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P_3

Pure Maths_3

Vectors in 2D
Notes_1

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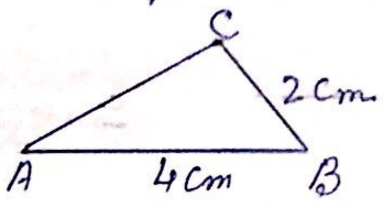
P-3

Vectors in 2 Dimensions

Notes-1

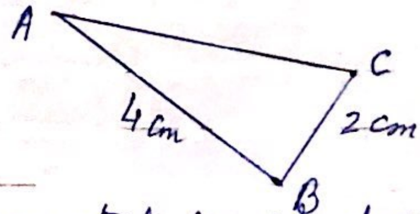
Scalar Quantities: A scalar quantity has magnitude (unit and a real number), length, mass, distance, speed, density are all scalar quantities.

Example:



Length of side $AB = 4$ cm.

Length of side $BC = 2$ Cm.



(Note: Here direction of the sides is not considered.)

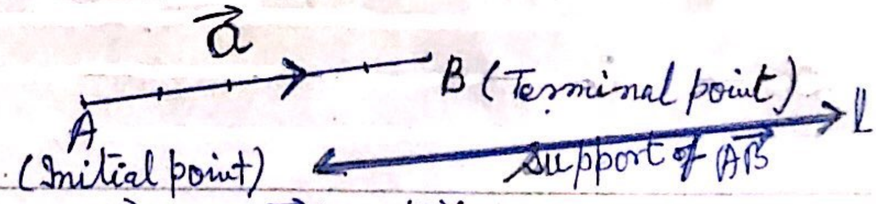
Vectors: A vector quantity has magnitude (unit and real no) and direction (support and sense).

Velocity, displacement, force are vector qualities.

Geometric representation of a Vector:

Vectors are represented by directed line segments.

Example: A force of 5 newtons is applied, it is denoted by "**a** (bold face letter) or $\vec{a}$ or $\underset{\sim}{a}$ or **AB** or $\overrightarrow{AB}$"



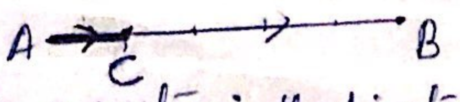
Magnitude of vector $\overrightarrow{AB} = |\overrightarrow{AB}|$ or $|\vec{a}| = 5$ Newton

{In this particular example)
Unit - Newton
Real no - 5

Direction of Vector $\overrightarrow{AB}$:

- support $AB \parallel$ line l (lies on line AB (on $\overleftrightarrow{AB}$))
- sense: A to B. (Shown by arrow)

Unit Vector:



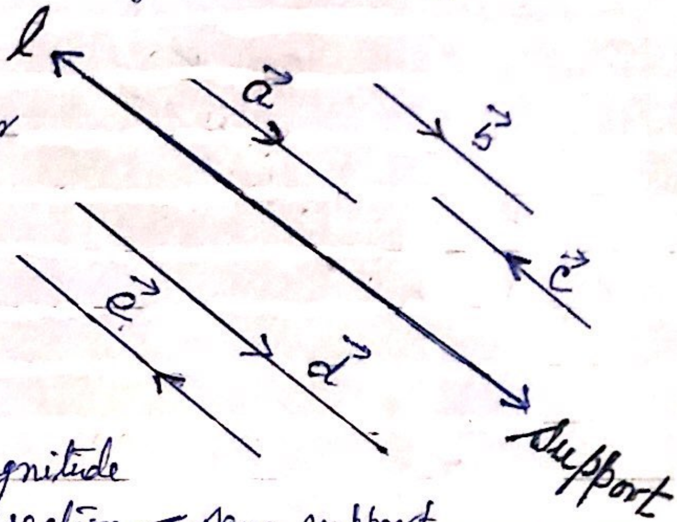
Given a Vector $\overrightarrow{AB}$, then a vector in the direction of $\overrightarrow{AB}$ and magnitude $=1$

Unit Vector $\overrightarrow{AC} = \dfrac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$.

Vectors Notes

Collinear Vectors: Vectors having the same support.

Vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ and $\vec{e}$ are all collinear vectors as they have the same support (line 'l')



(a) **Equal Vectors:** $\vec{a} = \vec{b}$

(i) same magnitude

and (ii) same direction ← same support, same sense

(b) **Parallel Vectors:** $\vec{m} = k\vec{n}$ $k \neq 0$ real no.

all the vectors $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}$ are parallel vectors

(i) if $k > 0$ same direction ; $\vec{d} = 2\vec{a}$

Vectors $\vec{a}, \vec{b}, \vec{d}$ are in the same direction.

and $\vec{c}$ and $\vec{e}$ are also in the same direction.

(ii) If $k < 0$

Vectors $\vec{a}$ and $\vec{e}$ are in opposite directions

as $\vec{e} = -\frac{3}{2}\vec{a}$

and $\vec{d} = -2\vec{c}$

(c) **Opposite Vectors:**

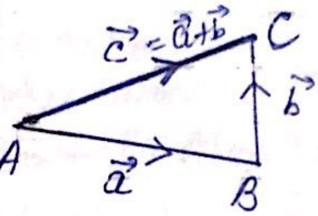
$\vec{b} = -\vec{c}$

(i) same magnitude but opposite direction (same magnitude and same support but opp. sense)

Vectors Notes

Addition of Vectors (Triangle Law of Vector addition):

$$\vec{AB} + \vec{BC} = \vec{AC} \quad [\vec{a}+\vec{b}=\vec{c}]$$

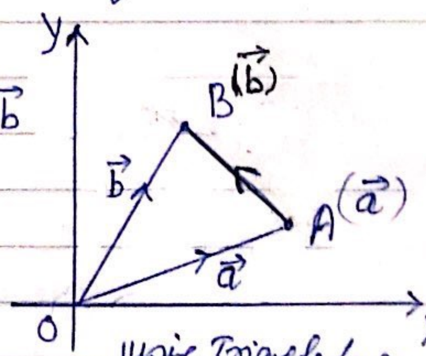


If two vectors are denoted by two sides of a triangle with their magnitude and directions taken in same order then the third side denotes their vector sum by magnitude and direction in opposite order.

[or The head of vector $\vec{a}$ connects to the tail of vector $\vec{b}$, Then the vector obtained by joining the tail of $\vec{a}$ to the head of vector $\vec{b}$ gives their vector sum)

Position Vector of a point:

Let $\vec{OA}=\vec{a}$ and $\vec{OB}=\vec{b}$



Let the reference point is origin O.

Then the position vector of point $A=\vec{a}$

and the position vector of point $B=\vec{b}$

Note: $\vec{AB} = \vec{b} - \vec{a}$

(Position Vector Terminal point) − (Position vector of Initial point)

Using Triangle law;

∵ In ΔOAB

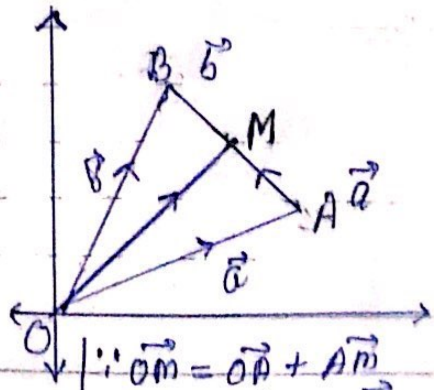
$\vec{OA}+\vec{AB}=\vec{OB}$

$\therefore \vec{AB}=\vec{OB}-\vec{OA}$

$=\vec{b}-\vec{a}$

$$\boxed{\vec{AB}=\vec{b}-\vec{a}}$$

Position Vector of Mid point:



Given the position vectors of points A and B (with reference to origin O) are $\vec{a}$ and $\vec{b}$. Let M is the mid point of segment AB.

Then the position vector of the mid point,

$$\vec{OM} = \left(\frac{\vec{a}+\vec{b}}{2}\right)$$

$\because \vec{OM} = \vec{OA} + \vec{AM}$

$= \vec{a} + \frac{1}{2}\vec{AB}$

$= \vec{a} + \frac{1}{2}(\vec{b}-\vec{a})$

$= \frac{(\vec{a}+\vec{b})}{2}$

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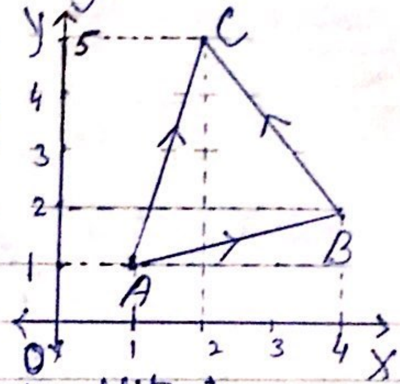
Vectors (In two Dimensions) Notes

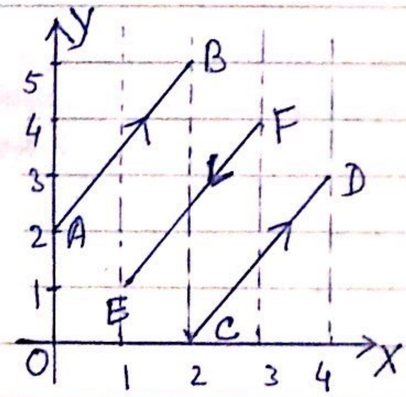
§ **Column Vector:** A column vector indicates a Translation (or a Shift) in the direction of x-axis and along y-axis.



(i) $\vec{AB} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$; $\vec{BC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$: $\vec{AC} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

Now $\begin{pmatrix} 3 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

$\therefore$ $\vec{AB} + \vec{BC} = \vec{AC}$ [Triangle Law of Vector addition)



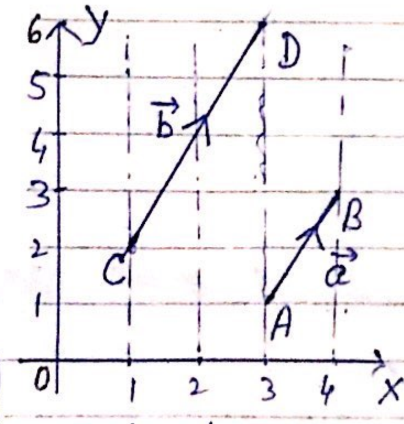
(ii) $\vec{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$; $\vec{CD} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\vec{FE} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

$\therefore$ $\vec{AB} = \vec{CD}$ Equal Vectors

$\vec{FE} = \begin{pmatrix} -2 \\ -3 \end{pmatrix} = -1\begin{pmatrix} 2 \\ 3 \end{pmatrix} = -\vec{AB} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$

or $\vec{AB} = -\vec{FE}$

$\therefore$ $\vec{AB}$ and $\vec{FE}$ are opposite vectors.



§ **Scalar Multiplication:**

$\vec{AB} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$; $\vec{CD} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$2 \cdot \vec{AB} = 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \vec{CD}$

$\vec{AB}$ & $\vec{CD}$ are two vectors in the same direction.

[$\vec{a} = k\vec{b}$ and $k > 0$
$\vec{a}$ and $\vec{b}$ are in the same direction]

§ **Magnitude of a Column Vector:**

$\vec{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$ Then magnitude of $\vec{AB} = |\vec{AB}| = \sqrt{x^2+y^2}$

Example: $\vec{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{1^2+2^2} = \sqrt{5}$

$\vec{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow |\vec{b}| = \sqrt{3^2+4^2} = 5$

Vectors in two dimensions. Notes

Unit Vector: A vector whose magitude is one unit.

Given a vector $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(i) Find a unit vector in the direction of $\vec{a}$

(ii) Find a vector $\vec{b}$ in the direction of $\vec{a}$ and $|\vec{b}| = 7$

Solution (i) $\vec{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \Rightarrow |\vec{a}| = \sqrt{3^2+4^2} = 5$

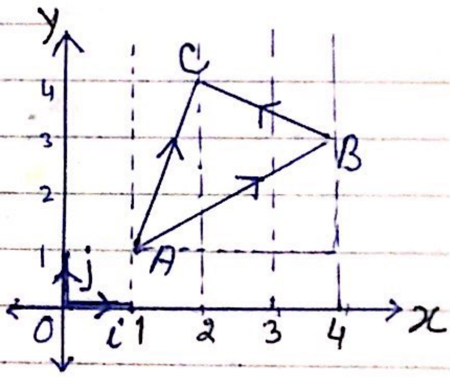
$\therefore$ Unit Vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|} = \dfrac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$.

(ii) $\vec{b} = 7 \times$ Unit Vector along $\vec{a}$

$= 7 \times \dfrac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} \frac{21}{5} \\ \frac{28}{5} \end{pmatrix}$

Basic Unit Vectors (in two dimensions)

$i = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $j = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$



$\left.\begin{array}{l} \vec{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3i + 2j \\ \vec{BC} = \begin{pmatrix} -2 \\ 1 \end{pmatrix} = -2i + j \end{array}\right\}$ $\vec{AC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\vec{AC} = (i+3j)$

(i) Now $\vec{AB} + \vec{BC}$

$= (3i+2j) + (-2i+j) = (3-2)i + (2+1)j$

$= i + 3j = \vec{AC}$

$\boxed{\vec{AB} + \vec{BC} = \vec{AC}}$ $\longrightarrow$ (Verifies triangle law of Vector addition)

(ii) $\vec{AB} = 3i+2j$

$\therefore |\vec{AB}| = \sqrt{3^2+2^2} = \sqrt{13}$

Unit Vector along $\vec{AB} = \dfrac{1}{\sqrt{13}}\vec{AB} = \dfrac{1}{\sqrt{13}}(3i+2j)$ ✓

(iii) $3\vec{AB} + 2\vec{BC} = 3(3i+2j) + 2(-2i+j)$

$= (9i+6j) + (-4i+2j) = (5i+8j)$

$\left.\begin{array}{l} \text{(iv) Position Vector } B = \vec{OB} = (4i+3j) \\ \text{(v) Position Vector } A = \vec{OA} = (i+j) \end{array}\right\}$ $\vec{AB} = \vec{OB} - \vec{OA}$

$= (4i+3j) - (i+j)$

$= (3i+2j)$ ✓

Vectors

Example 1(a). $\overrightarrow{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

Find (i) $5\overrightarrow{GH}$ --- [1]

(ii) $\overrightarrow{HG}$ --- [1]

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$ find the value of y. -- [1]

0580/S-17/21/Q18

Solution: (a)(i) $\overrightarrow{GH} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$

$$\therefore 5\overrightarrow{GH} = 5\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} 30 \\ -20 \end{pmatrix} ✓$$

(ii) $\overrightarrow{HG} = -\overrightarrow{GH} = -\begin{pmatrix} 6 \\ -4 \end{pmatrix} = \begin{pmatrix} -6 \\ 4 \end{pmatrix} ✓$

(b) $\begin{pmatrix} 6 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$.

or $\begin{pmatrix} 6+2 \\ 7+y \end{pmatrix} = \begin{pmatrix} 8 \\ 3 \end{pmatrix} \Rightarrow 7+y=3 \Rightarrow \underline{y=-4} ✓$

Example 2(a) D is a point $(2,-5)$ and $\overrightarrow{DE} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$.

Find the co-ordinates of the point E. --- [1]

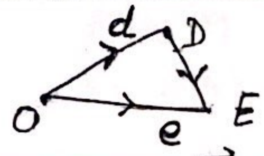
0580/W-17/21/Q14

(b) $\mathbf{v} = \begin{pmatrix} t \\ 12 \end{pmatrix}$ and $|\mathbf{v}| = 13$

work out the value of t, where t is negative. --- [2]

Solution (a) Let $E(x, y)$, $D = (2,-5)$

$\overrightarrow{OE} = \begin{pmatrix} x \\ y \end{pmatrix}$, $\overrightarrow{OD} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$



$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \end{pmatrix}$ $\qquad \overrightarrow{DE} = e - d$

$= \begin{pmatrix} x-2 \\ y+5 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$ Given $\overrightarrow{DE} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

$\Rightarrow x-2=7 \Rightarrow x=9$ $\therefore \underline{E(9,-4)} ✓$

$y+5=1 \Rightarrow y=-4$

(b) $V = \begin{pmatrix} t \\ 12 \end{pmatrix}$ $\therefore |V| = \sqrt{t^2+12^2} = 13$ Given

$\Rightarrow t^2+144=169$

$t^2 = 25$

$t = 5^{\times}, -5 ✓$ $\qquad$ t is negative

$\therefore \underline{t=-5} ✓$

Vectors

Example 3 (a) A vector **v** has a magnitude of 102 units and has the same direction as $\begin{pmatrix}8\\-15\end{pmatrix}$. Find **v** in the form $\begin{pmatrix}a\\b\end{pmatrix}$, where a and b are integers. ---[2]

(b) Vector $\mathbf{c}=\begin{pmatrix}4\\3\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}p-q\\5p+q\end{pmatrix}$ are such that $\mathbf{c}+2\mathbf{d}=\begin{pmatrix}p^2\\27\end{pmatrix}$.

Find the possible values of the constants p and q. [M-17 0606 | 12 | Q7] ---[6]

Solution (a) $\mathbf{v}=\begin{pmatrix}a\\b\end{pmatrix}$ has the same direction as $\begin{pmatrix}8\\-15\end{pmatrix}$

unit vector in the direction of $\begin{pmatrix}8\\-15\end{pmatrix}$

$$\text{Unit Vector} = \frac{\text{Vector}}{\text{Magnitude}} \Rightarrow = \frac{1}{17}\begin{pmatrix}8\\-15\end{pmatrix}$$

$$\left\{ \begin{aligned} \left|\begin{pmatrix}8\\-15\end{pmatrix}\right| &= \sqrt{8^2+(-15)^2} \\ &= \sqrt{64+225} \\ &= \sqrt{289} \\ \text{Mangitue} &= 17 \end{aligned} \right.$$

magnitude of v is 102

$$\therefore \mathbf{v}=\begin{pmatrix}a\\b\end{pmatrix} = 102\times\frac{1}{17}\begin{pmatrix}8\\-15\end{pmatrix}$$

$$\left\{ \text{Vector} = \text{Magnitude} \times \text{Unit Vector} \right.$$

$$= 6\begin{pmatrix}8\\-15\end{pmatrix} = \begin{pmatrix}48\\-90\end{pmatrix}$$

or $\mathbf{v}=\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}48\\-90\end{pmatrix}$ ✓

(b) $\mathbf{c}=\begin{pmatrix}4\\3\end{pmatrix}$ and $\mathbf{d}=\begin{pmatrix}p-q\\5p+q\end{pmatrix}$

Given $\mathbf{c}+2\mathbf{d}=\begin{pmatrix}p^2\\27\end{pmatrix}$

or $\begin{pmatrix}4\\3\end{pmatrix}+2\begin{pmatrix}p-q\\5p+q\end{pmatrix}=\begin{pmatrix}p^2\\27\end{pmatrix}$

or $\begin{pmatrix}4+2p-2q\\3+10p+2q\end{pmatrix}=\begin{pmatrix}p^2\\27\end{pmatrix}$

$$\Rightarrow \left\{ \begin{aligned} 4+2p-2q &= p^2 \quad \text{—(1)} \\ 3+10p+2q &= 27 \quad \text{—(2)} \end{aligned} \right.$$

from (2) $2q=(24-10p)$ —(3)

or $q=12-5p$ —(4)

Put the value of 2q in (1)

$4+2p-(24-10p)=p^2$

or $p^2-12p+20=0$

$(p-2)(p-10)=0$

$\Rightarrow p=2$ or $p=10$

$p=2,\ q=2$ ✓

$p=10,\ q=-38$

$$\left\{ \begin{aligned} &\text{from (4)} \\ &q=12-5p \end{aligned} \right.$$

$$\left. \begin{aligned} &\therefore p=2 \text{ and } q=2 \\ &\text{or } p=10 \text{ and } q=-38 \end{aligned} \right\} ✓$$

Vectors

Example 4. Given that $p = 2i - 5j$ and $q = i - 3j$, find the unit vector in the direction of $3p - 4q$. $\boxed{0606/S-18/11/Q8(a)}$ [4]

Solution:

$$3p - 4q = 3(2i - 5j) - 4(i - 3j)$$
$$= (2i - 3j)$$

$\therefore$ Magnitude of $3p - 4q = |3p - 4q| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$

$\therefore$ Unit Vector in the direction of $3p - 4q$

$$= \frac{1}{\sqrt{13}}(2i - 3j) = \frac{2i - 3j}{\sqrt{13}}$$

Example 5. Given that $a = 2i + 3j$, $b = i - 5j$ and $c = 3i + 11j$, find
(i) the exact value of $|a + c|$

Solution $a + c = (2i + 3j) + (3i + 11j) = (5i + 14j)$

$\therefore |a + c| = \sqrt{5^2 + 14^2} = \sqrt{221}$

(ii) Find the value of constant m such that $a + mb$ is parallel to j.

Solution: $a + mb = (2i + 3j) + m(i - 5j)$
$= (2 + m)i + (3 - 5m)j$ ——— ①

Given $a + mb$ is parallel to j or $0i + 1j$

$\therefore$ coeff of i in ① $= 0$

$\Rightarrow 2 + m = 0 \Rightarrow m = -2$

(iii) Find the value of constant n, such that $na - b = c$

Solution: $na - b = c$

$\Rightarrow n(2i + 3j) - (i - 5j) = (3i + 11j)$

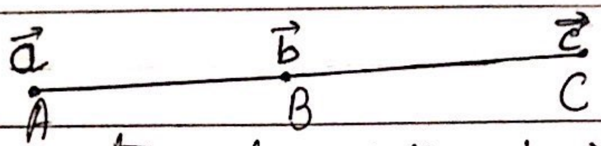
or $(2n - 1)i + (3n + 5)j = (3i + 11j)$ ——— ②

Equating the coeff. of i in ② on both the sides.

$2n - 1 = 3 \Rightarrow n = 2$

Vectors

§ <u>To verify that the three points in a plane are collinear, given their position vectors.</u>



Given the position vectors of three points A, B and C are a, b and c.

find vector $\vec{AB} = \vec{b} - \vec{a}$

and $\vec{AC} = \vec{c} - \vec{a}$

verify that $\vec{AC} = K\,\vec{AB}$; $K \in R, K \neq 0$

$\Rightarrow$ $\vec{AC}$ is parallel to $\vec{AB}$

and A is the common initial point, proves that the three points A, B and C are collinear.

<u>Example: 6.</u> Given the position vectors of three points A, B and C $a = (2i - j)$, $b = 4i + 3j$ and $c = 3i + j$ respectively, prove that the three points of collinear.

<u>Solution</u>

$$\vec{AB} = \vec{b} - \vec{a} = (4i+3j) - (2i-j) = (2i+4j) \quad \text{—} (1)$$

and $$\vec{AC} = \vec{c} - \vec{a} = (3i+j) - (2i-j) = (i+2j) \quad \text{—} (2)$$

Now $$\vec{AB} = (2i+4j) = 2(i+2j)$$
$$= 2\,\vec{AC} \qquad \text{from (1) \& (2)}$$

as $\vec{AB} = 2\vec{AC}$

vectors $\vec{AB}$ and $\vec{AC}$ are parallel,

but both the vectors have a common initial point, and $\vec{AB}$ and $\vec{AC}$ lie on the same line or

The three points A, B and C are collinear.

Vectors

Example 7: The vectors a, b and c are such that $a = \begin{pmatrix} 5 \\ -6 \end{pmatrix}$, $b = \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ and $3a + c = b$.

(i) Find c ---[1]

(ii) Find a unit vector in the direction of b. $\boxed{0606/S\text{-}17/23/Q4(a)}$ --[2]

Solution: (i) $3a + c = b$

$$\text{or } c = b - 3a$$
$$= \begin{pmatrix} 11 \\ -15 \end{pmatrix} - 3\begin{pmatrix} 5 \\ -6 \end{pmatrix}$$
$$\text{or } c = \begin{pmatrix} 11 \\ -15 \end{pmatrix} + \begin{pmatrix} -15 \\ +18 \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$$

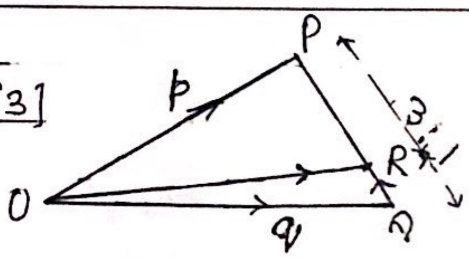
(ii) $b = \begin{pmatrix} 11 \\ -15 \end{pmatrix}$ —— ①

$\therefore$ magnitude of $|b| = \sqrt{(11)^2 + (-15)^2} = \sqrt{121 + 225} = \sqrt{346}$

Now unit vector in the direction of $b = \dfrac{b}{|b|} = \dfrac{1}{\sqrt{346}}\begin{pmatrix} 11 \\ -15 \end{pmatrix}$

Example 8. In the diagram $\overrightarrow{OP} = p$ and $\overrightarrow{OQ} = q$. The point R lies on PQ such that $PR = 3RQ$.
Find $\overrightarrow{OR}$ in terms of p and q.
Simplify your answer. ----[3]

$\boxed{0606/S\text{-}17/23/Q4(b)}$



Solution:

$$\overrightarrow{QP} = \overrightarrow{OP} - \overrightarrow{OQ} = p - q \text{ —— ①}$$

Now $PR = 3RQ \Rightarrow QR = \frac{1}{4} QP \Rightarrow \overrightarrow{QR} = \frac{1}{4}\overrightarrow{QP}$

In Triangle OQR $= \frac{1}{4}(p - q)$ —— ②

$$\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR}$$
$$= q + \frac{1}{4}(p - q) \quad \text{from ②}$$
$$= \frac{1}{4}(3q + p)$$
$$\text{or} \quad \overrightarrow{OR} = \frac{3}{4} q + \frac{1}{4} p$$

Vectors

Example 9. The vector p has a magnitude of 39 units and has the same direction as $-10i+24j$

(i) Find p in terms of i and j. ---[2]

(ii) Find the vector q such that $2p+q$ is parallel to the positive Y-axis and has a magnitude of 12 units. ---[3]

(iii) Hence show that $|q| = k\sqrt{5}$, where k is an integer to be found. ---[2]

0606 / S-17 / 11 / Q5(b)

Solution Given a vector $(-10i+24j) = r$ (let)

(i)

magnitude of $r = |r| = \sqrt{(-10)^2+(24)^2} = \sqrt{100+576} = \sqrt{676}$
$= 26$ ✓

$\therefore$ Unit Vector along $r = \frac{r}{|r|} = \frac{1}{26}(-10i+24j) = \frac{1}{13}(-5i+12j)$ ✓

Now p is in the direction r and magnitude 39.

$\therefore p = 39 \times$ Unit Vector r

or $p = 39 \times \frac{1}{13}(-5i+12j) = 3(-5i+12j) = (-15i+36j)$ ✓ — (1)

(ii)

Vector q is such that $2p+q$ is parallel to the positive Y-axis and magnitude 12, so the i-component is zero.

or $2p+q = 12j$

or $2(-15i+36j)+q = 12j$ from (1)

or $q = 30i-60j$ ✓

(iii)

$|q| = \sqrt{(30)^2+(-60)^2} = \sqrt{900+3600} = \sqrt{4500} = \sqrt{900\times5}$

or $|q| = 30\sqrt{5}$ ✓

Vectors

Example 10. O, P, Q and R are four points such that $\vec{OP} = p$, $\vec{OQ} = q$ and $\vec{OR} = 3q - 2p$.

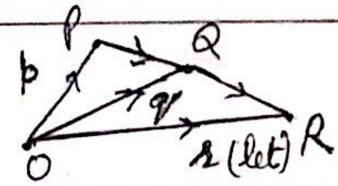
(i) Find in terms of p and q, (a) $\vec{PQ}$ (b) $\vec{QR}$ --- [2]

(ii) Justify your answer, what can be said about the positions of the points P, Q, R. --- [2]

(iii) Given that $\vec{OP} = i + 3j$ and $\vec{OQ} = 2i + j$, find the unit vector in the direction of $\vec{OR}$. --- [3]

$\boxed{0606 \; S\text{-}16/21/Q7}$

(i) Let $\vec{OR} = 3q - 2p = \vec{r}$ (let)



(a) $\vec{PQ} = \vec{OQ} - \vec{OP} = q - p$ —— ①

$\vec{QR} = \vec{OR} - \vec{OQ} = (3q - 2p) - q$

$= (2q - 2p)$

or $\vec{QR} = 2(q - p)$ —— ②

(ii) from ① & ② we find.

$$\vec{QR} = 2(q - p) = 2\vec{PQ}$$

or $$\vec{QR} = 2\vec{PQ}$$

$\therefore$ Vectors $\vec{QR}$ and $\vec{PQ}$ are parallel,

but these two vectors have a common point Q,

$\therefore$ points P, Q, R lie in a line or <u>collinear points</u>

(iii) $\vec{OR} = 3q - 2p$ $\quad$ [Since $\vec{OP} = p = i + 3j$ & $\vec{OQ} = q = 2i + j$

$= 3(2i + j) - 2(i + 3j)$

or $\vec{OR} = (4i - 3j)$ —— ③

and $|\vec{OR}| = \sqrt{4^2 + (-3)^2} = 5$ ✓

$\therefore$ <u>a unit vector along $\vec{OR}$</u> $= \dfrac{\vec{OR}}{|\vec{OR}|} = \dfrac{1}{5}(4i - 3j)$

$= \left(\dfrac{4}{5}i - \dfrac{3}{5}j\right)$ ✓

Example 11. Vectors a, b and c are such that $a=\begin{pmatrix}2\\y\end{pmatrix}$, $b=\begin{pmatrix}1\\3\end{pmatrix}$ and $c=\begin{pmatrix}-5\\5\end{pmatrix}$.

(i) Given that $|a|=|b-c|$, find the possible values of y ---[3]

(ii) Given that $\mu(b+c)+4(b-c)=\lambda(2b-c)$, find the values of λ and μ. 0606 / S-16/12/Q3/ ---[3]

Solution: (i) $a=\begin{pmatrix}2\\y\end{pmatrix} \Rightarrow |a|=\sqrt{2^2+y^2}=\sqrt{4+y^2}$ —— ①

and $b-c=\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}-5\\5\end{pmatrix}=\begin{pmatrix}6\\-2\end{pmatrix}$

$\therefore |b-c|=\sqrt{6^2+(-2)^2}=\sqrt{36+4}=\sqrt{40}$ —— ②

given $|a|=|b-c|$

$\sqrt{4+y^2}=\sqrt{40}$ from ① & ②

squaring

$4+y^2=40$

$y^2=36 \Rightarrow y=\pm 6$ ✓

(ii) $\mu(b+c)+4(b-c)=\lambda(2b-c)$

$\mu\left[\begin{pmatrix}1\\3\end{pmatrix}+\begin{pmatrix}-5\\5\end{pmatrix}\right]+4\left[\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}-5\\5\end{pmatrix}\right]=\lambda\left[2\begin{pmatrix}1\\3\end{pmatrix}-\begin{pmatrix}-5\\5\end{pmatrix}\right]$

$\mu\begin{pmatrix}-4\\8\end{pmatrix}+4\begin{pmatrix}6\\-2\end{pmatrix}=\lambda\begin{pmatrix}7\\1\end{pmatrix}$

or $\begin{pmatrix}-4\mu+24\\8\mu-8\end{pmatrix}=\begin{pmatrix}7\lambda\\\lambda\end{pmatrix}$

Equating the x-component & y-components

$\left.\begin{aligned}-4\mu+24&=7\lambda\\ \text{or } 8\mu-8&=\lambda\end{aligned}\right\}$ or $7\lambda+4\mu=24$ —— ③

$\lambda-8\mu=-8$ —— ④

Solving ③ & ④

$\mu=4/3$ ✓ and $\lambda=\frac{8}{3}$ ✓

Example 12: The four points O, A, B and C are such that $\vec{OA}=5a$, $\vec{OB}=15b$ and $\vec{OC}=24b-3a$. Show that B lies on line AC. 0606 /S-15/21/Q7(a)/ - [3]

Solution: $\vec{AB}=\vec{OB}-\vec{OA}=15\vec{b}-5\vec{a}=5(3b-a)$ —— ①

and $\vec{AC}=\vec{OC}-\vec{OA}=(24b-3a)-5a=24b-8a=8(3b-a)$ —— ②

from ① & ② $\vec{AB}=\frac{5}{8}\vec{AC} \Rightarrow$ AB and AC are parallel, but A is a common point.

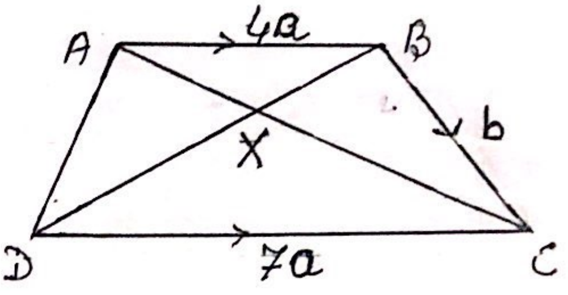
Collinear points $\therefore$ A, B, C lie in a line or B lies on line AC.

IG-Maths 0606

Vectors

Example 13: In the diagram $\overrightarrow{AB}=4a$, $\overrightarrow{BC}=b$ and $\overrightarrow{DC}=7a$. The lines AC and DB intersect at the point X.

Find in terms of a and b,



(a) $\overrightarrow{DB}$ ---[1]

(b) $\overrightarrow{DA}$ ---[1]

Given that $\overrightarrow{AX}=\lambda\overrightarrow{AC}$, find in terms of a, b and λ,

(c) $\overrightarrow{AX}$ ---[1]

(d) $\overrightarrow{DX}$ ---[2]

Given that $\overrightarrow{DX}=\mu\overrightarrow{DB}$

(e) find the value of λ and of μ. SP-20/02/Q9 ---[4]

Solution (a) $\overrightarrow{DB}=\overrightarrow{DC}+\overrightarrow{CB}$ (In ΔDBC)

$= 7a+(-b)$ ($\because \overrightarrow{BC}=\vec{b}$)

$=(7a-b)$ ✓ —— (1)

(b) $\overrightarrow{DA}+\overrightarrow{AB}=\overrightarrow{DB}$ (In ΔDAB)

$\overrightarrow{DA}=\overrightarrow{DB}-\overrightarrow{AB}$

$=(7a-b)-4a$

$=3a-b$ —— (2)

(c) $\overrightarrow{DA}+\overrightarrow{AC}=\overrightarrow{DC}$ (In ΔDAC)

$\overrightarrow{AC}=\overrightarrow{DC}-\overrightarrow{DA}$

$=7a-(3a-b)$ fr (2)

$=(4a+b)$ —— (3)

Now $\overrightarrow{AX}=\lambda\overrightarrow{AC}$ (Given)

or $\overrightarrow{AX}=\lambda(4a+b)$ fm (3)

(d) $\overrightarrow{DX}=\overrightarrow{DA}+\overrightarrow{AX}$ (In ΔDAX)

$=(3a-b)+\lambda(4a+b)$ fm (2) & (3) —— (4)

(e) Given $\overrightarrow{DX}=\mu\overrightarrow{DB}$

or $(3a-b)+\lambda(4a+b)=\mu(7a-b)$ from (4) & (1)

or $(3+4\lambda)a+(\lambda-1)b=7\mu a-\mu b$ —— (5)

fm (5) Equating the coefficients of vectors a & b,

$3+4\lambda=7\mu$ & $\lambda-1=-\mu$

or $\begin{cases} 4\lambda-7\mu=-3 & (6) \\ \lambda+\mu=1 & (7) \end{cases}$

Solving (6) and (7)

$\lambda=\frac{4}{11}$ and $\mu=\frac{7}{11}$ ✓

(IG-0606 Maths)

Vectors

Example 14. The diagram shows the quadrilateral OABC, such that $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$ and $\vec{OC} = \mathbf{c}$. It is given that AM : MC = 2 : 1 and OM : MB = 3 : 2

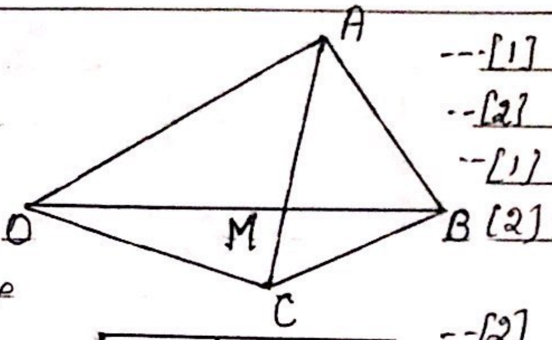
(i) Find $\vec{AC}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. --- [1]

(ii) Find $\vec{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$ -- [2]

(iii) Find $\vec{OM}$ in terms of $\mathbf{b}$ -- [1]

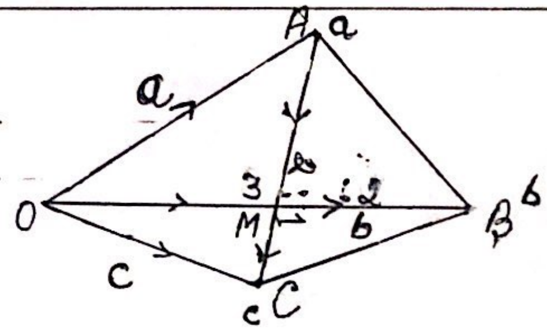
(iv) Find $5\mathbf{a} + 10\mathbf{c}$ in terms of $\mathbf{b}$ [2]

(v) Find $\vec{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$, giving your answer in its simplest form. -- [2]



M-18/12/Q6

Solution (i) $\vec{AC} = \mathbf{c} - \mathbf{a}$ —— (1)



(ii) $\vec{OM} + \vec{MC} = \vec{OC}$

$\therefore \vec{OM} = \vec{OC} - \vec{MC}$ $\left(\because AM : MC = 2 : 1,\ \vec{MC} = \frac{1}{3}\vec{AC}\right)$

or $\vec{OM} = \vec{OC} - \frac{1}{3}\vec{AC}$

$= \mathbf{c} - \frac{1}{3}(\mathbf{c} - \mathbf{a})$ from (1)

$= \left(\frac{2}{3}\mathbf{c} + \frac{1}{3}\mathbf{a}\right)$ —— (2)

(iii) OM : MB = 3 : 2

$\vec{OM} = \frac{3}{5}\vec{OB} = \frac{3}{5}\mathbf{b}$ ✓ —— (3)

(iv) $5\mathbf{a} + 10\vec{c} = 5(\mathbf{a} + 2\mathbf{c})$

$= 5 \times 3\vec{OM}$ [from (2) $(\mathbf{a} + 2\mathbf{c}) = 3\vec{OM}$]

$= 15 \times \frac{3}{5}\mathbf{b}$ [$\vec{OM} = \frac{3}{5}\mathbf{b}$ from (3)

$= 9\mathbf{b}$ —— (4)

(v) $\vec{AB} = \mathbf{b} - \mathbf{a}$

$= \frac{5}{9}(\mathbf{a} + 2\mathbf{c}) - \mathbf{a}$ from (2) & (3) [$\vec{OM} = \frac{3}{5}\mathbf{b} = \frac{1}{3}(\mathbf{a} + 2\mathbf{c})$ $\Rightarrow \mathbf{b} = \frac{5}{9}(\mathbf{a} + 2\mathbf{c})$

$= \left(-\frac{4}{9}\mathbf{a} + \frac{10}{9}\mathbf{c}\right)$ ✓