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Probability and Statistics_1

Discrete Random Variable

Ex_1. Solution (Revision)

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Discrete Random Variable - Formulae.

Important Notes

1. **Probability Distribution:**

X	: x_1	x_2	x_3	---	x_n
$P(X=x)$	: p_1	p_2	p_3	---	p_n

Note (i) $\Sigma p_i = 1$

(ii) Expected value $E(X) = \Sigma p_i x_i$

(iii) Variance of the random variable X: $Var(X) = \sigma^2 = \Sigma p_i \cdot (x_i - E(X))^2$

or $\sigma^2 = \Sigma p_i x_i^2 - (E(X))^2$

2. **Binomial Prob. distribution:** $P(X=r) = {}^nC_r p^r q^{n-r}$ $(q = 1-p)$

$X \sim B(n, p)$: $0 \leq r \leq n$

Expected Value $E(x) = np$

Variance $\sigma^2 = npq$

3. **The Geometric Distribution:** $X \sim Geo(p)$

$p,\ qp,\ q^2 \cdot p,\ q^3 \cdot p, \ldots q^{r-1} \times p \ldots$

(i) $P(X=r) = q^{r-1} \cdot p$ and $E(X) = \frac{1}{p}$ ✓

(ii) $P(X \leq r) = 1 - q^r$; $P(X < r) = 1 - q^{r-1}$

(iii) $P(X > r) = q^r$

Discrete random variables.
(Binomial and Geometric - distributions)
E(X) and Var(X)

Example 1: A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random varible X represents the number of paperback books she chooses.

(a) Show that the probability that she chooses exactly 2 paperback books is $3/14$. ---[2]

(b) Draw up the prob. distribution table for X. --[3]

(c) You are given that $E(X)=3$; Find $Var(X)$. --[2]

SP-20/05/Q3

Solution (a) Paperback books = 6, Hardback books = 2; Total No = 8

No of books chosen = 4

$$P(\text{Exactly 2 paperback}) = {}^6C_2/{}^8C_4 = 15/70 = \frac{3}{14}$$

(b)

x	2	3	4
Prob(x)	$\frac{{}^6C_2 \times {}^2C_2}{{}^8C_4} = \frac{3}{14}$	$\frac{{}^6C_3 \times {}^2C_1}{{}^8C_4} = \frac{8}{14}$	$\frac{{}^6C_4 \times {}^2C_0}{{}^8C_4} = \frac{3}{14}$

Total no of books chosen = 4

No. of hardback = 2

∴ Minimum Paperback = 2 } 4

(c) $Var(X) = \sum p_i x_i^2 - (E(x))^2$

$$= \frac{3}{14} \times 2^2 + \frac{8}{14} \times 3^2 + \frac{3}{14} \times 4^2 - 3^2 \text{ (Given } E(X)=3)$$

$$= \frac{12}{14} + \frac{72}{14} + \frac{48}{14} - 9 = \frac{3}{7} \text{ or } 0.429$$

• **Example 2:** A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 3 is obtained. ---[2]

Find the probability that obtaining a 3 requires fewer than 7 throws.

SP-20/05/Q5(c)

Solution: $P(\text{getting 3 in each throw}) = p = \frac{1}{6}$

$$q = 1 - \frac{1}{6} = 5/6$$

To find $P(X<7) = (1-q^6) = 1 - \left(\frac{5}{6}\right)^6 = 0.665$ $[P(X \le r) = 1 - q^r]$

$$\left[P(X<7) = P(X \le 6) = p + qp + qqp + qqqp + q^4p + q^5p \right.$$
$$= p[1 + q + q^2 + \cdots + q^5] = p\left[\frac{1 \times (1-q^6)}{1-q}\right] \quad n=6$$
$$\left. = \frac{p(1-q^6)}{p} = (1-q^6) \right]$$

∴ • 'Geometric distibution'

Example 3: An ordinary fair die is thrown repeatedly until 1 or 6 is obtained

(a) Find the prob. that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6. --[3]

On another occasion die is thrown 3 times. The random variable X is the number of times that a 1 or a 6 is obtained.

(b) Draw up the prob. distribution table for X. --[3]

(c) E(X) [M-20/52/Q2] --[2]

Solution: Sample space $S = \{1,2,3,4,5,6\}$

In a single throw $p = P(1 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

and $q = 1 - \frac{1}{3} = 2/3$

(a) atleast 3 but not more than 5 throws to get 1 or 6

$$P(3 \leq X \leq 5) = q^2p + q^3p + q^4p$$

$$= q^2p(1+q+q^2) = \left(\frac{2}{3}\right)^2 \times \frac{1}{3}\left(1+\frac{2}{3}+\left(\frac{2}{3}\right)^2\right)$$

$$= \frac{4}{27} \times \frac{19}{9} = \frac{76}{243}$$

Alternate method:

Using Geometric Distribution.

$P(X \leq r) = 1 - q^r$

$$\therefore P(3 \leq X \leq 5) = P(X \leq 5) - P(X \leq 2)$$

$$= (1-q^5) - (1-q^2) = (q^2 - q^5) = q^2(1-q^3)$$

$$= \left(\frac{2}{3}\right)^2\left(1-\left(\frac{2}{3}\right)^3\right) = \frac{4}{9}\left(1-\frac{8}{27}\right)$$

$$\frac{4}{9} \times \frac{19}{27} = 76/243$$

(b) Using Binomial distribution:

$n = 3,\ p = 1/3,\ q = 2/3$

as the prob of getting a 1 or a 6 is same in each throw.

X	0	1	2	3
P(X)	${}^nC_0\, p^0 q^3 = \left(\frac{2}{3}\right)^3$ ${}^3C_0 \cdot 1 \times \left(\frac{2}{3}\right)^3 = \frac{8}{27}$	${}^3C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^2$ $= 12/27$	${}^3C_2 \cdot \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1$ $= \frac{6}{27}$	${}^3C_3 \times \left(\frac{1}{3}\right)^3$ $= \frac{1}{27}$

(c) $E(X) = \sum p_i x_i = 0 \times \frac{8}{27} + 1 \times \frac{12}{27} + 2 \times \frac{6}{27} + 3 \times \frac{1}{27} = \frac{27}{27} = 1$

Alternate method:

for Binomial distribution $E(X) = np = 3 \times \frac{1}{3} = 1$

Example 4: In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.
Find the probability that the number of adults in this sample who own a car is less than 6. [M-20/52/Q5(a)] --[3]

Solution: P(adult has a car) = 70% ⇒ $p = 0.7 \rightarrow q = 1-0.7 = 0.3$; $n = 8$

$$P(r<6) = 1 - P(6,7,8) = 1 - \{{}^8C_6 (0.7)^6 (0.3)^2 + {}^8C_7 (0.7)^7 (0.3)^1 + 0.7^8\}$$
$$= 1 - 0.55177$$
$$= \underline{0.448}$$

(using Binomial prob.)
$\{P(r) = {}^nC_r\, p^r . q^{n-r}\}$

5. A fair spinner with 5 sides numbered 1, 2, 3, 4, 5, is spun repeatedly. The score on each spin is the number on the side on which the spinner lands.
(a) Find the prob. that a score of 3 is obtained for the first time on the 8th spin. --[1]
(b) Find the prob. that fewer than 6 spins are required to obtain a score of 3 for the first time. --[2]
[M-21/52/Q1]

Solution (a) Score 3 on the 8th spin (Dicrete Geo. Prob. distribution)
$p = P(\text{Score } 3) = \frac{1}{5}$, $q = 1 - \frac{1}{5} = \frac{4}{5}$

$$P(r=8) = q^7 . p = \left(\frac{4}{5}\right)^7 . \frac{1}{5} = \underline{0.0419}$$

(b) $P(r<6) = P(r \leq 5) = 1 - q^5 = 1 - \left(\frac{4}{5}\right)^5 = \underline{0.672}$

6. The random variable X takes the values 1, 2, 3, 4 only. The probability that X takes the value x is $kx(5-x)$, where k is constant.

(a) Draw up the prob. distribution table for X, in terms of k. --- [2]

(b) Show that $Var(X) = 1.05$ --- [4]

[M-21/52/Q4]

Solution: $P(x) = kx(5-x)$, $X = 1, 2, 3, 4$

(a) Prob. distribution in terms

X	1	2	3	4
P(X)	4k	6k	6k	4k

--- [2]

(b) $\Sigma p_i = 20k = 1 \Rightarrow k = 0.05$

$E(X) = \Sigma p_i x_i = k[1\times4 + 2\times6 + 3\times6 + 4\times4] = 0.05\times 50$
$= 2.5$ ✓

$E(X^2) = \Sigma p_i x_i^2 = k[4\times1^2 + 6\times2^2 + 6\times3^2 + 4\times4^2] = 0.05\times146 = 7.3$ ✓

$\therefore Var(X) = E(X^2) - (E(X))^2$ $[\text{or } \Sigma p_i x_i^2 - \mu^2]$

$= 7.3 - (2.5)^2 = 1.05$ ✓

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7 A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the prob. distribution table for X. ---[3]

(b) Given $E(X)=0.25$, find the value of $Var(X)$. --[2]

[M-22 / 52 / Q1]

Solution: X = Sum of two number on red & Blue.

Red \ Blue	−3	−2	−1	−1
1	−2	−1	0	0
2	−1	0	1	1
2	−1	0	1	1
3	0	1	2	2

(a)

X	−2	−1	0	1	2
P(X)	1/16	3/16	5/16	5/16	2/16

Prob. distribution

(b) Given $E(X)=0.25$; $Var(X)=\sum p_i x_i^2 - [E(X)]^2$

Now $\sum p_i x_i^2 = \frac{1}{16}\times(-2)^2 + \frac{3}{16}(-1)^2 + \frac{5}{16}\times 0^2 + \frac{5}{16}\times 1^2 + \frac{2}{16}\times 2^2 = \frac{20}{16}$ ✓

$\therefore Var(X) = \sum p_i x_i^2 - (E(X))^2 = \frac{20}{16} - \left(\frac{1}{4}\right)^2 = \frac{19}{16}$ ✓ $[E(X)=0.25=1/4]$

8. In a certain country, the prob. of more than 10 cm of rain on any particular day is 0.18, independently of any other day.

(a) Find the prob. that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. --[3]

(b) For 3 randomly chosen 7-day periods, find the prob. that exactly two of these periods have atleast one day with more than 10 cm of rain. --[3]

[M-22 / 52 / Q2]

Solution

(a) $p = P(\text{rain more than } 10\text{cm on any day}) = 0.18$

Using Binomial Prob. $P(X=r) = {}^nC_r\, p^r q^{n-r}$

$p=0.18$; $q=1-0.18=0.82$; $n=7$

$P(r>2) = 1 - P(0,1,2)$

$= 1 - [\,{}^7C_0 \cdot .18^0 \times .82^7 + {}^7C_1 \cdot .18^1 \times .82^6 + {}^7C_2 \cdot .18^2 \times .82^5\,]$

$= 1 - [0.249285 + 0.383048 + 0.252251]$

$= 1 - 0.88458 = \underline{0.115}$ ✓

(b) $P(\text{atleast 1 day of rain out of 7}) = P(r \geq 1)$

$= 1 - P(0) = 1 - (0.82)^7$

$= 0.7507$

Now $n=3$, $p=0.7507$, $q=(1-0.7507)$

$P(\text{Exactly 2 Period}) = {}^3C_2\, p^2 \cdot q$

$= 3\times(0.7507)^2 \cdot (1-0.7507)$

$= \underline{0.421}$ ✓

9. A factory produces chocholates in three flavours: lemon, orange and straberry in the ratio 3 : 5 : 7 repectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocholate with lamon flavour that Nell chooses is the 7_{th} chocholate that she chocks. ---[1]

(b) Find the prob. that the first chocolate with lemon flavour that Nell chooses is after she has cheiked atleast 6 chocholates. [2]

'Surprise' boxes of chocholates each contain 15 chocolates: 3 are lemon, 5 are oranges and 7 are straberry.

Petra has a box of Surprise chocholetes. She chooses 3 chocolates at random from the box. She eats each chocotate before choosing the next one.

(c) Find the prob. that none of Petra's 3 chocolates has orange flavour. ---[2]

(d) Find the prob. that each of Petra's 3 chocolates has a different flavour [3]

(e) Find the prob. that atleast 2 of Petra's 3 chocolates have straberry flavour given that none of them has orange flavour [4]

M-22/52/Q6

Solution: Lemon : Orange : straberry = 3 : 5 : 7

(a) $p = P(\text{Lemon}) = 3/15 = \frac{1}{5}$; $q = 1 - \frac{1}{5} = \frac{4}{5}$

Using Geo. Prob. $P(x) = q^{x-1} \cdot p$ ✓

$P(7_{th}\ \text{Lemon}) = (4/5)^6 \cdot 1/5 = \frac{4096}{78125}$ (0.0524)

(b) $P(\text{Lemon}, x > 6] = q^6 = (4/5)^6 = \frac{4096}{15625}$ ✓

(c) Orange = 5, Rest = 10

$P(\text{None orange}) = \frac{10}{15} \times \frac{9}{14} \times \frac{8}{13} = \frac{24}{91}$ ✓

(or ${}^{10}C_3 / {}^{15}C_3$)

(d) P(3 with diff colours)

$= 7/15 \times \frac{5}{14} \times \frac{3}{13} \times 3!$

$= 3/13$ (0.231)

(e) $P(\text{None Orange}) = \frac{24}{91}$ (Part (c))

P(atleast two straberry and None orange)

= P(2 straberry) + P(3 straberry)

$= 7/15 \times \frac{6}{14} \times \frac{3}{13} \times 3 + \frac{7}{15} \times \frac{6}{14} \times \frac{5}{13} = 14/65$

P(atleast two straberry / None orange)

$= \frac{14}{65} \Big/ \frac{24}{91}$

$= \frac{14}{65} \times \frac{91}{24}$

$= \frac{49}{60}$ ✓ (0.817)

10. Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. ---[3]

(b) Complete the following prob. distribution table for X. ---[2]

x	0	1	2	3	4
$P(X=x)$	0.05	0.225			0.075

(c) Given that $E(X)=2.1$, find the value of $Var(X)$ ---[2]

[M-23/52/Q2]

Solution (a) P(Exactly one head) = P(one head of biased coin) × (No head on fairs coins) + P(No head on biased coins) × (one head on a fair coin)

$= 0.6\times(0.5)^3 + 0.4\times(.5)^3\times 3 = \underline{0.225}$

(b) P(2) = P(one baised) × (One fair) × (No head on two fair) + P(No head on biased) × P(Two heads on fair coins)

$= 0.6\times 0.5\times(0.5)^2\times 3 + 0.4\times 3\times(.5)^2\times 0.5 = \underline{0.375}$

P(3) = P(one baised) × (2 fair) + P(No baised) × (3 fair) $= 0.6\times 3\times(0.5)^2\times 0.5 + 0.4\times(.5)^3 = \underline{0.275}$

(c) $E(X) = 2.1$

$\Sigma p_i x_i^2 = 0 + 1^2\times 0.225 + 2^2\times 0.375 + 3^2\times 0.275 + 4^2\times 0.075$
$= 5.4$

$Var\,X = \Sigma p_i x_i^2 - E(x)^2$
$= 5.4 - (2.1)^2$
$= \underline{0.99}$

11. 80% of the residents of Kunwawa are in favour of a leisure centre being built in the town.

20 residents of Kunwawa are chosen at random and asked, in turn, whether they are in favour of the leisure centre.

(a) Find the prob. that more than 17 of these residents are in favour of the leisure centre. ---[3]

(b) Find the prob. that the 5_{th} person asked is the first person who is not in favour of the leisure centre. ---[1]

(c) Find the prob. that the 7_{th} person asked is the second person who is not in the favour of the leisure centre. ---[2]

[M-23/52/Q3]

Solution (a) $p = 0.8$ (80%), $q = 0.2$, $n = 20$

$P(X>17) = P(18, 19, 20)$

$= {}^{20}C_{18} \cdot (0.8)^{18} \cdot (0.2)^2 + {}^{20}C_{19} \cdot (0.8)^{19} \cdot (.2)^1 + {}^{20}C_{20} \cdot (0.8)^{20}$

$= 0.13691 + 0.05765 + 0.01153 = \underline{0.206}$ ✓

(b) $P(\text{not in favour}) = p_1 (\text{Now}) = 0.2$; $P(\text{in favour}) = q_1 = 0.8$

$P(5_{th} \text{ person was in favour}) = q_1^4 \cdot p_1 = (.8)^4 \times 0.2 = \underline{0.08192}$ ✓

(c) $P(7_{th} \text{ person is the second person not in favour})$

$= q_1^5 \times p_1^2 \times 6$

$= (0.8)^5 \cdot (0.2)^2 \times 6$

$= \underline{0.0786}$ ✓

$\left\{ \begin{array}{l} q_1, q_1 p, q_1^2 p, q^3 p, q^4 p, \\ q^5 \times p, \; q^5 p^2 k \\ \quad 2 \le k \le 7 \end{array} \right.$

Example 12. The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the prob. that the score is 4 is $\frac{1}{12}$. --- [1]

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws is denoted by the random variable X.

(b) Find mean of X -- [1]

(c) Find the prob. that a score of 4 is first obtained on the 6_{th} throw. -- [1]

(d) Find $P(X<8)$ [S-20/51/Q7] -- [2]

Solution: when a pair of dice is thrown $n=36$

(a)

1, 2, 3, 4, 5, 6

3, 2, 1

Score is 4 → (1,3), (2,2), (3,1) → 3 time

$\therefore P(\text{Score } 4) = 3/36 = \frac{1}{12} \checkmark = p$

(b) Geometric Prob, $p = \frac{1}{12}$

Mean of $X = \frac{1}{p} = \frac{1}{\frac{1}{12}} = 12 \checkmark$

(c) $P(4 \text{ on } 6_{th} \text{ throw}) = q^5 \cdot p$ $\quad p = \frac{1}{12}$

$= \left(\frac{11}{12}\right)^5 \cdot \frac{1}{12}$ $\quad [\because q = \frac{11}{12}$

$= 0.0539 \checkmark$

(d) $P(X<8) = P(X \leq 7)$

$= 1 - q^7$

$= 1 - \left(\frac{11}{12}\right)^7$

$= 0.456 \checkmark$

(Using Geometric Prob.)

Example 13 A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jameel chooses 3 sweets at random from a box.

(a) Draw up the prob. distribution table for the number of jellies that Jameel chooses. --[4]

The company also produces large boxes of sweets. For any large box, the prob that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the prob. that no more than 7 of these boxes contain more jellies than chocolates. [S-20/51/Q3] --[3]

Solution: No of jellies = 5

(a) No of Chocholates = 3 } 3 sweets are chosen. → Combination ${}^nC_r$

$${}^8C_3 = \frac{8!}{3!5!} = \frac{8\times7\times6}{3\times2\times1} = 56$$

No. of Jellies x	0	1	2	3
$P(x)$	$\frac{{}^5C_0}{{}^8C_3}$	$\frac{{}^5C_1 \times {}^3C_2}{{}^8C_3}$	$\frac{{}^5C_2 \times {}^3C_1}{{}^8C_3}$	$\frac{{}^5C_3}{{}^8C_3}$
	$\frac{1}{56}$	$15/56$	$\frac{30}{56}$	$\frac{10}{56}$

(b) $p = 0.64$, $q = 1 - 0.64 = 0.36$, $n = 10$

$P(X \leq 7) = 1 - P(8, 9, 10)$

$= 1 - \{ {}^{10}C_8 (0.64)^8 . (0.36)^2 + {}^{10}C_9 (0.64)^9 . (0.32)^1 + 0.64^{10} \}$

$= \underline{0.759}$ ✓

[Using Binomial Prob.
$P(X = r) = {}^nC_r \, p^r . q^{n-r}$

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Example 14. A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbers 1, 1, 2, 2, 3, Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X=3) = \frac{7}{15}$ --[2]

(b) Draw up the prob. distribution table for X --[3]

(c) Find $E(X)$ and $Var(X)$. [S-20/52/Q5] --[3]

Solution: First spinner = {1, 2, 3}
Second spinner = {1, 1, 2, 2, 3}

(a)

	1	1	2	2	3
1	(1,1) 1	(1,1) 1	(1,2) 2	(1,2) 2	(1,3) 3
2	(2,1) 2	(2,1) 2	(2,2) 2	(2,2) 2	(2,3) 3
3	(3,1) 3	(3,1) 3	(3,2) 3	(3,2) 3	(3,3) 3

n = Total number of outcomes = $3 \times 5 = 15$
$X = 3$ has come → 7 times

$P(X=3) = \frac{7}{15}$ ✓

(b)

x	1	2	3
$P(x)$	2/15	6/15	7/15

(c) $E(X) = \Sigma p_i x_i$

$= 1 \times \frac{2}{15} + 2 \times \frac{6}{15} + 3 \times \frac{7}{15}$

$E(X) = \frac{35}{15} = \frac{7}{3}$ ✓

$Var(X) = \Sigma p_i x_i^2 - (E(X))^2$

$= \frac{2}{15} \times 1^2 + \frac{6}{15} \times 2^2 + \frac{7}{15} \times 3^2 - \left(\frac{7}{3}\right)^2$

$= \frac{89}{15} - \frac{49}{9} = \frac{22}{45}$

$Var(X) = \frac{22}{45}$ (or 0.489) ✓

Example 15. On any given day, the prob. that Moena messages her friend Pasha is 0·72.

(a) Find the prob. that for a random sample of 12 days Moena messages Pasha on no more than 9 days. ---[3]

(b) Moena messages Pasha on 1 January. Find the prob. that the next day on which she message Pasha is 5 January. --[1]

[S-20/52/Q7(a)(b)]

Solution: $p = 0.72 \rightarrow q = 1 - 0.72 = 0.28,\ n = 12.$ (Using Binomial probability $P(X=r) = {}^{n}C_{r} \cdot p^{r} q^{n-r}$)

(a) $P(X \leq 9) = 1 - P(10, 11, 12)$

$= 1 - \{ {}^{12}C_{10} (0.72)^{10} (0.28)^{2} + {}^{12}C_{11} (0.72)^{11} (0.28) + 0.72^{12} \}$

$= \underline{0.696}$ ✓

(b)

Jan 1	Jan 2	Jan 3	Jan 4	Jan 5
p	q	qq	qqq	$qqq \cdot p$

(Using Geometric Prob.)

$P(\text{message on 5 Jan}) = q^3 p = (0.28)^3 \times 0.72 = 0.0158$ ✓

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Example 16. In a certain large college, 22% of students own a car.

(a) 3 students from the college are chosen at random. Find the prob. that all three students own a car. --[1]

(b) 16 students from the college are chosen at random. Find the prob. that the number of those student who own a car is at least 2 and at most 4. --[3]

S-20/53/Q2

Solution: Using Binomial Prob. $P(X=r) = {}^nC_r\, p^r q^{n-r}$

(a) $n=3,\ p=0.22,$

$P(X=3) = {}^3C_3\, p^3 = (0.22)^3$

$= 0.0106$ ✓

(b) $n=16,\ p=0.22,\ q=0.78$

$P(2 \le r \le 4) = P(2, 3, 4)$

$= {}^{16}C_2 (.22)^2 (.78)^{14} + {}^{16}C_3 (.22)^3 (.78)^{13} + {}^{16}C_4 (.22)^4 (.78)^{12}$

$= 0.631$ ✓

Example 17. A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered −2, −1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random varible X is the sum of the two numbers that have been noted.

(a) Draw up the prob. distribution table for X. --[3]

(b) Find Var(X). [S-20/53/Q4] --[3]

Solution:

(a)

	1	2	2	3
−2	−1	0	0	1
−1	0	1	1	2
1	2	3	3	4

for the sum of number $n = 4\times 3 = 12$

X	−1	0	1	2	3	4
P(X)	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

(b) $E(X) = \Sigma p_i x_i$

$= \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$

$\therefore Var(X) = \Sigma p_i x_i^2 - (E(X))^2$

$= \frac{1}{12}(1+0+3+8+18+16) - \left(\frac{4}{3}\right)^2$

$= \frac{37}{18} (= 2.06)$

Example 18. A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random varible X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of x. --[1]

(b) Find the prob. that exactly 3 throws are required to obtain a pair of tails. --[1]

(c) Find the prob. that fewer than 6 throws are required to obtain a pair of tails. [S-20/53/Q5a,b,c] -[2]

Solution: A pair of coins is thrown –

(a) $S = \{HH, HT, TH, TT\}$

$p = P(\text{both TT}) = \frac{1}{4}$. $\therefore p = \frac{1}{4}, q = \frac{3}{4}$

$\therefore E(X) = \frac{1}{p} = \frac{1}{\frac{1}{4}} = 4$

(Geometric Probability)

(b) $P(X=3) = qqp = \frac{3}{4}\times\frac{3}{4}\times\frac{1}{4} = \frac{9}{64} (= 0.141)$

(c) $P(X<6) = P(X\leq 5) = 1 - q^5$

$= 1 - (3/4)^5$

$= 0.763$

$(\because P(X \leq r) = 1 - q^r)$

19. In Questa, 60% of the adults travel to work by car.
A random sample of 12 adults from Questa is taken.
Find the probability that the number who travel to work by car is less than 10. [S-21/51/Q6(a)] -- [3]

Solution: P(An adult travels by car), $p = \frac{60}{100} = 0.6$

$q = 1 - 0.6 = 0.4$

$n = 12.$

P(Adults travelling by car less than 10) $= P(0, 1, --- 9)$

$= 1 - P(10, 11, 12)$

$= 1 - \{ {}^{12}C_{10} \cdot 0.6^{10} \cdot 0.4^{2} + {}^{12}C_{11}\, 0.6^{11} \times 0.4^{1} + {}^{12}C_{12} \cdot 0.6^{12}]$

$= 1 - 0.08334 = 0.917$

$= \underline{0.917}$

20. Sharma knows that she has 3 tins of carrots, 2 tins of peas, and 2 tins of sweetcorns in her cupboard. All the tins are the same shape and size, but the labels have been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrot. The random variable X is the number of tins that she needs to open.

(a) Show that $P(X=3) = 6/35$ ---[2]

(b) Draw up the probability distribution table for X. ---[4]

(c) Find $Var(X)$. --[3]

S-21/51/Q7

Solution: Number of Carrot tins = 3; Number of peas tins = 2; Sweetcorns = 2

Total Numbers of tins = 3+2+2 = 7

(a) $P(X=3) = P(\text{Not Carrot}) \times P(\text{Not Carrot}) \times P(\text{Carrot tin})$

$= \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} = \frac{6}{35}$ ✓

(b) Probability distribution table for X.

X	1	2	3	4	5
p	p_1 $\frac{3}{7}$ $= \frac{15}{35}$	$q_1 p_1$ $\frac{4}{7}\times\frac{3}{6}$ $= \frac{10}{35}$	$q_1 q_2 p_1$ $\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$ $= \frac{6}{35}$	$q_1 q_2 q_3 p_1$ $\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{3}{4}$ $= \frac{3}{35}$	$q_1 q_2 q_3 q_4 p_1$ $\frac{4}{7}\times\frac{3}{6}\times\frac{2}{5}\times\frac{1}{4}\times\frac{3}{3}$ $= \frac{1}{35}$

(c) $Var(X) = \Sigma p_i x_i^2 - [E(x)]^2$ ---(i)

$E(X) = \Sigma p_i x_i$

$= \frac{15}{35}\times 1 + \frac{10}{35}\times 2 + \frac{6}{35}\times 3 + \frac{3}{35}\times 4 + \frac{1}{35}\times 5 = \frac{70}{35} = 2$ ✓ ---(ii)

$\Sigma p_i x_i^2 = \frac{15}{35}\times 1^2 + \frac{10}{35}\times 2^2 + \frac{6}{35}\times 3^2 + \frac{3}{35}\times 4^2 + \frac{1}{35}\times 5^2 = \frac{15+40+54+48+5}{35} = \frac{182}{35}$ ✓ --(iii)

from (ii) and (iii) in (i)

$Var(X) = \frac{182}{35} - 2^2 = \frac{42}{35} = \frac{6}{5}$ ✓ (or 1.2)

21. An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X.

(a) Write down the mean of X. ---[1]

(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. ---[2]

(c) Find the probability that a 5 is first obtained in fewer than 10 throws. $\boxed{S\text{-}21 \,|\, 52 \,|\, Q1}$ ---[2]

Solution: When an ordinary dice is thrown $\{1, 2, 3, 4, 5, 6\}$

$P(\text{throwing } 5): p = \frac{1}{6}, \quad q = 1 - \frac{1}{6} = \frac{5}{6}$

when die is repeatedly thrown:
$$\begin{cases} P(\text{getting success in } r_{th} \text{ trial}) = q^{r-1} \cdot p & X = r \\ P(X < r) = 1 - q^{r-1} \\ P(X \leq r) = 1 - q^{r} \\ P(X > r) = q^{r} \end{cases}$$

$E(X) = \frac{1}{p}$

(a) Mean of $X = E(X) = \frac{1}{p} = \frac{1}{\frac{1}{6}} = \underline{6}$ ✓

(b) $P(3 < r < 8) = P(r = 4, 5, 6, 7) = q^3p + q^4p + q^5p + q^6p$

$= P(r > 3) - P(r > 7)$

$= q^3 - q^7$ ✓

$$\begin{cases} = q^3p[1 + q + q^2 + q^3] \\ = q^3 \cancel{p} \cdot \frac{(1-q^4)}{\cancel{(1-q)}} = q^3 - q^7 \\ = \left(\frac{5}{6}\right)^3 - \left(\frac{5}{6}\right)^7 = 0.2997 \\ = \underline{0.3} ✓ \end{cases}$$

(c) $P(r < 10) = 1 - q^9$ $\quad [P(X < r) = 1 - q^{r-1}]$

$= 1 - \left(\frac{5}{6}\right)^9$

$= \underline{0.806}$ ✓

22. A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered −2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution for X. ---[3]

(b) Find E(X) and Var(X). ---[3]

[S-21/52/Q4]

Solution (a) X denotes the sum of the numbers on the two spinners.

	−2	0	1
1	−1	1	2
2	0	2	3
2	0	2	3

Number of outcomes $= 3\times 3 = 9$ ✓

(a) Probability distribution of X:

X	−1	0	1	2	3
P(X)	1/9	2/9	1/9	3/9	2/9

(b) $E(X) = \sum p_i x_i = \frac{1}{9}\times(-1) + \frac{2}{9}\times 0 + \frac{1}{9}\times 1 + \frac{3}{9}\times 2 + \frac{2}{9}\times 3$

$= \frac{-1+0+1+6+6}{9} = \frac{12}{9} = \frac{4}{3}$ ✓ ---(i)

Now $Var(X) = \sum p_i x_i^2 - (E(X))^2$ ----(ii)

Now $\sum p_i x_i^2 = \frac{1}{9}\cdot(-1)^2 + \frac{2}{9}\times 0^2 + \frac{1}{9}\times 1^2 + \frac{3}{9}\times 2^2 + \frac{2}{9}\times 3^2$

$= \frac{1+0+1+12+18}{9} = \frac{32}{9}$ ----(iii)

from (ii) $Var(X) = \frac{32}{9} - \left(\frac{4}{3}\right)^2$ [from (i) and (iii)]

$= \frac{32-16}{9} = \frac{16}{9}$

$\therefore Var(X) = \frac{16}{9}$ (or 1.78) ✓

23. Every day Richard takes a flight between Aston and Begin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrives late. ---[1]

(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early atleast 3 times. ---[3]

S-21/52/Q5

Solution: P(arrives early) = 0.15; P(on time) = 0.55; P(late) = 0.3

(a) P(late) = 0.3 ⟶ $P(\underset{\text{any day}}{\text{does not arrives late}}) = 1 - 0.3 = 0.7$ ✓

$n = 3$ day

$P(\underset{\text{for 3 days.}}{\text{does not arrive late}}) = (0.7)^3 = \underline{0.343}$ ✓

(b) Now $n = 9$, $p = P(\text{arrives early}) = 0.15$

atleast 3 days ⟶ $q = 1 - 0.15 = 0.85$ ✓

$P(r \geq 3) = 1 - P(0, 1, 2)$

$= 1 - \{0.85^9 + {}^9C_1 \times 0.15^1 \times 0.85^8 + {}^9C_2 \times 0.15^2 \times 0.85^7\}$

$= 1 - \{0.231617 + 0.367862 + 0.259667\}$

$= \underline{0.141}$ ✓

24. The randomly variable X can only takes values $-2, -1, 0, 1, 2$. The probability of X is given in the following table:

x	-2	-1	0	1	2
$P(X=x)$	p	p	0.1	q	q

--- [4]

Given that $P(X \geq 0) = 3P(X<0)$, find the values of p and q.

[S-21 | 53 | Q2]

<u>Solution</u>: $\Sigma p_i = 1 \Rightarrow p+p+0.1+q+q = 1$

$\Rightarrow 2p+2q = 0.9$ --- (i)

and $P(X \geq 0) = 3P(X<0) \Rightarrow 0.1+2q = 3(2p)$

$\Rightarrow 6p - 2q = 0.1$ --- (ii)

add equations (i) and (ii) $\Rightarrow 8p = 1 \Rightarrow p = \frac{1}{8}$ ✓ (0.125)

from (i) $2 \times \frac{1}{8} + 2q = 0.9$

$\Rightarrow q = 0.325$ ✓

---

25. Three fair six-sided dice, each with faces marked, 1, 2, 3, 4, 5 and 6 are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

Find the probability that a score of 18 is obtained for the first time on the 5_{th} throw of the three dice. --- [3]

[S-21 | 53 | Q4(b)]

<u>Solution</u>: When 3 dice are thrown the total number of out comes $= 6 \times 6 \times 6$
$= 216$

Total $18 = 6+6+6$ will be obtained only once.

$\therefore P(\text{Total } 18) = \frac{1}{216} = p(\text{let}) \Rightarrow q = 1 - \frac{1}{216} = \frac{215}{216}$ ✓

$P(\text{Getting total 18 in the } 5_{th} \text{ throw}) = q^4 \cdot p$ [Geometric prob distribution]

$= \left(\frac{215}{216}\right)^4 \cdot \frac{1}{216}$

$= 0.00454$ ✓

26. In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.
10 households in Arka are chosen at random.
Find the probability that fewer than 3 of these households have no broadband service. ---[3]
[S-21/53/Q 7(b)(i)]

Solution: P(No broadband service) = 0.35 (35%)

$\therefore p = 0.35,\ q = 0.65$ and $n = 10$

Binomial prob. distribution;

$P(r<3) = P(0,1,2)$ $\quad [P(r) = {}^nC_r \cdot p^r \cdot q^{n-r}]$

$= {}^{10}C_0 (0.35)^0 \cdot 0.65^{10} + {}^{10}C_1 (0.35)^1 (0.65)^9 + {}^{10}C_2 (0.35)^2 \cdot (0.65)^8$

$= 0.013463 + 0.072492 + 0.17565 = \underline{0.262}$ ✓

27. Jacob has four coins. One of the coins is baised such that when it is thrown the prob. of obtaining a head is 7/10. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The prob. distribution table for X is as follows:

x	0	1	2	3	4
$P(X=x)$	3/80	a	b	c	7/80

(a) Show that $a = 1/5$ and find the values of b and c. ---[4]

(b) Find $E(X)$ [1]

Jacob throws all four coins together 10 times.

(c) Find the prob. that he obtains exactly one head on fewer than 3 occasions. ---[3]

(d) Find the prob. that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. --[2]

[S-22/51/Q4]

Solution (a) $a = P(1\text{ head})$

$= P(1 \text{ baised head and 3 tails})$
$+ P(1 \text{ fair head and 1 baised tail and 2 fair tails})$

$= 0.7 \cdot (0.5)^3 + 0.3 \times (0.5)^3 \times 3 = 1/5$ ✓

$b = P(2\text{ heads}) = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 \times 3 = 3/8$ ✓

$c = P(3\text{ head}) = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 = 3/10$ ✓

(b) $E(X) = \sum p_i x_i = \frac{3}{80} \times 0 + \frac{1}{5} \times 1 + \frac{3}{8} \times 2 + \frac{3}{10} \times 3 + \frac{7}{80} \times 4 = 176/80$ ✓

(c) $p = P(\text{Exactly one head}) = 0.2,\ q = 0.8$
$n = 10$
$P(r < 3) = P(0, 1, 2)$
$= {}^{10}C_0 \cdot (.2)^0 (.8)^{10} + {}^{10}C_1 (.2)^1 (.8)^9 + {}^{10}C_2 (.2)^2 \cdot (.8)^8$

$= 0.107374 + 0.268435 + 0.301980$
$= 0.678$ ✓

[Using Binomial dis. $P(X=r) = {}^nC_r\, p^r \cdot q^{n-r}$

(d) Using Geo. Prob. distributn.
$p = 0.2,\ q = 0.8,\ r = 7 \text{ or } 8$
$P(\text{getting in } r_{th} \text{ time}) = q^{r-1} \cdot p$
$= (0.8)^6 \times 0.2 + (.8)^7 \times 0.2$
$= 0.0524288 + 0.041943$
$= 0.0944$ ✓

28. A fair six sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the prob. distribution table for X. ---[3]

(b) Find E(X) and Var(X). ---[3]

[S-22/52/Q2]

Solution (a)

x	2	3	4	5	6
$P(x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$

	1	2	2	3	3	3
1	2	3	3	4	4	4
2	3	4	4	5	5	5
2	3	4	4	5	5	5
3	4	5	5	6	6	6
3	4	5	5	6	6	6
3	4	5	5	6	6	6

(b) $E(X) = \Sigma p_i x_i$

$= \frac{1}{36}\times 2 + \frac{4}{36}\times 3 + \frac{10}{36}\times 4 + \frac{12}{36}\times 5 + \frac{9}{36}\times 6$

$= \frac{168}{36} = \frac{14}{3}$ ✓

$\Sigma p_i x_i^2 = \frac{1}{36}\times 2^2 + \frac{4}{36}\times 3^2 + \frac{10}{36}\times 4^2 + \frac{12}{36}\times 5^2 + \frac{9}{36}\times 6^2 = \frac{824}{36} = \frac{206}{9}$ ✓

$\therefore Var(X) = \Sigma p_i x_i^2 - (E(X))^2 = \frac{206}{9} - \left(\frac{14}{3}\right)^2 = \frac{10}{9}$ ✓

29. The random variable X takes the values −2, 1, 2, 3. It is given that $P(X=x) = kx^2$, where k is a constant.

(a) Draw up the prob. distribution table for X. ---[3]

(b) Find E(X) and Var(X). ---[3]

[S-22/53/Q3]

Solution: Prob. distribution for $P(x) = kx^2$

(a)

x	−2	1	2	3
$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

$\Sigma p_i = 1$

$k(-2)^2 + k\times 1^2 + k\times 2^2 + k\times 3^2$

$\Rightarrow 4k + k + 4k + 9k = 1$

$\Rightarrow 18k = 1$

$\Rightarrow k = \frac{1}{18}$ ✓

(b) $E(X) = \Sigma p_i x_i = \frac{4}{18}\times(-2) + \frac{1}{18}\times 1 + \frac{4}{18}\times 2 + \frac{9}{18}\times 3 = \frac{28}{18} = \frac{14}{9}$ ✓

$\Sigma p_i x_i^2 = \frac{4}{18}\times(-2)^2 + \frac{1}{18}\times 1^2 + \frac{4}{18}\times 2^2 + \frac{9}{18}\times 3^2 = \frac{114}{18} = \frac{57}{9}$ ✓

$\therefore Var(X) = \Sigma p_i x_i^2 - (E(X))^2 = \frac{57}{9} - \left(\frac{14}{9}\right)^2 = \frac{57}{9} - \frac{196}{81}$

$= \frac{317}{81}$ ✓ $\left(3\frac{74}{81}\right)$

30. Ramesh throws an ordinary fair 6-sided die.

(a) Find the prob. that he obtains a 4 for the first time on his 8_{th} throw. --[1]

(b) Find the prob. that it takes no more than 5 throws for Ramesh to obtain a 4. --[2]

Ramesh now repeatedly throws to ordinary 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the prob. that Ramesh obtains a total of less than 4 on at least three throws.

[S-22/53/Q4] --[4]

Solution: Using Geo. Prob. distribution: $p = P(4) = 1/6$

(a) $P(\text{getting in } r_{th} \text{ throw}) = q^{r-1} \cdot p$ $\{ q = 5/6$

$P(\text{getting 4 in } 8_{th} \text{ throw}) = \left(\frac{5}{6}\right)^7 \cdot \frac{1}{6} = \underline{0.0465}$

(b) $P(r \leq 5) = 1 - q^5 = 1 - \left(\frac{5}{6}\right)^5 = \underline{0.598}$

[Using Bino Prob. $P(X=r) = {}^nC_r\, p^r \cdot q^{n-r}$]

(C) Two dice $= \{(1,1), (1,2), \ldots (1,6) \ldots (6,6)\}$

$n(S) = 36$

$P(\text{Total less than } 4) = P(2) + P(3)$

$P\{(1,1), (1,2), (2,1)\}$

$p = 3/36 = \frac{1}{12}$;

$q = 1 - \frac{1}{12} = \frac{11}{12}$

$n = 10$;

$P(r \geq 3) = 1 - P(0,1,2)$

$= 1 - \left\{ {}^{10}C_0 \left(\frac{1}{12}\right)^0 \left(\frac{11}{12}\right)^{10} + {}^{10}C_1 \left(\frac{1}{12}\right)^1 \left(\frac{11}{12}\right)^9 + {}^{10}C_2 \left(\frac{1}{12}\right)^2 \left(\frac{11}{12}\right)^8 \right\}$

$= 1 - (0.418904 + 0.380822 + 0.155791)$

$= 0.0455$

31. Eli has four fair 4-sided dice with sided labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X=3) = 3/64$ ... [2]

(b) Complete the following probability distribution table for X.

x	0	1	2	3	4
$P(X=x)$	$81/256$			$3/64$	$\frac{1}{256}$

....[2]

(c) Find $E(X)$ ....[2]

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the prob. that he obtains at least two 2s on fewer than 20 of these occasions. ...[5]

[S-23/51/Q6]

<u>Solution</u>(a) $S = \{1,2,3,4\}$, $n=4$, $p = P(\text{getting } 2) = \frac{1}{4}$, $q = 1-\frac{1}{4} = 3/4$

$P(X=3) = {}^4C_3 \cdot \left(\frac{1}{4}\right)^3 \cdot \frac{3}{4} = 4 \times \frac{1}{64} \times \frac{3}{4} = \frac{3}{64}$ ✓ $\left(P(X=r) = {}^nC_r \cdot p^r \cdot q^{n-r}\right)$

(b)

x	0	1	2	3	4
$P(X=x)$	$81/256$	$27/64$ ✓	$27/128$ ✓	$3/64$	$\frac{1}{256}$

$P(X=1) = {}^4C_1 \cdot p^1 \cdot q^3 = 4 \times \frac{1}{4} \times \left(\frac{3}{4}\right)^3 = \frac{27}{64}$ ✓

$P(X=2) = {}^4C_2\, p^2 q^2 = 6 \times \left(\frac{1}{4}\right)^2 \times \left(\frac{3}{4}\right)^2 = 27/128$ ✓

(c) $E(X) = \sum p_i x_i = 0 \times \frac{81}{256} + 1 \times \frac{27}{64} + 2 \times \frac{27}{128} + 3 \times \frac{3}{64} + 4 \times \frac{1}{256} = 0 + \frac{27}{64} + \frac{54}{128} + \frac{36}{256} + \frac{4}{256} = 1$ ✓

(Note: As the prob. distribution is a Binomial prob. dis $\rightarrow$ $E(x) = np = 4 \times \frac{1}{4} = 1$ ✓

(d) Now $n = 96$, The Binomial distribution $B(n, p) \longrightarrow N(\mu, \sigma^2)$ Normal distribution,

$P(X \geq 2) = 1 - \{P(0) + P(1)\}$
$= 1 - \left(\frac{81}{256} + \frac{27}{64}\right)$

Now, $p = \frac{67}{256}$

Here $\mu = np = 96 \times \frac{67}{256} = 25.125$ ✓
and $\sigma^2 = npq = 96 \times \frac{67}{256} \times \frac{189}{256} = 18.549$

Using Normal prob. distribution.

$B(n,p) \rightarrow N(\mu, \sigma^2)$
Continuity Correction
$X < 20 \rightarrow X \leq 19.5$

Now $P(X < 20) = P\left(Z < \frac{19.5 - 25.125}{\sqrt{18.549}}\right)$

$= P(Z < -1.306)$

$= 1 - \phi(1.306)$

$= 1 - 0.9042$

$= 0.0958$ ✓

32. A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.
Sahim buys one copy of the magazine every Monday.

(a) Find the probability that the first time the free gift is an elephant is before the 6th Monday. ---[2]

(b) Find the prob. that Sahim will get more than two leopards in the 12 magazines ---[3]

(c) Find the prob. that after 5 weeks Sahim has exactly one of each animals... [3]

S-23/51/Q7

Solution $p = P(\text{elephant}) = \frac{1}{5} = 0.2 \Rightarrow q = 1 - 0.2 = 0.8$

(a) $P(X < 6) = P(X \leq 5) = 1 - q^5 = 1 - (0.8)^5 = \underline{0.672}$ ✓

(b) $P(\text{more than two leopards}) = 1 - P(0, 1, 2)$ $\left\{ \begin{array}{l} \text{Using Binomial dist.} \\ p = 0.2, q = 0.8, n = 12 \end{array} \right.$

$= 1 - \{ {}^{12}C_0 (0.8)^{12} + {}^{12}C_1 (0.2)^1 (0.8)^{11} + {}^{12}C_2 (0.2)^2 \cdot (0.8)^{10} \}$

$= 1 - \{0.06872 + 0.20615 + 0.28347\}$

$= \underline{0.442}$ ✓

(c) $P(\text{any one animal}) = 0.2$

$\therefore P(\text{exactly one animal after 5 weeks}) = (0.2)^5 \times 5!$

$= \underline{0.0384}$ ✓

33. The random variable X takes values −2, 2 and 3. It is given that: $P(X=x) = k(x^2-1)$, where k is a constant.

(a) Draw up the prob. distribution table for X, giving the prob. as numerical fractions. ...[3]

(b) Find E(X) and Var(X). ...[3]

[S-23/52/Q1]

Solution (a) $P(X=x) = k(x^2-1)$

x	−2	2	3
$P(x)$	$3k = 3/14$	$3k$ $3/14$	$8k$ $8/14$ ✓

Now $\sum p_i = 1$
$\Rightarrow 3k+3k+8k=1 \Rightarrow k = 1/14$

$E(X) = \sum x_i \cdot p_i = -2\times\frac{3}{14} + 2\times\frac{3}{14} + 3\times\frac{8}{14} = \frac{24}{14} = \frac{12}{7}$ ✓

Consider $\sum x_i^2 \cdot p_i = 4\times\frac{3}{14} + 4\times\frac{3}{14} + 9\times\frac{8}{14} = \frac{96}{14} = \frac{48}{7}$ ✓

$\therefore Var(X) = \sum x_i^2 \cdot p_i - (E(X))^2 = \frac{48}{7} - \left(\frac{12}{7}\right)^2 = \frac{48}{7} - \frac{144}{49} = \frac{192}{49}$ ✓
$= 3.92$ ✓

34. A sport event is taking place for 4 days, beginning on Sunday. The prob. that it will rain on Sunday is 0.4. On any subsequent day, the prob. that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the prob. that it does not rain on any of the four days of the event. ...[1]

(b) Find the prob. that the first day on which it rains during the event is Tuesday. [2]

(c) Find the prob. that it rains on exactly one of the four days of the event. ...[3]

[S-23/52/Q2]

Solution: P(rain on Sunday) = 0.4
P(No rain on Sunday) = 0.6

P(No rain on day next if no rain on previous day) = 1 − 0.2 = 0.8

(a) P(No rain on any of the four days)
$= 0.6\times(0.8)^3 = 0.3072$ ✓

(b) P(first day it rains is Tuesday)
= P(Not Sunday)·(Not Monday)·(Rain on Tuesday)
$= 0.6\times0.8\times0.2 = 0.096$ ✓

(c) P(rains on exactly one day)
= P(RDDD) + P(DRDD) + P(DDRD) + P(DDDR)
$= 0.4\times0.3\times0.8\times0.8 + 0.6\times0.2\times0.3\times0.8 + 0.6\times0.8\times0.2\times0.3 + 0.6\times0.8\times0.8\times0.2$
$= 0.0768 + 0.0288 + 0.0288 + 0.0768$
$= 0.2112$ ✓

{D → Dry
R → Rain}

35. A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable X denotes the number of spins required.

(a) Find $P(X=4)$ ---[1]

(b) Find $P(X<6)$ ...[2]

Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5 are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the prob. that the score is greater than 0, given that the score is not equal to 2. ---[3]

The two spinners are spun at the same time repeatedly.

(d) For 9 randomly chosen spins of the spinners, find the prob. that the score is greater than 2 on at least 3 occasions. --[3]

$\boxed{S\text{-}23/52/Q4}$

Solution (a) $p = P(2) = \frac{1}{5} = 0.2,\ q = 0.8$; (Geo(p), $P(X=r) = q^{r-1}\cdot p$

$P(X=4) = q^3 p = (0.8)^3 \times 0.2 = \underline{0.1024}$ ✓

(b) $P(X<6) = P(X \leq 5) = 1 - q^5$

$= 1-(0.8)^5 = \underline{0.672}$ ✓

(c) $P(X>0 / X\neq 2) = \dfrac{P(X>0 \cap X\neq 2)}{P(X\neq 2)}$

$= \dfrac{14/25}{19/25} = \dfrac{14}{19} = \underline{0.737}$ ✓

×	1	2	3	4	5
1	0	1	2	3	4 ✓
2	1	0	1	2	3 ✓
3	2	1	0	1	2
4	3 ✓	2	1	0	1
5	4 ✓	3 ✓	2	1	0

(d) $P(X \geq 3) = P(3,4,5,6,7,8,9)$, $\left\{ \text{with } p = \frac{6}{25},\ q = \frac{19}{25} \right.$; $p = (X>2)$ In a single throw

$= 1 - P(0,1,2)$

$= 1 - \left\{ {}^9C_0 \left(\frac{19}{25}\right)^9 + {}^9C_1 \left(\frac{6}{25}\right)^1 \left(\frac{19}{25}\right)^8 + {}^9C_2 \left(\frac{6}{25}\right)^2 \left(\frac{19}{25}\right)^7 \right.$

$= 1 - (0.08459 + 0.2404 + 0.3037)$

$= \underline{0.371}$ ✓

36. Two fair coins are thrown at the same time repeatedly unit a pair of heads is obtained. The numbers of throws taken is denoted by the random variable X.

(a) State the value of E(X). ---[1]

(b) Find the prob. that exactly 5 throws are required to obtain a pair of heads. ---[1]

(c) Find the prob. that fewer than 7 throws are required to obtain a pair of heads. S-23/53/Q1 ---[2]

Solution: Two coins are tossed $S=\{HH, HT, TH, TT\} \Rightarrow P(HH)=p=\frac{1}{4},\ q=\frac{3}{4}$

(a) $E(X)=\frac{1}{p}=\frac{1}{1/4}=\underline{4}$ ✓

(b) $P(X=5)=q^4 \cdot p=\left(\frac{3}{4}\right)^4 \times \frac{1}{4}=0.0791$ ✓

(c) $P(X<7)=P(X\le 6)=1-q^6$

$=1-\left(\frac{3}{4}\right)^6=0.822$ ✓

37. The random variable X takes values 1, 2, 3, 4. It is given that $P(X=x)=kx(x+a)$ where k and a are constants.

(a) Given that $P(X=4)=3P(X=2)$, find the value of a and the value of k. ---[4]

(b) Draw up the prob. distribution table for X, giving the prob. as numerical fractions. ---[1]

(c) Given that $E(X)=3.2$, find Var(X). ---[2]

S-23/53/Q3

Solution $P(X=x)=kx(x+a)$ ---(1)

(a) Given $P(X=4)=3 \cdot P(X=2)$

$\Rightarrow 4k(4+a)=3\times 2k(2+a)$

$\Rightarrow 16k+4ak=12k+6ak$

$\Rightarrow 4k=2ak \Rightarrow a=2$ ✓

from (1) $P(X=x)=kx(x+2)$

$\Sigma p_i=1$

$P(X=1)+P(X=2)+P(X=3)+P(X=4)=1$

$\Rightarrow 3k+8k+15k+24k=1$ ---(2)

$\Rightarrow 50k=1 \Rightarrow k=\frac{1}{50}$ ✓

Put $k=\frac{1}{50}$ in (2)

(b)

X	1	2	3	4
P(X=x)	3/50	8/50	15/50	24/50

(c) Given $E(X)=3.2$

$\Sigma x_i^2 p_i = 1^2\times\frac{3}{50}+2^2\times\frac{8}{50}+3^2\times\frac{15}{50}+4^2\times\frac{24}{50}$

$=\frac{554}{50}=11.08$

$Var\,X = \Sigma x_i^2 p_i-(E(X))^2$

$=11.08-(3.2)^2$

$=\underline{0.84}$ ✓

38. Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

(a) Find the probability that Kayla takes more than 6 throws to achieve a success. ---[2]

(b) Find the prob. that, for a random sample of 10 throws, Kayla achieves at least 3 success. [W-20/51/Q3] --[3]

Solution (a) $p = 0.25$, $q = 1 - 0.25 = 0.75$ [Geometric Prob distribution]

$P(X > 6) = (0.75)^6 = \underline{0.178}$ ✓ [$\because P(X > r) = q^r$]

(b) $p = 0.25$, $q = 0.75$, $n = 10$ [Binomial Prob distribution]

$P(r \geq 3) = 1 - P(0, 1, 2)$ [$P(X = r) = {}^nC_r\, p^r q^{n-r}$]

$= 1 - [0.75^{10} + {}^{10}C_1 \times 0.25 \times 0.75^9 + {}^{10}C_2 \times 0.25 \times 0.75^8)$

$= 1 - [0.0563135 + 0.1877117 + 0.2815676)$

$= \underline{0.474}$ ✓

---

39. A random variable X takes each of the values 1, 2, 3, 4, with prob. $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the prob. distribution table for Y. --[4]

(b) Find the prob of $Y = 2$ given that Y is even. [W-20/51/Q4] --[2]

Solution (a)

Y	1	2	3	4
P(Y)	$7 \times \frac{1}{4} \times \frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

X	1	2	3	4
1	1	1	2	3
2	1	2	1	2
3	2	1	3	1
4	3	2	1	4

$\}$ Y

$(1,1) \to 1$; $(1,2) \to 1$; $(1,4) \to 3$

$(2,2) \to 2$; $(1,3) \to 2$; $(4,4) \to 4$

(b) $P(Y=2/\text{even}) = \dfrac{\frac{5}{16}}{\frac{5}{16} + \frac{1}{16}} = \frac{5}{6}$ (or 0.833)

40 A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. --[2]

On another occasion, the die is thrown 10 times. [W-20 | 52 | Q1]

(b) Find the prob. that a 4 is obtained at least 3 times. --[3]

Solution (a) $p = \frac{1}{6}, \; q = 1 - \frac{1}{6} = \frac{5}{6}$,

$P(r < 6) = 1 - q^5 = 1 - (\frac{5}{6})^5 = \underline{0.598}$ ✓

(b) $p = \frac{1}{6}, \; q = \frac{5}{6}, \; n = 10$

$P(r \geq 3) = 1 - P(0, 1, 2) = 1 - \{(\frac{5}{6})^{10} + {}^{10}C_1 \cdot \frac{1}{6} \cdot (\frac{5}{6})^9 + {}^{10}C_2 \cdot (\frac{1}{6})^2 \cdot (\frac{5}{6})^8\}$

$= 1 - (0.1615056 + 0.3230111 + 0.290710)$

$= \underline{0.225}$ ✓

41. A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability, that Sadie takes exactly 1 red ball is $\frac{15}{56}$ ---[2]

(b) Draw up the prob. distribution table for X. --[3]

(c) Given that $E(X) = \frac{15}{8}$, find $Var(X)$. [W-20 | 52 | Q2] --[2]

Solution (a) $P(1 \text{ red}) = \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times 3 = \underline{\frac{15}{56}}$ ✓

Red – 5, Blue – 3 } Total 8.
[RBB + BRB + BBR]

(b)

x	0	1	2	3
$P(x)$	$\frac{1}{56}$	$15/56$	$\frac{30}{56}$	$\frac{10}{56}$
	0.0179	0.268	0.536	0.179

(c) $Var X = \sum p_i \cdot x_i^2 - (E(X))^2$

$= 0^2 \times \frac{1}{56} + 1^2 \times \frac{15}{56} + 2^2 \times \frac{30}{56} + 3^2 \times \frac{10}{56} - (\frac{15}{8})^2$

$= \frac{15 + 120 + 90}{56} - (\frac{15}{8})^2 = \frac{225}{448} = \underline{0.502}$ ✓

42 An ordinary fair dice is thrown until a 6 is obtained.
(a) Find the probability that obtaining a 6 takes more than 8 throws. --- [2]
Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.
(b) Find the expected value of X. --- [1]
(c) Find the prob. that obtaining a pair of 6s takes either 10 or 11 throws. [W-20/53/Q2] -- [2]

Solution:(a) $p = \frac{1}{6}$, $q = \frac{5}{6}$, $\left[P(6) = \frac{1}{6}\right]$

$P(r > 8) = q^8 = \left(\frac{5}{6}\right)^8 = 0.233$ [Geometric Prob. distribution]

(b) $P(X) = \frac{1}{p} = \frac{1}{\frac{1}{36}} = \underline{36}$ ✓ $\left[P(\text{a pair of } 6) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}\right]$

(c) $P(X=10) + P(X=11) = \left(\frac{35}{36}\right)^9 \times \frac{1}{36} + \left(\frac{35}{36}\right)^{10} \times \frac{1}{36} = \underline{0.0425}$ ✓

---

43. Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.
(a) Find the prob. that two tails are obtained for the first time on the 7th throw. --- [2]
(b) Find the prob. that takes more than 9 throws to obtain two tails for the first time. --- [2]
[W-21/51/Q1]

Solution: Two coins are tosses = {HH, HT, TH, TT}
(a) $p = P(TT) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ ✓; $q = 1 - \frac{1}{4} = 3/4$ ✓
In Geometric prob. distribution:
$P(X = r) = q^{r-1} \cdot p$
$P(\text{getting two tails in 7th throw}) = \left(\frac{3}{4}\right)^6 \cdot \frac{1}{4}$
$= \underline{0.0445}$ $\left(\text{or } \frac{729}{16384}\right)$

(b) $P(X > r) = q^r$
$P(X > 9) = \left(\frac{3}{4}\right)^9$
$= \underline{0.0751}$; $\left(\text{or } \frac{19683}{262144}\right)$ ✓

44. A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered −1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the prob. distribution table for X. ---[3]

(b) Find Var(X) ---[3]

[W-21/51/Q4]

Solution (a)

x	−1	0	1	2	3
p	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{4}{12}$	$\frac{3}{12}$	$\frac{2}{12}$

	0	1	2	2
−1	−1	0	1	1
0	0	1	2	2
1	1	2	3	3

(b) $E(X) = \Sigma p_i x_i = -\frac{1}{12} + 0 + \frac{4}{12} + \frac{6}{12} + \frac{6}{12} = \frac{15}{12} = \frac{5}{4}$

$\Sigma p_i x_i^2 = \frac{1}{12} + 0 + \frac{4}{12} + \frac{12}{12} + \frac{18}{12} = 35/12$

$Var(x) = \Sigma p_i x_i^2 - (E(x))^2 = \frac{35}{12} - \left(\frac{5}{4}\right)^2 = \frac{35}{12} - \frac{25}{16} = \frac{140-75}{48} = \frac{65}{48}$

45. A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement.

(a) Show that the prob. that exactly one marble is yellow is 5/14. ---[3]

The random variable X is the number of yellow marbles selected.

(b) Draw up the prob. distribution table for X. ---[3]

(c) Find E(X). [W-21/52/Q3] ---[1]

Solution: Yellow − 5 ; Green − 4 ; Total − 9

(a) For one yellow, without replacement

YGG, GYG, GGY

$P(\text{Exactly one yellow}) = \frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} \times 3 = \frac{180}{504} = \frac{5}{14}$

$\left(\text{or } \frac{{}^5C_1 \times {}^4C_2}{{}^9C_3}\right)$

(b) "Prob. distribution"

X	0	1	2	3
P(X)	$\frac{{}^5C_0 \times {}^4C_3}{{}^9C_3}$	$\frac{{}^5C_1 \times {}^4C_2}{{}^9C_3}$	$\frac{{}^5C_2 \times {}^4C_1}{{}^9C_3}$	$\frac{{}^5C_3 \times {}^4C_0}{{}^9C_3}$
	$= \frac{24}{504}$	$\frac{180}{504}$	$\frac{240}{504}$	$\frac{60}{504}$

(c) $E(X) = \Sigma p_i x_i$

$= 0 \times \frac{24}{504} + 1 \times \frac{180}{504} + 2 \times \frac{240}{504} + 3 \times \frac{60}{504}$

$= \frac{840}{504} = 5/3$

$\therefore E(X) = 5/3$

46 In a certain region, the prob. that any given day in October is wet is 0·16, independently of other days.

(a) Find the prob. that, in a 10-day period in October, fewer than 3 days will be wet. ---[3]

(b) Find the prob. that the first wet day in October is 8 October --[2]

(c) For 4 randomly chosen years, find the prob. that exactly 1 of these years the first wet day in October is 8 October --[2]

W-21/52/Q5

Solution: Prob. of any one day in October is wet $p = 0.16$, P(not wet) $q = 1 - 0.16 = 0.84$

(a) $P(r < 3) = P(0) + P(1) + P(2)$ out of 10 days $\left\{ n(X = r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r} \right.$ (Binomial prob. distribution)

$= {}^{10}C_{0} \cdot 0.16^{0} \times 0.84^{10} + {}^{10}C_{1} \cdot 0.16^{1} \times 0.84^{9} + {}^{10}C_{2} \times 0.16^{2} \times 0.84^{8}$

$= 0.17490 + 0.333145 + 0.28555 = \underline{0.794}$

(b) Using Geometric Prob. dis. $P(X = r) = q^{r-1} \cdot p$

$\therefore$ P(first time 8_{th} oct is wet) $= (0.84)^{7} \cdot 0.16 = \underline{0.0472}$

(c) P(Exactly 1 out of 4 year, first wet day on 8_{th} Oct);

from Part (b) $\left\{ \begin{array}{l} p_1 = 0.0472 \\ q_1 = 1 - 0.472 \\ \quad = 0.9528 \end{array} \right.$

$= {}^{4}C_{1} \times p_1^{1} \times q_1^{3}$

$= 4 \times (0.0472) \cdot (0.9528)^{3}$

Using Binomial dis.

$= \underline{0.163}$

47. In a game, Tim throws three darts at a board. This is called a 'turn'. The centre of the board is called bull's-eye.
The random variable X is the number of darts in a turn that hit the bull's-eye. The prob. distribution of X is given as:

x	0	1	2	3
$P(X=x)$	0.6	p	q	0.05

It is given that $E(X) = 0.55$

(a) Find the values of p and q. ---[4]

(b) Find $Var(X)$ ---[2]

Tim is practising for a competition and he repeatedly throws three darts at the board.

(c) Find the prob. that $X=1$ in at least 3 of 12 randomly chosen turns. ---[3]

(d) Find the prob. that Tim first succeeds in hitting the bulls-eye with all three darts on his 9_{th} turn. ---[1]

[W-21/53/Q6]

Solution (a) $\Sigma p_i = 0.6 + p + q + 0.05 = 1$
$\Rightarrow p + q = 0.35$ ---- (1)

$E(X) = \Sigma p_i x_i$
$\Rightarrow 0 \times 0.6 + 1 \times p + 2 \times q + 3 \times 0.05 = 0.55$ (Given)
$\Rightarrow p + 2q = 0.4$ --- (2)

solving (1) & (2) $p = 0.3$; $q = 0.05$

(b) $\Sigma p_i x_i^2 = 0.6 \times 0^2 + 0.3 \times 1^2 + 0.05 \times 2^2 + 0.05 \times 3^2$
$= 0.95$

$Var(X) = \Sigma p_i \cdot x_i^2 - (E(X))^2$
$= 0.95 - (0.55)^2 = 0.6475$

(c) $p = P(X=1) = 0.3$, $q = 1 - 0.3 = 0.7$; $n = 12$
$P(\text{atleast 3 out of 12}) = P(r \geq 3)$
$= 1 - P(r = 0, 1, 2)$ $[P(X=r) = {}^{n}C_r\, p^r q^{n-r}]$
$= 1 - [{}^{12}C_0 \times .3^0 \times .7^{12} + {}^{12}C_1 \times .3^1 \times .7^{11}$
$+ {}^{12}C_2 \times .3^2 \times .7^{10}]$
$= 1 - [0.01384 + 0.07118 + 0.16779] = 0.747$

(d) Using Geo. Prob. dis.
$P(r_{th} \text{ success}) = q_1^{r-1} \cdot p_1$ --- (3)
$P(X=3) = 0.05$, $q_1 = 0.95$
from (3)
$P(\text{success in } 9_{th} \text{ turn}) = q^8 \cdot p$
$= 0.95^8 \times 0.05$
$= 0.0332$

48. The prob. distribution table for a random variable X is shown below:

x	-2	-1	0.5	1	2
$P(X=x)$	0.12	p	q	0.16	0.3

--- [4]

Given that $E(X) = 0.28$, find the value of p and the value of q.

[W-22/51/Q1]

Solution: $0.12 + p + q + 0.16 + 0.3 = 1 \quad (\because \Sigma p_i = 1)$

$\Rightarrow \quad p + q = 0.42$ --- (1)

$E(X) = \Sigma x_i \cdot p_i = 0.12\times(-2) + p(-1) + q(0.5) + 0.16\times 1 + 0.3\times 2 = 0.28$ (given)

$\Rightarrow \quad -p + 0.5q = -0.24$ --- (2)

add (1) & (2) $\quad 1.5q = 0.18 \Rightarrow q = \frac{0.18}{1.5} = 0.12$ ✓

from (1) $p + 0.12 = 0.42 \Rightarrow p = 0.3$ ✓

$\therefore \quad \underline{p = 0.3}$ and $\underline{q = 0.12}$.

49. Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces.

(a) Find the probability that a score of 17 or more is first obtained on the 6_{th} throw. ---[3]

(b) Find the prob. that a score of 17 or more is obtained in fewer than 8 throws. [W-22 / 52 / Q3] -[2]

<u>Solution</u> (a) $P(17 \text{ or } 18) = \frac{4}{216} = \frac{1}{54} = p, \; q = \frac{53}{54}$

$\left\{ \begin{array}{l} (6,6,5), (6,5,6), (5,6,6), (6,6,6) \\ 4 \text{ out comes out of} \\ \text{Total } 6 \times 6 \times 6 = 216 \checkmark \end{array} \right.$

$P(17 \text{ or } 18 \text{ is in } 6_{th} \text{ throw}) = q^5 \cdot p$

$= \left(\frac{53}{54}\right)^5 \cdot \frac{1}{54} = 0.0169 \checkmark$

b. $P[17 \text{ or } 18 \text{ in less than } 8 \text{ throws}] = 1 - q^7 = 1 - \left(\frac{53}{54}\right)^3 = 0.123 \checkmark$

50 Three fair 4-sided spinners each have sides 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which each spinner lands is recorded. The random variable X denotes the highest number recorded.

(a) Show that $P(X=2)=\frac{7}{64}$ --- [3]

(b) Complete the prob. distribution table for X.

x	1	2	3	4
$P(X=x)$		$7/64$	$19/64$	

--- [2]

On another occasion, one of the fair 4-sided spinners is spun repeated until 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) $P(Y=6)$ --- [1]

(d) Find $P(Y>4)$ --- [2]

W-22/53/Q4

Solution (a) $P(X=2)=$ P(2 on all spinners) + P(2 on two spinners and 1 on one spinner) + P(2 on one and 1 on two spinners)

$$= \left(\frac{1}{4}\right)^3 + {}^3C_2\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + {}^3C_1\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) = \frac{(1+3+3)}{64} = \frac{7}{64}$$

(b) $P(X=1)=\left(\frac{1}{4}\right)^3=\frac{1}{64}$; and $P(X=4)=\left[1-\left(\frac{1}{64}+\frac{7}{64}+\frac{19}{64}\right)\right]=\frac{37}{64}$

(c) $P(Y=6)=\left(\frac{3}{4}\right)^5\times\frac{1}{4}=0.0593$

$\left\{ P(3)=\frac{1}{4}=p;\ q=\frac{3}{4} \right.$

Geo(p). $P(Y=r)=q^{r-1}\cdot p$

(d) $P(Y>4)=q^4=\left(\frac{3}{4}\right)^4=\frac{81}{256}$

$=0.316$

$\left\{ P(Y>r)=q^r \right.$

Geo. distribution