambda$ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle *OBP* (or *OCP*), *or* setting the derivative of $\left\|\overrightarrow{OP}\right\|$ to zero | **M1** |
| | Solve and obtain $\lambda = -\frac{5}{9}$ | **A1** |
| | Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent | **A1** |

| | |
|---|---|
| **Alternative method for question 9(c)** | |
| Use a scalar product to find the projection *CN* (or *BN*) of *OC* (or *OB*) on *BC* | **M1** |
| Obtain answer $CN = \frac{5}{3}\left(\text{or } BN = \frac{14}{3}\right)$ | **A1** |
| Use Pythagoras to find *ON* | **M1** |
| Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent | **A1** |
| | **4** |

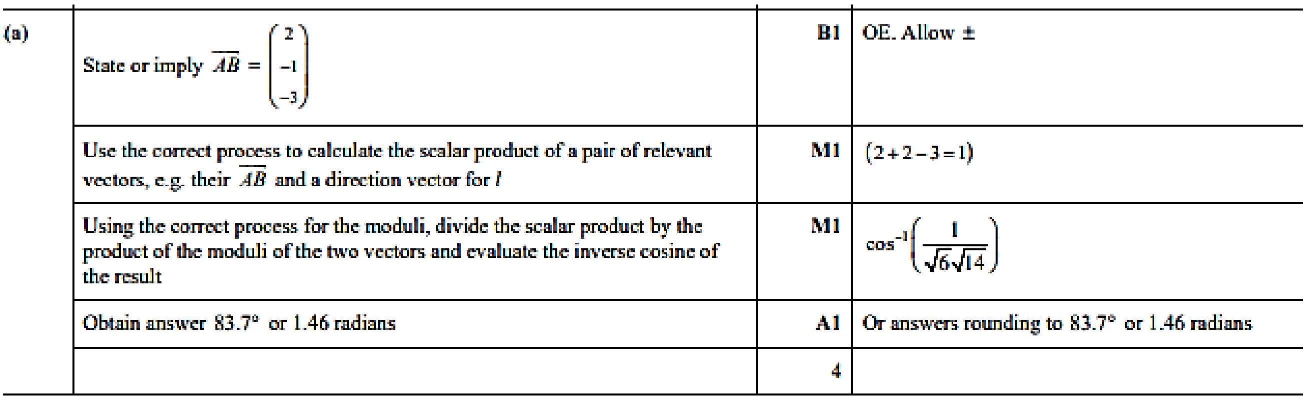
## 7) JUNE-2020 _9709_32 Q10

| | | |
|---|---|---|
| (a) | State that the position vector of $M$ is $3\mathbf{i}+\mathbf{j}$ | **B1** |
| | Use a correct method to find the position vector of $N$ | **M1** |
| | Obtain answer $\frac{10}{3}\mathbf{i}+2\mathbf{j}+2\mathbf{k}$ | **A1** |
| | Use a correct method to form an equation for $MN$ | **M1** |
| | Obtain correct answer in any form, e.g. $\mathbf{r}=3\mathbf{i}+\mathbf{j}+\lambda\left(\frac{1}{3}\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$ | **A1** |
| | | **5** |
| (b) | State or imply $\mathbf{r}=\mu(2\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ as equation for $OB$ | **B1** |
| | Equate sufficient components of $MN$ and $OB$ and solve for $\lambda$ or for $\mu$ | **M1** |
| | Obtain $\lambda=3$ or $\mu=2$ and position vector $4\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ for $P$ | **A1** |
| | | **3** |
| (c) | Carry out correct process for evaluating the scalar product of direction vectors for $OP$ and $MP$, or equivalent | **M1** |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result | **M1** |
| | Obtain answer 21.6° | **A1** |
| | | **3** |

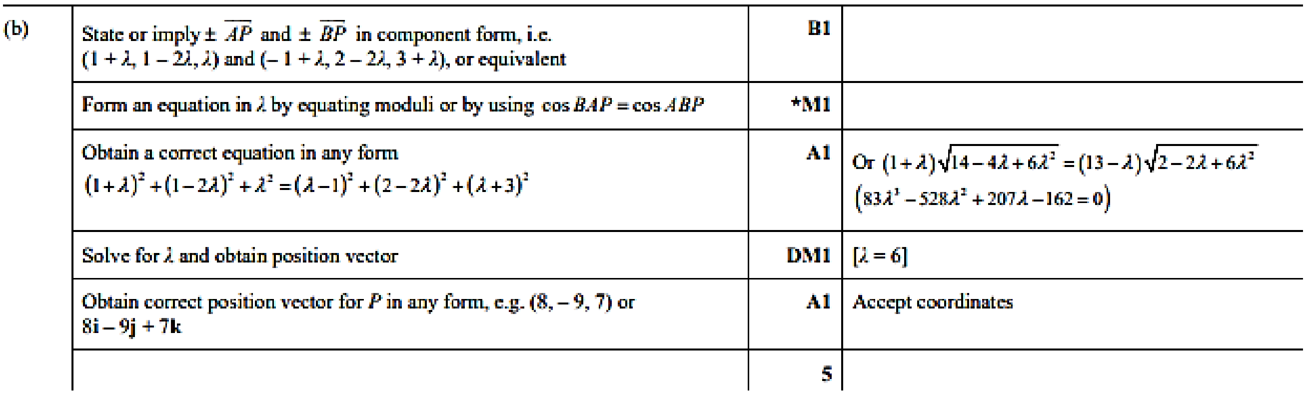
## 8) JUNE-2020 _9709_33 Q8

| | | |
|---|---|---|
| (a) | State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form | **B1** |
| | Use a correct method for finding the position vector of $C$ | **M1** |
| | Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent | **A1** |
| | Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. $AB$ and $AD$ | **M1** |
| | Show that $ABCD$ has a pair of unequal adjacent sides | **A1** |
| | **Alternative method for question 8(a)** | |
| | State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form | **B1** |
| | Use a correct method for finding the position vector of $C$ | **M1** |
| | Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent | **A1** |
| | Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent | **M1** |
| | Show that the diagonals of $ABCD$ are not perpendicular | **A1** |
| | | **5** |
| (b) | Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$ | **M1** |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result | **M1** |
| | Obtain answer 100.3° | **A1** |
| | | **3** |
| (c) | Use a correct method to calculate the area, e.g. calculate $AB.AC\sin BAD$ | **M1** |
| | Obtain answer 11.0<br>**(FT on angle BAD)** | **A1 FT** |
| | | **2** |

## 9) JUNE-2021 _9709_31 Q8

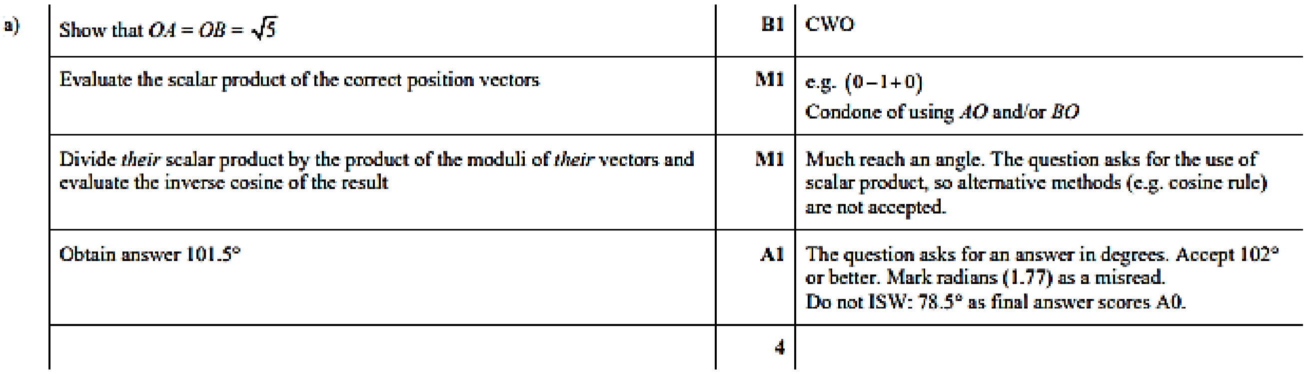


| | | | |
|---|---|---|---|
| (a) | State or imply $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$ | B1 | OE. Allow ± |
| | Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. their $\overrightarrow{AB}$ and a direction vector for $l$ | M1 | $(2+2-3=1)$ |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result | M1 | $\cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{14}}\right)$ |
| | Obtain answer 83.7° or 1.46 radians | A1 | Or answers rounding to 83.7° or 1.46 radians |
| | | 4 | |



| | | | |
|---|---|---|---|
| (b) | State or imply $\pm\ \overrightarrow{AP}$ and $\pm\ \overrightarrow{BP}$ in component form, i.e. $(1 + \lambda, 1 - 2\lambda, \lambda)$ and $(-1 + \lambda, 2 - 2\lambda, 3 + \lambda)$, or equivalent | B1 | |
| | Form an equation in $\lambda$ by equating moduli or by using $\cos BAP = \cos ABP$ | *M1 | |
| | Obtain a correct equation in any form<br>$(1+\lambda)^2+(1-2\lambda)^2+\lambda^2=(\lambda-1)^2+(2-2\lambda)^2+(\lambda+3)^2$ | A1 | Or $(1+\lambda)\sqrt{14-4\lambda+6\lambda^2}=(13-\lambda)\sqrt{2-2\lambda+6\lambda^2}$<br>$\left(83\lambda^3-528\lambda^2+207\lambda-162=0\right)$ |
| | Solve for $\lambda$ and obtain position vector | DM1 | $[\lambda=6]$ |
| | Obtain correct position vector for $P$ in any form, e.g. $(8, -9, 7)$ or $8\mathbf{i} - 9\mathbf{j} + 7\mathbf{k}$ | A1 | Accept coordinates |
| | | 5 | |

## 10) JUNE-2021 _9709_32 Q11

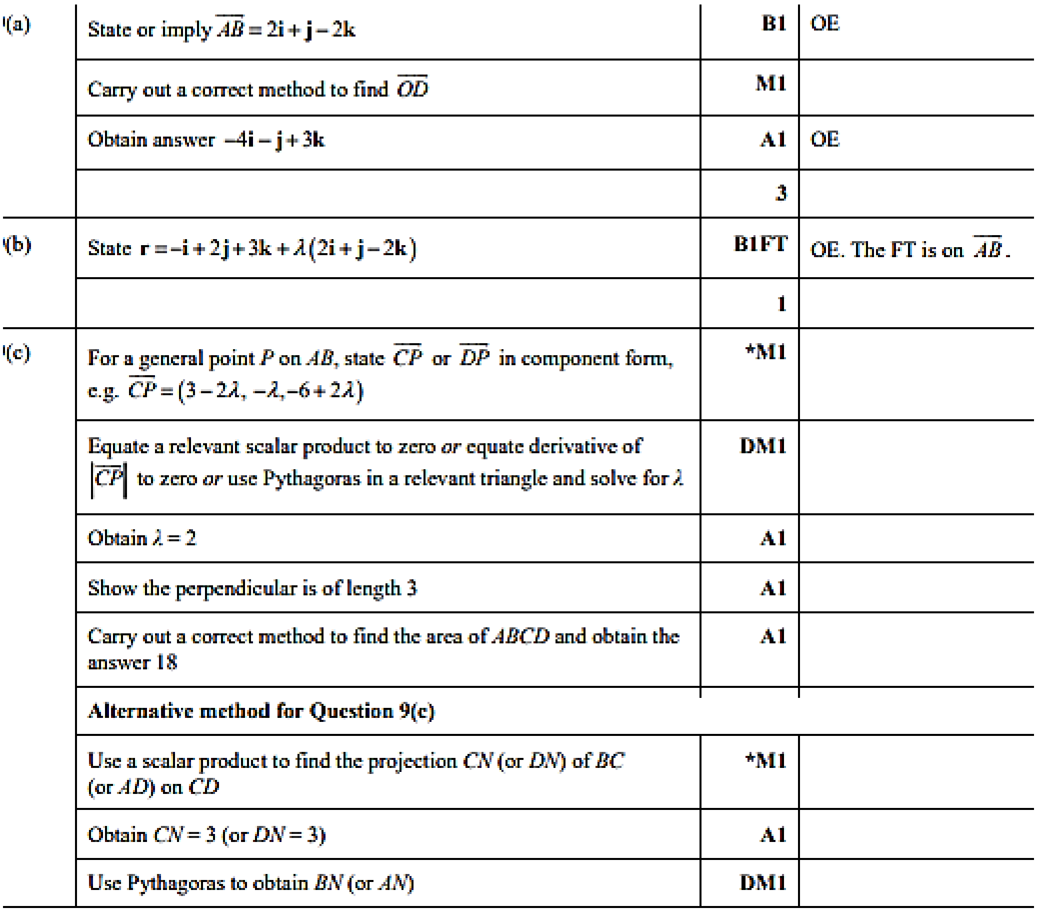


| | | | |
|---|---|---|---|
| a) | Show that $OA = OB = \sqrt{5}$ | B1 | CWO |
| | Evaluate the scalar product of the correct position vectors | M1 | e.g. $(0-1+0)$<br>Condone of using $AO$ and/or $BO$ |
| | Divide *their* scalar product by the product of the moduli of *their* vectors and evaluate the inverse cosine of the result | M1 | Much reach an angle. The question asks for the use of scalar product, so alternative methods (e.g. cosine rule) are not accepted. |
| | Obtain answer 101.5° | A1 | The question asks for an answer in degrees. Accept 102° or better. Mark radians (1.77) as a misread.<br>Do not ISW: 78.5° as final answer scores A0. |
| | | 4 | |

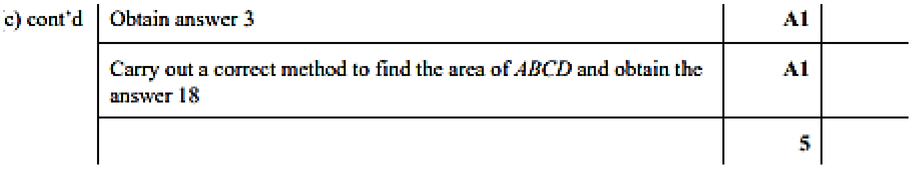
| | | | |
|---|---|---|---|
| (b) | State or imply $M$ has position vector **i** – **k** | **B1** | OE |
| | Taking a general point of $OM$ to have position vector $\lambda$**i** – $\lambda$**k**, express $AP = \sqrt{7}\ OA$ as an equation in $\lambda$ | ***M1** | $\lambda\left(their\ \overline{OM}\right)$ |
| | State a correct equation in any form | **A1** | e.g. $\sqrt{(-2+\lambda)^2+1+(-\lambda)^2} = \sqrt{7}\sqrt{5}$ |
| | Reduce to $\lambda^2 - 2\lambda - 15 = 0$ | **A1** | OE |
| | Solve a quadratic and state a position vector | **DM1** | |
| | Obtain answers 5**i** – 5**k** and – 3**i** + 3**k** | **A1** | Accept coordinates |
| | **Alternative method for Question 11(b)** | | |
| | State or imply that $OP = \gamma\sqrt{2}$ | **B1** | |
| | State or imply that $\cos\frac{1}{2}AOB = \sqrt{\frac{2}{5}}$ and use cosine rule to form an equation in $\gamma$ | ***M1** | Allow $\cos\frac{1}{2}AOB = 0.632...$ |
| | State a correct equation in any form | **A1** | e.g. $35 = 5 + 2\gamma^2 - 2\sqrt{5}.\gamma\sqrt{2}.\frac{\sqrt{2}}{\sqrt{5}}$ |
| | Reduce to $\gamma^2 - 2\gamma - 15 = 0$ | **A1** | OE |
| | Solve a quadratic and state a position vector | **DM1** | |
| | Obtain answers 5**i** – 5**k** and – 3**i** + 3**k** | **A1** | Accept coordinates |

| | | | |
|---|---|---|---|
| (b) | **Alternative method for Question 11(b)** | | |
| | State or imply $M$ has position vector **i** – **k** | **B1** | OE |
| | State or imply that $AM = \sqrt{3}$ | **B1** | |
| | Use Pythagoras to find $MP$ | ***M1** | $MP = \sqrt{35 - (AM)^2}$ |
| | Obtain $MP = 4\sqrt{2}$ | **A1** | |
| | Correct method to find a position vector | **DM1** | $(\mathbf{i}-\mathbf{k}) \pm 4(\mathbf{i}-\mathbf{k})$ |
| | Obtain answers 5**i** – 5**k** and – 3**i** + 3**k** | **A1** | Accept coordinates |
| | | **6** | |

## **11)** JUNE-2021 _9709_33 Q9



| | | | |
|---|---|---|---|
| (a) | State or imply $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ | **B1** | OE |
| | Carry out a correct method to find $\overrightarrow{OD}$ | **M1** | |
| | Obtain answer $-4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ | **A1** | OE |
| | | **3** | |
| (b) | State $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ | **B1FT** | OE. The FT is on $\overrightarrow{AB}$. |
| | | **1** | |
| (c) | For a general point *P* on *AB*, state $\overrightarrow{CP}$ or $\overrightarrow{DP}$ in component form, e.g. $\overrightarrow{CP} = (3 - 2\lambda,\ -\lambda, -6 + 2\lambda)$ | ***M1** | |
| | Equate a relevant scalar product to zero *or* equate derivative of $\lvert\overrightarrow{CP}\rvert$ to zero *or* use Pythagoras in a relevant triangle and solve for $\lambda$ | **DM1** | |
| | Obtain $\lambda = 2$ | **A1** | |
| | Show the perpendicular is of length 3 | **A1** | |
| | Carry out a correct method to find the area of *ABCD* and obtain the answer 18 | **A1** | |
| | **Alternative method for Question 9(c)** | | |
| | Use a scalar product to find the projection *CN* (or *DN*) of *BC* (or *AD*) on *CD* | ***M1** | |
| | Obtain *CN* = 3 (or *DN* = 3) | **A1** | |
| | Use Pythagoras to obtain *BN* (or *AN*) | **DM1** | |



| | | | |
|---|---|---|---|
| c) cont'd | Obtain answer 3 | **A1** | |
| | Carry out a correct method to find the area of *ABCD* and obtain the answer 18 | **A1** | |
| | | **5** | |

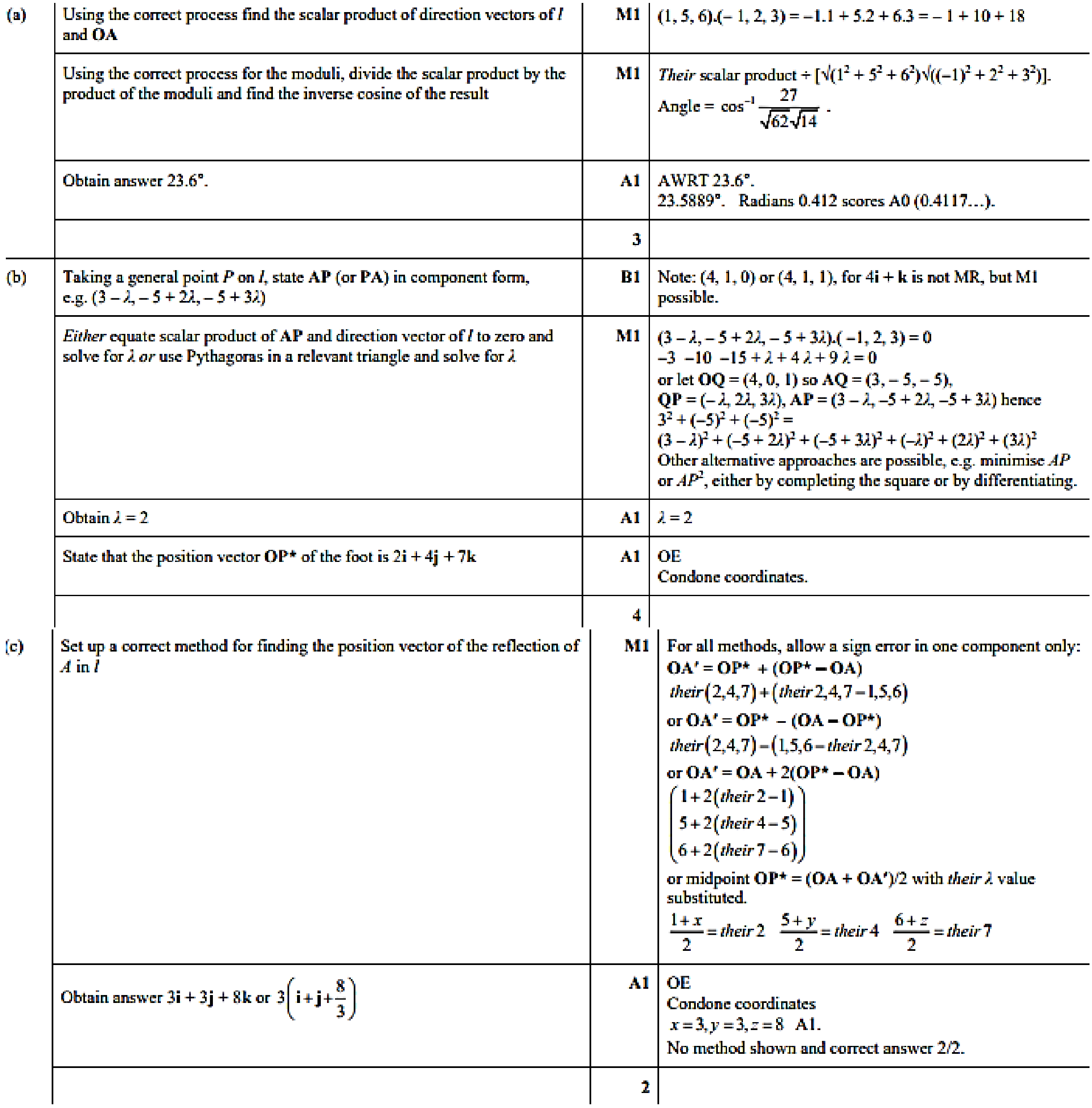
## **12)** JUNE-2022 _9709_31 Q9

|  |  |  |  |
|---|---|---|---|
| (a) | Obtain $\overrightarrow{OM} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ | B1 |  |
|  | Use a correct method to find $\overrightarrow{MN}$ | M1 | e.g. $\overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AN}$ or $\overrightarrow{MO} + \overrightarrow{ON}$ |
|  | Obtain $\overrightarrow{MN} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ | A1 | Accept any notation. |
|  |  | 3 |  |
| (b) | Use a correct method to form an equation for $MN$ | M1 | Allow without $\mathbf{r} = \ldots$ |
|  | Obtain $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ | A1 FT | OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$<br>Must have $\mathbf{r} = \ldots$ Follow *their* answers to part 9(a). |
|  |  | 2 |  |
| (c) | State $\overrightarrow{OP}$ for a general point $P$ on $MN$ in component form, e.g. $(2+\lambda, 3+\lambda, -2\lambda)$ | B1 |  |
|  | Equate scalar product of $\overrightarrow{OP}$ and a direction vector for $MN$ to zero and solve for $\lambda$ | M1 |  |
|  | Obtain $\lambda = -\frac{5}{6}$ | A1 | OE e.g. $\mu = \frac{1}{6}$ |
|  | Obtain $\sqrt{\frac{53}{6}}$ correctly | A1 | AG e.g. from $\sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{13}{6}\right)^2 + \left(\frac{5}{3}\right)^2}$ |
|  |  | 4 |  |

**13)** JUNE-2022 _9709_32 Q9

|  |  |  |  |
|---|---|---|---|
| (a) | Express general point of $l$ or $m$ in component form, i.e.<br>$(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$ | B1 |  |
|  | Equate components and eliminate either $\lambda$ or $\mu$ | M1 | e.g. $\mu = \frac{2}{1-b}$, $\lambda = \frac{-1-b}{1-b}$, $\mu = \frac{-4}{2+a}$, $\lambda = \frac{a+6}{a+2}$ |
|  | Eliminate the other parameter or obtain a second expression in the first | M1 | $\lambda$ and $\mu$ are not required to be the subject of the equations. |
|  | Show intermediate steps to obtain $2b - a = 4$ | A1 | AG |
|  | **Alternative method for question 9(a)** |  |  |
|  | Express general point of $l$ or $m$ in component form, i.e.<br>$(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$ | B1 |  |
|  | Express $a$ or $b$ in terms of $\lambda$ and $\mu$ | M1 | $a = \dfrac{2\lambda - 6}{\mu}$, $b = \dfrac{-1-\lambda}{\mu}$ |
|  | Use $\lambda = 1 - \mu$ | M1 |  |
|  | Obtain $2b - a = 4$ | A1 | AG |
|  |  | 4 |  |
| (b) | Using the correct process equate the scalar product of the direction vectors to zero | *M1 | $(2\mathbf{i} - \mathbf{j} - \mathbf{k}).(a\mathbf{i} + b\mathbf{j} + \mathbf{k}) = 0$ SOI. |
|  | Obtain $2a - b - 1 = 0$ | A1 | OE e.g. $2(2b-4) - b - 1 = 0$ |
|  | Solve simultaneous equations for $a$ or for $b$ | DM1 |  |
|  | Obtain $a = 2, b = 3$ | A1 |  |
|  |  | 4 |  |
| (c) | Substitute found values in component equations and solve for $\lambda$ or for $\mu$ | M1 |  |
|  | Obtain answer $3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ from either $\lambda = 2$ or $\mu = -1$ | A1 | Accept as coordinates or equivalent. |
|  |  | 2 |  |

## **14)** JUNE-2022 _9709_33 Q9



| | | | |
|---|---|---|---|
| (a) | Using the correct process find the scalar product of direction vectors of *l* and **OA** | **M1** | $(1, 5, 6).(-1, 2, 3) = -1.1 + 5.2 + 6.3 = -1 + 10 + 18$ |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result | **M1** | *Their* scalar product $\div [\sqrt{(1^2 + 5^2 + 6^2)}\sqrt{((-1)^2 + 2^2 + 3^2)}]$.<br>Angle $= \cos^{-1} \frac{27}{\sqrt{62}\sqrt{14}}$ . |
| | Obtain answer 23.6°. | **A1** | AWRT 23.6°.<br>23.5889°. Radians 0.412 scores A0 (0.4117…). |
| | | **3** | |
| (b) | Taking a general point *P* on *l*, state **AP** (or **PA**) in component form, e.g. $(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)$ | **B1** | Note: (4, 1, 0) or (4, 1, 1), for **4i + k** is not MR, but M1 possible. |
| | *Either* equate scalar product of **AP** and direction vector of *l* to zero and solve for $\lambda$ *or* use Pythagoras in a relevant triangle and solve for $\lambda$ | **M1** | $(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda).(-1, 2, 3) = 0$<br>$-3\ -10\ -15 + \lambda + 4\lambda + 9\lambda = 0$<br>or let **OQ** = (4, 0, 1) so **AQ** = (3, – 5, – 5),<br>**QP** $= (-\lambda, 2\lambda, 3\lambda)$, **AP** $= (3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)$ hence<br>$3^2 + (-5)^2 + (-5)^2 =$<br>$(3 - \lambda)^2 + (-5 + 2\lambda)^2 + (-5 + 3\lambda)^2 + (-\lambda)^2 + (2\lambda)^2 + (3\lambda)^2$<br>Other alternative approaches are possible, e.g. minimise *AP* or $AP^2$, either by completing the square or by differentiating. |
| | Obtain $\lambda = 2$ | **A1** | $\lambda = 2$ |
| | State that the position vector **OP*** of the foot is **2i + 4j + 7k** | **A1** | OE<br>Condone coordinates. |
| | | **4** | |
| (c) | Set up a correct method for finding the position vector of the reflection of *A* in *l* | **M1** | For all methods, allow a sign error in one component only:<br>**OA′ = OP* + (OP* – OA)**<br>$\textit{their}(2,4,7) + (\textit{their}\ 2,4,7 - 1,5,6)$<br>or **OA′ = OP* – (OA – OP*)**<br>$\textit{their}(2,4,7) - (1,5,6 - \textit{their}\ 2,4,7)$<br>or **OA′ = OA + 2(OP* – OA)**<br>$\begin{pmatrix} 1+2(\textit{their}\ 2-1) \\ 5+2(\textit{their}\ 4-5) \\ 6+2(\textit{their}\ 7-6) \end{pmatrix}$<br>or midpoint **OP* = (OA + OA′)/2** with *their* $\lambda$ value substituted.<br>$\frac{1+x}{2} = \textit{their}\ 2 \quad \frac{5+y}{2} = \textit{their}\ 4 \quad \frac{6+z}{2} = \textit{their}\ 7$ |
| | Obtain answer **3i + 3j + 8k** or $3\left(\mathbf{i}+\mathbf{j}+\frac{8}{3}\right)$ | **A1** | OE<br>Condone coordinates<br>$x = 3, y = 3, z = 8$ A1.<br>No method shown and correct answer 2/2. |
| | | **2** | |

## **15)** JUNE-2023 _9709_31 Q6

| | | | |
|---|---|---|---|
| (a) | Obtain a vector for one side of the parallelogram | B1 | e.g. $\overline{AB}=\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overline{BC}=\begin{pmatrix}-1\\-5\\-6\end{pmatrix}$. |
| | Correct method to obtain $\pm\overline{OD}$ | M1 | e.g. $\overline{OD}=\overline{OA}+\overline{BC}$.<br>M0 if use $\overline{AB}=\overline{CD}$ or $\overline{BC}=\overline{DA}$. |
| | Obtain $\overline{OD}=\mathbf{i}-4\mathbf{j}-3\mathbf{k}$ | A1 | Any equivalent form. Accept coordinates. |
| | | 3 | |
| (b) | Using the correct process, evaluate the scalar product $\overline{BA}.\overline{BC}$ | M1 | $(2+10-6)$ Scalar product of two relevant vectors. OE |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli. | M1 | $\dfrac{2+10-6}{\sqrt{9}\times\sqrt{62}}$. |
| | Obtain answer $\dfrac{2}{\sqrt{62}}$ | A1 | ISW Or simplified equivalent i.e. $\dfrac{\sqrt{62}}{31}$. |
| | | 3 | |
| (c) | State or imply $\sin\theta=\sqrt{\dfrac{58}{62}}$ | B1 FT | Follow *their* $\cos\theta$. |
| | Use correct method to find the area of $ABCD$ | M1 | e.g. $2\times\frac{1}{2}BA\times BC\sin\theta$. Condone decimals. |
| | Correct unsimplified expression for the area | A1 FT | e.g. $2\times\frac{1}{2}\times3\times\sqrt{62}\times\sin\theta$. Condone decimals.<br>Follow *their* sides and angle. |
| | Obtain answer $3\sqrt{58}$ | A1 | Correct only. |
| | | 4 | |

## **16)** JUNE-2023 _9709_32 Q11

<table>
<tr><td>(a)</td><td>Carry out correct method for finding a vector equation for <i>AB</i></td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain [r =] i + 2j – 2k + λ (i – 3j + 3k)</td><td>A1</td><td>OE e.g. $\mathbf{r}=2\mathbf{i}-\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}+3\mathbf{j}-3\mathbf{k})$.</td></tr>
<tr><td></td><td>Equate two pairs of components of general points on <i>their AB</i> and <i>l</i> and evaluate λ or μ</td><td>M1</td><td>$\begin{pmatrix}1+\lambda\\2-3\lambda\\-2+3\lambda\end{pmatrix}=\begin{pmatrix}1+2\mu\\-1-3\mu\\3+4\mu\end{pmatrix}$.</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. λ = –1, μ = –2</td><td>A1</td><td>Correct value from two correct component equations.</td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect (≠ is sufficient justification e.g. 0 ≠ –3).</td><td>A1</td><td>Conclusion needs to follow correct values.<br>Hybrid versions are possible e.g. using <b>j</b> and <b>k</b> to get one parameter and then <b>i</b> to obtain the other.<br>or e.g. solving two pairs of simultaneous equations and showing that the results are not the same.<br>Alternatives:
<table>
<tr><td><i>A</i></td><td>λ</td><td>μ</td><td></td><td><i>B</i></td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>2</td><td>1</td><td>4 ≠ 7</td><td>ij</td><td>1</td><td>1</td><td>4 ≠ 7</td></tr>
<tr><td>ik</td><td>5</td><td>5/2</td><td>–13≠-17/2</td><td>ik</td><td>4</td><td>5/2</td><td>–13≠-17/2</td></tr>
<tr><td>jk</td><td>–1</td><td>–2</td><td>0 ≠ –3</td><td>jk</td><td>–2</td><td>–2</td><td>0 ≠ –3</td></tr>
</table>
</td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
</table>

| | | | |
|---|---|---|---|
| (b) | Find $\overline{AP}$ for a general point $P$ on $l$, e.g. $-3\mathbf{j}+5\mathbf{k}+\mu(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})$ | **B1** | Or equivalent e.g. $\overline{PA}=-2\mu\mathbf{i}+(3\mu+3)\mathbf{j}-(4\mu+5)\mathbf{k}$. |
| | Calculate scalar product of *their* $\overline{AP}$ and a direction vector for $l$ and equate the result to zero | **M1** | e.g. $4\mu+(9+9\mu)+(20+16\mu)=0$.<br>M0 if using $\overline{OP}$.<br>M0 if using parallel line through $A$. |
| | Obtain $\mu=-1$ | **A1** | |
| | Obtain answer $-\mathbf{i}+2\mathbf{j}-\mathbf{k}$ | **A1** | Accept coordinates in place of position vector. |
| | **Alternative Method for Question 11(b)** | | |
| | Find $\overline{AP}$ for a general point $P$ on $l$, e.g. $-3\mathbf{j}+5\mathbf{k}+\mu(2\mathbf{i}-3\mathbf{j}+4\mathbf{k})$ | **B1** | Or equivalent e.g. $\overline{PA}=-2\mu\mathbf{i}+(3\mu+3)\mathbf{j}-(4\mu+5)\mathbf{k}$. |
| | Use Pythagoras and differentiate with respect to $\mu$ to obtain value of $\mu$ corresponding to minimum distance. (No need to prove it is a minimum) | **M1** | $\frac{\mathrm{d}}{\mathrm{d}\mu}\left(4\mu^2+9(\mu+1)^2+(4\mu+5)^2\right)=0$. |
| | Obtain $\mu=-1$ | **A1** | |
| | Obtain answer $-\mathbf{i}+2\mathbf{j}-\mathbf{k}$ | **A1** | Accept coordinates in place of position vector. |
| | | **4** | |

## **17)** JUNE-2023 _9709_33 Q9

| | | | |
|---|---|---|---|
| )(a) | Perform scalar product of direction vectors and set result equal to zero | **M1** | $2c+6+4=0$. |
| | Use $P$ to find the value of $\lambda$ | **M1** | $3-2\lambda=7\Rightarrow\lambda=-2$ $[a+\lambda c=4, b+4\lambda=-2]$.<br>Equation for line $l$ may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda=2$ all marks available. |
| | Obtain $c=-5$ or $b=6$ | **A1** | |
| | $a=-6, b=6$ and $c=-5$ all correct | **A1** | |
| | | **4** | **SC1:** Use $P$ to find the value of $\lambda$ M1<br>Substitute $\lambda=-2$ into point $P$, so $a-2c=4$, and put $\mu=-1$ and $\lambda=-1$ into $l$ so $a-c=-1$, then solve to obtain $a=-6$, $b=6$ and $c=-5$.<br>All 3 values correct A1.<br>Max 2/4. |
| (b) | Find $\overline{PQ}$ (or $\overline{QP}$) for a general point $Q$ on $m$<br>$=\pm((1+2\mu, 2-3\mu, 3+\mu)-(a+\lambda c, 3-2\lambda, b+4\lambda))$ | **B1** | $\left[\overline{PQ}\text{ or }\overline{QP}=\pm\begin{pmatrix}-3+2\mu\\-5-3\mu\\5+\mu\end{pmatrix}\right]$<br>Could be *their* $a$, $b$, $c$ and $\lambda$ values provided M1 M1 gained in **(a)**.<br>Allow expression in answer column. |
| | Equate the scalar product of $\overline{PQ}$ (or $\overline{QP}$) and a direction vector for $m$ to zero and obtain an equation in $\mu$ | **M1*** | $(2(-3+2\mu)-3(-5-3\mu)+(5+\mu))=0$.<br>Allow $\overline{PQ}=\overline{OQ}+\overline{OP}$ sign problem. |
| | Solve and obtain $\mu=-1$ | **A1** | $PQ^2=(-3+2\mu)^2+(-5-3\mu)^2+(5+\mu)^2$.<br>$[=14(\mu+1)^2+45]$.<br>Min when $\mu=-1$ or by differentiation. |
| | Obtain $\overline{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ or $\overline{PQ}=-5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$<br>Must be labelled correctly | **A1** | The working may be in **(a)** provided at least this result is used in **(b)**. |
| | Carry out a method to find the position vector of $R$<br><br>**Alternative method for DM1**<br>$\overline{OR}=(4, 7, -2)+t(-5, -2, 4)$ $\overline{QR}=\overline{OR}-\overline{OQ}$<br>Solve $\lvert QR\rvert^2=\frac{9}{4}\lvert PQ\rvert^2$ or $\lvert QR\rvert=\frac{3}{2}\lvert PQ\rvert$ $t=2.5$ | **DM1** | e.g. Use $\overline{OR}=\overline{OP}+\frac{5}{2}\overline{PQ}$ or $\overline{OR}=\overline{OQ}+\frac{3}{2}\overline{PQ}$ or<br>$\overline{OR}=\frac{5}{2}\overline{OQ}-\frac{3}{2}\overline{OP}$ or $2\overline{QR}=2(\overline{OR}-\overline{OQ})=3\overline{PQ}$<br>where $\overline{OR}=(x, y, z)$.<br>$\overline{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overline{QP}$ for $\overline{PQ}$. |

|  |  |  |  |
|---|---|---|---|
| (b) | Obtain $-\frac{17}{2}i+2j+8k$ from correct working | A1 | Accept coordinates.<br>Don't accept $-\frac{17}{2}\mathbf{i}+\frac{4}{2}\mathbf{j}+\frac{16}{2}\mathbf{k}$. |
|  |  | 6 | SC2 Equate lines, attempt to find $\mu=-1$ or $\lambda=-1$ M1*<br>$\overline{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ A1.<br>Attempt to find $\overline{OQ}$ using other parameter value DM1.<br>$\overline{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ therefore intersect A1.<br>Then use main scheme for the final DM1 A1. |
|  |  |  | First **DM1 A1** are available if they show the 3 coordinates are consistent for the 2 parameter values instead of attempting to find $\overline{OQ}$ using the other parameter value and then showing intersection |

**18)** OCT 2020_9709_31 Q11

|  |  |  |  |
|---|---|---|---|
| (a) | Express general point of at least one line correctly in component form, i.e.<br>$(1+a\lambda,\ 2+2\lambda,\ 1-\lambda)$ or $(2+2\mu,\ 1-\mu,\ -1+\mu)$ | B1 |  |
|  | Equate at least two pairs of corresponding components and solve for $\lambda$ or for $\mu$ | M1 | May be implied<br>$1+a\lambda=2+2\mu$ $\ 2+2\lambda=1-\mu$ $\ 1-\lambda=-1+\mu$ |
|  | Obtain $\lambda=-3$ or $\mu=5$ | A1 |  |
|  | Obtain $a=-\frac{11}{3}$ | A1 | Allow $a=-3.667$ |
|  | State that the point of intersection has position vector<br>$12\mathbf{i}-4\mathbf{j}+4\mathbf{k}$ | A1 | Allow coordinate form $(12, -4, 4)$ |
|  |  | 5 |  |

|  |  |  |  |
|---|---|---|---|
| (b) | Use correct process for finding the scalar product of direction vectors for the two lines | M1 | $(a, 2, -1).(2, -1, 1)=2a-2-1$ or $2a-3$ |
|  | Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$ | *M1 |  |
|  | State a correct equation in $a$ in any form,<br>e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}}=\pm\frac{1}{6}$ | A1 |  |
|  | Solve for $a$ | DM1 | Solve 3-term quadratic for $a$ having expanded $(2a-3)^2$ to produce 3 terms e.g.<br>$36(2a-3)^2=6(a^2+5)$ $\ 138a^2-432a+294=0$<br>$23a^2-72a+49=0$ $\ (23a-49)(a-1)=0$ |
|  | Obtain $a=1$ | A1 |  |
|  | Obtain $a=\frac{49}{23}$ | A1 | Allow $a=2.13$ |

| b) | **Alternative method for question 11(b)** | | |
|---|---|---|---|
| | $\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$<br>$- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert . \lvert 2^2 + (-1)^2 + 1^2\rvert]$ | **M1** | Use of cosine rule. Must be correct vectors. |
| | Equate the result to $\pm\frac{1}{6}$ | ***M1**<br>**A1** | Allow M1* here for any two vectors |
| | Solve for $a$ | **DM1** | Solve 3-term quadratic for $a$ having expanded $(2a-3)^2$ to produce 3 terms e.g.<br>$36(2a-3)^2 = 6(a^2+5)$  $138a^2 - 432a + 294 = 0$<br>$23a^2 - 72a + 49 = 0$  $(23a-49)(a-1) = 0$ |
| | Obtain $a = 1$ | **A1** | |
| | Obtain $a = \frac{49}{23}$ | **A1** | Allow $a = 2.13$ |
| | | **6** | |

**19)** OCT 2020_9709_32 Q8

<table>
<tr><td>a)</td><td>Obtain $\overrightarrow{AB} = \begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td><b>B1</b></td><td>Or equivalent seen or implied</td></tr>
<tr><td></td><td>Use the correct process for calculating the modulus of both vectors to obtain $AB$ and $CD$</td><td><b>M1</b></td><td>$AB = \sqrt{24}, CD = \sqrt{6}$</td></tr>
<tr><td></td><td>Using exact values, verify that $AB = 2CD$</td><td><b>A1</b></td><td>Obtain given statement from correct work<br>Allow from $BA = 2DC$, OE</td></tr>
<tr><td></td><td></td><td><b>3</b></td><td></td></tr>
<tr><td>b)</td><td>Use the correct process to calculate the scalar product of the relevant vectors (<i>their</i> $\overrightarrow{AB}$ and $\overrightarrow{CD}$ )</td><td><b>M1</b></td><td>$\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}2\\1\\1\end{pmatrix}$ or $\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}4\\2\\2\end{pmatrix}$</td></tr>
<tr><td></td><td>Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result</td><td><b>M1</b></td><td></td></tr>
<tr><td></td><td>Obtain answer 99.6° (or 1.74 radians) or better</td><td><b>A1</b></td><td>Do not ISW if go on to subtract from 180°<br>(99.594…, 1.738...) Accept 260.4°</td></tr>
<tr><td></td><td></td><td><b>3</b></td><td></td></tr>
<tr><td>(c)</td><td>State correct vector equations for $AB$ and $CD$ in any form,<br>e.g. $(\mathbf{r} =)\begin{pmatrix}2\\1\\5\end{pmatrix} + \lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r} =)\begin{pmatrix}1\\1\\2\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td><b>B1ft</b></td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$<br>Alternative: $(\mathbf{r} =)\begin{pmatrix}4\\-1\\1\end{pmatrix} + \lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r} =)\begin{pmatrix}3\\2\\3\end{pmatrix} + \mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td></tr>
<tr><td></td><td>Equate at least two pairs of components of their lines and solve for $\lambda$ or for $\mu$</td><td><b>M1</b></td><td></td></tr>
<tr><td></td><td>Obtain correct pair of values from correct equations</td><td><b>A1</b></td><td>Alternatives when taking A or B as point on line
<table>
<tr><td><i>A</i></td><td>$\lambda$</td><td>$\mu$</td><td></td><td><i>B</i></td><td>$\lambda$</td><td>$\mu$</td><td></td></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3} \neq \frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3} \neq \frac{7}{3}$</td></tr>
<tr><td>ik</td><td>$\frac{1}{2}$</td><td>1</td><td>$0 \neq 2$</td><td>ik</td><td>$-\frac{1}{2}$</td><td>0</td><td>$0 \neq 2$</td></tr>
<tr><td>jk</td><td>$\frac{3}{2}$</td><td>-3</td><td>$5 \neq -5$</td><td>jk</td><td>$\frac{1}{2}$</td><td>-4</td><td>$5 \neq -5$</td></tr>
</table></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td><b>A1</b></td><td>CWO with conclusion<br>e.g. $\frac{17}{3} \neq \frac{7}{3}$ or $\frac{17}{3} = \frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td></td><td><b>4</b></td><td></td></tr>
</table>

**20)** OCT 2021_9709_31 Q9

| | | | |
|---|---|---|---|
| (a) | Use correct method to evaluate the scalar product of relevant vectors | M1 | $(-4-2+6)$ |
| | Obtain answer zero and deduce the given statement | A1 | Need a conclusion or a statement in advance that the scalar product will be zero. |
| | | 2 | |
| (b) | Express general point of *l* or *m* in component form,<br>e.g. $(3+4s, 2-s, 5+3s)$ or $(1-t, -1+2t, -2+2t)$ | B1 | |
| | Equate at least two pairs of components and solve for *s* or for *t* | M1 | |
| | Obtain correct answer $s=-1$ and $t=2$ | A1 | |
| | Verify that all three equations are satisfied | A1 | |
| | State position vector of the intersection $-\mathbf{i}+3\mathbf{j}+2\mathbf{k}$, or equivalent | A1 | Can come from 1 correct value and no contradictory statement. |
| | | 5 | |
| (c) | Taking a general point *P* on *m*, form an equation in *t* by ***either*** equating a relevant scalar product to zero, ***or*** equating the derivative of $\lvert\overrightarrow{OP}\rvert$ to zero, ***or*** taking a specific point *Q* on *m*, e.g. $(1, -1, -2)$, using Pythagoras in triangle *OPQ* | *M1 | e.g. $\begin{pmatrix}1-t\\-1+2t\\-2+2t\end{pmatrix}.\begin{pmatrix}-1\\2\\2\end{pmatrix}=0$ |
| | Obtain $t=\frac{7}{9}$ | A1 | |
| | Carry out correct method to find *OP* | DM1 | |
| | Obtain $\frac{\sqrt{5}}{3}$ | A1 | Obtain the given answer from full and correct working. |
| | **Alternative method for question 9(c)** | | |
| | Take a specific point *Q* on *m*, e.g. $(-1, 3, 2)$ and use a scalar product to find *QN*, the projection of *OQ* on *m* | *M1 | |
| | Obtain $QN=\frac{11}{3}$, or equivalent | A1 | |
| | Use Pythagoras to obtain *ON* | DM1 | |
| | Obtain the given answer correctly | A1 | |
| | | 4 | |

## **21)** OCT 2021_9709_32 Q10

| | | | |
|---|---|---|---|
| a) | Obtain direction vector $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ , or equivalent | **B1** | Accept answers as column vectors throughout. |
| | Use a correct method to form a vector equation | **M1** | |
| | State answer $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, or equivalent correct form | **A1** | e.g. $\mathbf{r}=\begin{pmatrix}0\\3\\1\end{pmatrix}+\mu\begin{pmatrix}1\\-1\\-2\end{pmatrix}$ Allow $\begin{pmatrix}x\\y\\z\end{pmatrix}$ for **r**. |
| | | **3** | |
| b) | Use a correct method to find the position vector of *C* | **M1** | e.g. $\mathbf{OC}=\mathbf{OA}+\mathbf{AC}=\begin{pmatrix}1-3\\2+3\\-1+6\end{pmatrix}$ |
| | Obtain answer $-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$, or equivalent | **A1** | Accept as coordinates. |
| | | **2** | |
| (c) | State $\overrightarrow{OP}$ in component form | **B1 FT** | |
| | Form an equation in $\lambda$ by equating the modulus of $OP$ to $\sqrt{14}$ , or equivalent | **M1** | |
| | Simplify and obtain $3\lambda^2-\lambda-4=0$ , or equivalent | **A1** | $3\lambda^2+\lambda-4=0$ if using $\mathbf{i}-\mathbf{j}-2\mathbf{k}$ in (a).<br>$3\mu^2+5\mu-2=0$ if using $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$ in (a) and *OB*. |
| | Solve a 3-term quadratic and find a position vector | **M1** | $\left(\lambda=-1,\frac{4}{3}\text{ or }\lambda=1,-\frac{4}{3}\text{ or }\mu=\frac{1}{3},-2\text{ or }\mu=-\frac{1}{3},2\right)$ |
| | Obtain answers $2\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i}+\frac{10}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$ , or equivalent | **A1** | Accept as coordinates. |
| | | **5** | |

## **22)** OCT 2021_9709_33 Q8

| | | |
|---|---|---|
| (a) | State $\overrightarrow{OM}=4\mathbf{i}+2\mathbf{j}$ | **B1** |
| | Use a correct method to find $\overrightarrow{ON}$ | **M1** |
| | Obtain answer $3\mathbf{j} + \mathbf{k}$ | **A1** |
| | Use a correct method to find a line equation for *MN* | **M1** |
| | Obtain answer $\mathbf{r} = 3\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} - \mathbf{j} - \mathbf{k})$, or equivalent | **A1** |
| | | **5** |

| | | | |
|---|---|---|---|
| (b) | Taking a general point $P$ on $MN$, form an equation in $\lambda$ by *either* equating a relevant scalar product to zero *or* equating the derivative of $\overline{OP}$ to zero *or* using Pythagoras in triangle $OPM$ or $OPN$ | **M1** | |
| | Obtain $\lambda = \frac{2}{9}$ | **A1** | OE |
| | Use correct method to find $OP$ | **M1** | |
| | Obtain the given answer correctly | **A1** | |
| | **Alternative method to Question 8(b)** | | |
| | Use a scalar product to find the projection of $OM$ (or $ON$) on $MN$ | **M1** | |
| | Obtain answer $\frac{14}{\sqrt{18}} \left( \text{or } \frac{4}{\sqrt{18}} \right)$ | **A1** | |
| | Use Pythagoras to obtain the perpendicular | **M1** | |
| | Obtain the given answer correctly | **A1** | |
| | | **4** | |

**23)** OCT 2022-9709_31 Q11

| | | | |
|---|---|---|---|
| (a) | State $\overrightarrow{OM} = 2\mathbf{i} + 2\mathbf{j}$ or equivalent | **B1** | Can be implied by $\overrightarrow{MB} = -2\mathbf{i} + 2\mathbf{j}$ or $\overrightarrow{MA} = 2\mathbf{i} - 2\mathbf{j}$. |
| | Obtain $\overrightarrow{MD} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ | **B1** | |
| | Use a correct method to find $\overrightarrow{ON}$ | **M1** | e.g. $\overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB}$ |
| | Obtain answer $3\mathbf{j} + \mathbf{k}$ | **A1** | |
| | | **4** | |
| (b) | Use the correct process for evaluating the scalar product of $\overrightarrow{MD}$ and $\overrightarrow{ON}$ | **M1** | |
| | Using the correct process for the moduli, divide the scalar product by the product of the moduli and reach the inverse cosine of the result | **M1** | $\cos^{-1}\left(\frac{-6+3}{\sqrt{10}\sqrt{17}}\right)$ |
| | Obtain final answer 103.3° | **A1** | |
| | | **3** | |

|  |  |  |  |
|---|---|---|---|
| (c) | Taking a general point *P* of *ON* to have position vector $\lambda(3\mathbf{j}+\mathbf{k})$, form an equation in $\lambda$ by *either* equating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{MP}$ to zero, *or* applying Pythagoras to triangle *OMP*, *or* equating the derivative of $\lvert\overrightarrow{MP}\rvert$ to zero | M1 | e.g. $\begin{pmatrix}-2\\-2+3\lambda\\\lambda\end{pmatrix}.\begin{pmatrix}0\\3\\1\end{pmatrix}=0$ |
|  | Solve and obtain $\lambda=\frac{3}{5}$ | A1 |  |
|  | Substitute for $\lambda$ and calculate *MP* | M1 | $\overrightarrow{MP}=-2\mathbf{i}-\frac{1}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$ |
|  | Obtain $\sqrt{\frac{22}{5}}$ | A1 | AG |
|  | **Alternative method for question 11(c)** |  |  |
|  | Use a scalar product to find the projection *OQ* of *OM* on *OM* | M1 |  |
|  | Obtain $OQ=\frac{6}{\sqrt{10}}$ | A1 |  |
|  | Use Pythagoras in triangle *OMQ* to find *MQ* | M1 |  |
|  | Obtain $\sqrt{\frac{22}{5}}$ | A1 | AG |
|  |  | 4 |  |

## 24) OCT 2022_9709_32 Q6

|  |  |  |  |
|---|---|---|---|
| a) | State or imply $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correctly in component form | B1 | $\left(\overrightarrow{AB}=2\mathbf{i}-2\mathbf{j}+\mathbf{k},\quad \overrightarrow{AC}=4\mathbf{i}-3\mathbf{k}\right)$. |
|  | Using the correct process with relevant vectors to evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, | M1 | or $\overrightarrow{BA}.\overrightarrow{CA}$ $(8-3=5)$.<br>M0 for $\overrightarrow{AB}.\overrightarrow{CA}$. |
|  | Using the correct process for the moduli, divide *their* scalar product by the product of *their* moduli to obtain $\cos\theta$ or $\theta$ | M1 | $\left(\frac{5}{\sqrt{9}\sqrt{25}}\right)$<br>Independent of the first M1. |
|  | Obtain answer $\frac{1}{3}$ | A1 | ISW. Need to see a value for $\cos\theta$.<br>Accept $\frac{5}{15}$ or 0.333 ($\cos^{-1}\frac{1}{3}$ alone is not sufficient) |
|  |  | 4 |  |
| (b) | Use correct method to find an **exact** value for the sine of angle *BAC* from *their* **(a)** | M1 | $\left(\sqrt{1-\frac{1}{9}}\right)$ |
|  | Obtain answer $\frac{2}{3}\sqrt{2}$, or equivalent | A1 |  |
|  | Use correct area formula to find the area of triangle *ABC* with *their* versions of relevant vectors | M1 | $\left(\frac{1}{2}\sqrt{9}\sqrt{25}\times \textit{their}\sin\theta\right)$ or $\frac{1}{2}\sqrt{9}\sqrt{25}\times\sin\left(\cos^{-1}\frac{1}{3}\right)$ |
|  | Obtain answer $5\sqrt{2}$ or $\sqrt{50}$ | A1 | Only ISW |
|  | **Alternative method 1 for question 6(b)** |  |  |
|  | Use correct method to find the perpendicular distance from *A* to *BC* (or *B* to *AC* or *C* to *AB*) | M1 | $\begin{pmatrix}2+2\lambda\\-2+2\lambda\\1-4\lambda\end{pmatrix}.\begin{pmatrix}2\\2\\-4\end{pmatrix}=0\Rightarrow\lambda=\frac{1}{6}$ |
|  | Obtain $\frac{1}{3}\sqrt{75}$ | A1 | $\left(\left\lvert\frac{7}{3}\mathbf{i}-\frac{5}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}\right\rvert\right)$ |
|  | Use correct area formula to find the area of triangle *ABC* | M1 | $\left(\frac{1}{2}\times \textit{their}\sqrt{24}\times \textit{their}\,\frac{1}{3}\sqrt{75}\right)$<br>The length they use for *their* base must be found correctly. |
|  | Obtain answer $5\sqrt{2}$ or $\sqrt{50}$ | A1 |  |

| | | | |
|---|---|---|---|
| (b) | **Alternative method 2 for question 6(b)** | | |
| | Correct method to find the semi-perimeter | **M1** | |
| | Obtain $4+\sqrt{6}$ | **A1** | |
| | Correct application of Hero's (Heron's) formula | **M1** | $\sqrt{(4+\sqrt{6})(1+\sqrt{6})(-1+\sqrt{6})(4-\sqrt{6})}$ |
| | Obtain answer $5\sqrt{2}$ or $\sqrt{50}$ | **A1** | |
| | | **4** | |

**25)** OCT 2022_9709_33 Q9

| | | | |
|---|---|---|---|
| (a) | State $\overrightarrow{OM} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ | **B1** | |
| | Use a correct method to find $\overrightarrow{ON}$ | **M1** | |
| | Obtain answer $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ | **A1** | |
| | | **3** | |
| (b) | Carry out a correct method to form a vector equation for *MN* | **M1** | |
| | Obtain a correct equation in any form, e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$ | **A1** | OE |
| | | **2** | |
| c) | State a correct vector equation for *AB* in any form, e.g. $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}$ | **B1** | |
| | Equate components of *AB* and *MN* and solve for $\lambda$ or for $\mu$ | **M1** | |
| | Obtain $\lambda = -3$ or $\mu = 2$ | **A1** | |
| | Obtain position vector $\begin{pmatrix} -1 \\ 10 \\ 3 \end{pmatrix}$, or equivalent, for *Q* | **A1** | |
| | | **4** | |