PROBABILITY AND STATISTICS -1
9709
(March, June and November series 2020 – 2023 With marking scheme)

Discrete Random variables

EXERCISE -1

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1) SP-2020_9709_5Q3

A book club sends 6 paperback and 2 hardback books to Mrs Hunt. She chooses 4 of these books at random to take with her on holiday. The random variable X represents the number of paperback books she chooses.

(a) Show that the probability that she chooses exactly 2 paperback books is $\frac{3}{14}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) You are given that $E(X) = 3$.

Find $\text{Var}(X)$. [2]

2) SP-2020 _9709_5 Q5(c)

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown 90 times.

On another occasion, the same die is thrown repeatedly until a 3 is obtained.

(c) Find the probability that obtaining a 3 requires fewer than 7 throws. [2]

3) MARCH 2020_9709_52 Q2

An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained.

(a) Find the probability that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6. [3]

On another occasion, this die is thrown 3 times. The random variable X is the number of times that a 1 or a 6 is obtained.

(b) Draw up the probability distribution table for X. [3]

(c) Find $E(X)$. [2]

4) MARCH 2020_9709_52 Q5(a)

In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

(a) Find the probability that the number of adults in this sample who own a car is less than 6. [3]

5) MARCH 2021_9709_52 Q1

A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands.

(a) Find the probability that a score of 3 is obtained for the first time on the 8th spin. [1]

(b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time. [2]

6) MARCH 2021_9709_52 Q4

The random variable X takes the values 1, 2, 3, 4 only. The probability that X takes the value x is $kx(5-x)$, where k is a constant.

(a) Draw up the probability distribution table for X, in terms of k. [2]

(b) Show that $\mathrm{Var}(X) = 1.05$. [4]

7) MARCH 2022_9709_52 Q1

A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

(b) Given that $E(X) = 0.25$, find the value of $\mathrm{Var}(X)$. [2]

8) MARCH 2022_9709_52 Q2

In a certain country, the probability of more than 10 cm of rain on any particular day is 0.18, independently of the weather on any other day.

(a) Find the probability that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. [3]

(b) For 3 randomly chosen 7-day periods, find the probability that exactly two of these periods have at least one day with more than 10 cm of rain. [3]

9) MARCH 2022_9709_52 Q6

A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

(d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

(e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

10) MARCH 2023_9709_52 Q2

Alisha has four coins. One of these coins is biased so that the probability of obtaining a head is 0.6. The other three coins are fair. Alisha throws the four coins at the same time. The random variable X denotes the number of heads obtained.

(a) Show that the probability of obtaining exactly one head is 0.225. [3]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	0.05	0.225			0.075

(c) Given that $E(X) = 2.1$, find the value of $Var(X)$. [2]

11) MARCH 2023_9709_52 Q3

80% of the residents of Kinwawa are in favour of a leisure centre being built in the town.

20 residents of Kinwawa are chosen at random and asked, in turn, whether they are in favour of the leisure centre.

(a) Find the probability that more than 17 of these residents are in favour of the leisure centre. [3]

(b) Find the probability that the 5th person asked is the first person who is **not** in favour of the leisure centre. [1]

(c) Find the probability that the 7th person asked is the second person who is **not** in favour of the leisure centre. [2]

12) JUNE 2020_9709_51 Q1

The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the mean of X. [1]

(c) Find the probability that a score of 4 is first obtained on the 6th throw. [1]

(d) Find $P(X < 8)$. [2]

13) JUNE 2020_9709_51 Q3

A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

(a) Draw up the probability distribution table for the number of jellies that Jemeel chooses. [4]

The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

14) JUNE 2020_9709_52 Q5

A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Find $E(X)$ and $Var(X)$. [3]

15) JUNE 2020_9709_52 Q7(a)(b)

On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

16) JUNE 2020_9709_53 Q2

In a certain large college, 22% of students own a car.

(a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car. [1]

(b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4. [3]

17) JUNE 2020_9709_53 Q4

A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered −2, −1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

(b) Find $Var(X)$. [3]

18) JUNE 2020_9709_53 Q5(a)(b)(c)

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

19) JUNE 2021_9709_51 Q6(a)

In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

20) JUNE 2021_9709_51 Q7

Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable X is the number of tins that she needs to open.

(a) Show that $P(X = 3) = \frac{6}{35}$. [2]

(b) Draw up the probability distribution table for X. [4]

(c) Find $\text{Var}(X)$. [3]

21) JUNE 2021_9709_52 Q1

An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X.

(a) Write down the mean of X. [1]

(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. [2]

(c) Find the probability that a 5 is first obtained in fewer than 10 throws. [2]

22) JUNE 2021_9709_52 Q4

A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered -2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $\text{Var}(X)$. [3]

23) JUNE 2021_9709_52 Q5

Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

24) JUNE 2021_9709_53 Q2

The random variable X can take only the values $-2, -1, 0, 1, 2$. The probability distribution of X is given in the following table.

x	-2	-1	0	1	2
$P(X = x)$	p	p	0.1	q	q

Given that $P(X \geqslant 0) = 3P(X < 0)$, find the values of p and q. [4]

25) JUNE 2021_9709_53 Q4(b)

Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

(b) Find the probability that a score of 18 is obtained for the first time on the 5th throw of the three dice. [3]

26) JUNE 2021_9709_53 Q7(b)(i)

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

27) JUNE 2022_9709_51 Q4

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

(b) Find $E(X)$. [1]

Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

28) JUNE 2022_9709_52 Q2

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

29) JUNE 2022_9709_53 Q3

The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

30) JUNE 2022_9709_53 Q4

Ramesh throws an ordinary fair 6-sided die.

(a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]

(b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

Ramesh now repeatedly throws two ordinary fair 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the probability that Ramesh obtains a total of less than 4 on at least three throws. [4]

31) JUNE 2023_9709_51 Q6

Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable X denotes the number of 2s obtained.

(a) Show that $P(X = 3) = \frac{3}{64}$. [2]

(b) Complete the following probability distribution table for X. [2]

x	0	1	2	3	4
$P(X = x)$	$\frac{81}{256}$			$\frac{3}{64}$	$\frac{1}{256}$

(c) Find $E(X)$. [2]

Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

32) JUNE 2023_9709_51 Q7

A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

(a) Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

(b) Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

(c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

33) JUNE 2023_9709_52 Q1

The random variable X takes the values -2, 2 and 3. It is given that

$$P(X = x) = k(x^2 - 1),$$

where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

(b) Find $E(X)$ and $Var(X)$. [3]

34) JUNE 2023_9709_52 Q2

A sports event is taking place for 4 days, beginning on Sunday. The probability that it will rain on Sunday is 0.4. On any subsequent day, the probability that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the probability that it does **not** rain on any of the 4 days of the event. [1]

(b) Find the probability that the first day on which it rains during the event is Tuesday. [2]

(c) Find the probability that it rains on exactly one of the 4 days of the event. [3]

35) JUNE 2023_9709_52 Q4

A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable X denotes the number of spins required.

(a) Find $P(X = 4)$. [1]

(b) Find $P(X < 6)$. [2]

Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5, are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the probability that the score is greater than 0 given that the score is **not** equal to 2. [3]

The two spinners are spun at the same time repeatedly .

(d) For 9 randomly chosen spins of the two spinners, find the probability that the score is greater than 2 on at least 3 occasions. [3]

36) JUNE 2023_9709_53 Q1

Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X.

(a) State the value of $E(X)$. [1]

(b) Find the probability that exactly 5 throws are required to obtain a pair of heads. [1]

(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads. [2]

37) JUNE 2023_9709_53 Q3

The random variable X takes the values 1, 2, 3, 4. It is given that $P(X = x) = kx(x + a)$, where k and a are constants.

(a) Given that $P(X = 4) = 3P(X = 2)$, find the value of a and the value of k. [4]

(b) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [1]

(c) Given that $E(X) = 3.2$, find $\text{Var}(X)$. [2]

38) OCT 2020_9709_51 Q3

Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

(a) Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

(b) Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

39) OCT 2020_9709_51 Q4

The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

(b) Find the probability that $Y = 2$ given that Y is even. [2]

40) OCT 2020_9709_52 Q1

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

41) OCT 2020_9709_52 Q2

A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

(b) Draw up the probability distribution table for X. [3]

(c) Given that $E(X) = \frac{15}{8}$, find $Var(X)$. [2]

42) OCT 2020_9709_53 Q2

An ordinary fair die is thrown until a 6 is obtained.

(a) Find the probability that obtaining a 6 takes more than 8 throws. [2]

Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the expected value of X. [1]

(c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

43) OCT 2021_9709_51 Q1

Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

(a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]

(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

44) OCT 2021_9709_51 Q4

A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered −1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

(b) Find $Var(X)$. [3]

45) OCT 2021_9709_52 Q3

A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement.

(a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$. [3]

The random variable X is the number of yellow marbles selected.

(b) Draw up the probability distribution table for X. [3]

(c) Find $E(X)$. [1]

46) OCT 2021_9709_52 Q5

In a certain region, the probability that any given day in October is wet is 0.16, independently of other days.

(a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet. [3]

(b) Find the probability that the first wet day in October is 8 October. [2]

(c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October. [2]

47) OCT 2021_9709_53 Q6

In a game, Jim throws three darts at a board. This is called a 'turn'. The centre of the board is called the bull's-eye.

The random variable X is the number of darts in a turn that hit the bull's-eye. The probability distribution of X is given in the following table.

x	0	1	2	3
$P(X = x)$	0.6	p	q	0.05

It is given that $E(X) = 0.55$.

(a) Find the values of p and q. [4]

(b) Find $\text{Var}(X)$. [2]

Jim is practising for a competition and he repeatedly throws three darts at the board.

(c) Find the probability that $X = 1$ in at least 3 of 12 randomly chosen turns. [3]

(d) Find the probability that Jim first succeeds in hitting the bull's-eye with all three darts on his 9th turn. [1]

48) OCT 2022_9709_51 Q1

The probability distribution table for a random variable X is shown below.

x	-2	-1	0.5	1	2
$P(X = x)$	0.12	p	q	0.16	0.3

Given that $E(X) = 0.28$, find the value of p and the value of q. [4]

49) OCT 2022_9709_52 Q3

Three fair 6-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time repeatedly. The score on each throw is the sum of the numbers on the uppermost faces.

(a) Find the probability that a score of 17 or more is first obtained on the 6th throw. [3]

(b) Find the probability that a score of 17 or more is obtained in fewer than 8 throws. [2]

50) OCT 2022_9709_53 Q4

Three fair 4-sided spinners each have sides labelled 1, 2, 3, 4. The spinners are spun at the same time and the number on the side on which each spinner lands is recorded. The random variable X denotes the highest number recorded.

(a) Show that $P(X = 2) = \frac{7}{64}$. [3]

(b) Complete the probability distribution table for X. [2]

x	1	2	3	4
$P(X = x)$		$\frac{7}{64}$	$\frac{19}{64}$	

On another occasion, one of the fair 4-sided spinners is spun repeatedly until a 3 is obtained. The random variable Y is the number of spins required to obtain a 3.

(c) Find $P(Y = 6)$. [1]

(d) Find $P(Y > 4)$. [2]

MARKING SCHEME

1) SP-2020_9709_5Q3

(a)	**EITHER** Solution 1 P (exactly 2) = $\frac{^6C_2}{^8C_4}$	1	(M1	$\frac{^6C_x}{^8C_y}$ seen
	$= \frac{15}{70} = \frac{3}{14}$	1	A1)	AG CWO
	OR Solution 2 $P(2) = \frac{6}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{1}{5} \times {}^4C_2$	1	(M1	$^4C_2$ multiplied by 4 fractions
	$= \frac{3}{14}$	1	A1)	AG CWO
	Available marks	2		
(b)	x: 2, 3, 4; Prob: $\frac{3}{14}$, $\frac{8}{14}$, $\frac{3}{14}$	1	B1	2, 3, 4 only in top line
		1	B1	one correct probability other than P(2)
		1	B1FT	third correct probability FT $\Sigma = 1$
		3		
(c)	$\text{Var}(X) = \frac{12}{14} + \frac{72}{14} + \frac{48}{14} - 3^2$	1	M1	using $\Sigma x^2 p - 3^2$
	$= \frac{3}{7} = 0.429$	1	A1	
		2		

Table in (b):

x	2	3	4
Prob	$\frac{3}{14}$	$\frac{8}{14}$	$\frac{3}{14}$

2) SP-2020 _9709_5 Q5(c)

c)	$1 - \left(\frac{5}{6}\right)^6$	1	M1	
	$= 0.665$	1	A1	
		2		

3) MARCH 2020_9709_52 Q2

(a)	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3 + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$	M1	One correct term with $0 < p < 1$
	$= \frac{4}{27} + \frac{8}{81} + \frac{16}{243}\left(= \frac{2432}{7776}\right)$	A1	Correct expression, accept unsimplified
	$= \frac{76}{243}$ or 0.313	A1	
		3	

(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$\frac{8}{27}$</td><td>$\frac{12}{27}$</td><td>$\frac{6}{27}$</td><td>$\frac{1}{27}$</td></tr></table>	B1	Probability distribution table with correct values of x, no additional values unless with probability of 0 stated, at least one non-zero probability included
	$P(0) = \left(\frac{2}{3}\right)^3$ $P(1) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \times 3$ $P(2) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2 \times 3$ $P(3) = \left(\frac{1}{3}\right)^3$	B1	1 correct probability seen (may not be in table) or 3 or 4 non-zero probabilities summing to 1
		B1	All probabilities correct
		3	
(c)	$E(X) = \left[0 \times \frac{8}{27}\right] + 1 \times \frac{12}{27} + 2 \times \frac{6}{27} + 3 \times \frac{1}{27}$ $= \left[\frac{0}{27}\right] + \frac{12}{27} + \frac{12}{27} + \frac{3}{27}$	M1	Correct method from *their* probability distribution table with at least 3 terms, $0 \leqslant$ *their* $P(x) \leqslant 1$, accept unsimplified
	$= 1$	A1	
		2	

4) MARCH 2020_9709_52 Q5(a)

(a)	$1 - P(6, 7, 8)$ $= 1 - ({}^8C_6\, 0.7^6 0.3^2 + {}^8C_7\, 0.7^7 0.3^1 + 0.7^8)$	M1	One term ${}^8C_x\, p^x(1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
	$= 1 - 0.55177$	A1	Correct unsimplified expression, or better
	$= 0.448$	A1	
	Alternative method for question 5(a)		
	$P(0, 1, 2, 3, 4, 5)$ $= 0.3^8 + {}^8C_1\, 0.7^1 0.3^7 + {}^8C_2\, 0.7^2 0.3^6 + {}^8C_3\, 0.7^3 0.3^5 + {}^8C_4 0.7^4 0.3^4 + {}^8C_5\, 0.7^5 0.3^3$	M1	One term ${}^8C_x\, p^x(1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
		A1	Correct unsimplified expression, or better
	$= 0.448$	A1	
		3	

5) MARCH 2021_9709_52 Q1

(a)	$\left[\left(\frac{4}{5}\right)^7 \frac{1}{5} =\right]$ $\frac{16384}{390625}$ or 0·0419[43…]	B1	Evaluated, final answer.
		1	
(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	M1	$1-p^n$ $n = 5,6$ or $p + pq + pq^2 + pq^3 + pq^4$ $(+ pq^5)$ $0 < p < 1$, $p + q = 1$, Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or 0·672[32]	A1	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5 + {}^5C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right) + {}^5C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 + {}^5C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 + {}^5C_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	M1	$(p)^5 + {}^5C_4(p)^4(q) + {}^5C_3(p)^3(q)^2 + {}^5C_2(p)^2(q)^3 + {}^5C_1(p)(q)^4$ or $(p)^6 + {}^6C_5(p)^5(q) + {}^6C_4(p)^4(q)^2 + {}^6C_3(p)^3(q)^3$ $+ {}^6C_2(p)^2(q)^4 + {}^6C_1(p)(q)^5$, $0 < p < 1, p + q = 1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or 0·672[32]	A1	Final answer.
		2	

6) MARCH 2021_9709_52 Q4

(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>prob</td><td>4k</td><td>6k</td><td>6k</td><td>4k</td></tr></table>	B1	Table with x values and one correct probability expressed in terms of k. Condone any additional x values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.
		2	
(b)	$[4k + 6k + 6k + 4k = 1]$ $k = \frac{1}{20}$ $(= 0.05)$	B1	Correct value for k SOI. May be calculated in **4(a)**. **SC B1** If denominator $20k$ used throughout.
	$E(X) = 1\times\frac{4}{20} + 2\times\frac{6}{20} + 3\times\frac{6}{20} + 4\times\frac{4}{20} = \frac{4}{20} + \frac{12}{20} + \frac{18}{20} + \frac{16}{20}$ $(= 2.5)$	M1	Accept unsimplified expression. Condone $4k + 12k + 18k + 16k$. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator $20k$.
	$\text{Var}(X) = 1^2\times\frac{4}{20} + 2^2\times\frac{6}{20} + 3^2\times\frac{6}{20} + 4^2\times\frac{4}{20} - \left(\textit{their}\, 2\frac{1}{2}\right)^2$ $= (4 + 24 + 54 + 64)\times \textit{their}\, 0.05 - (\textit{their}\, 2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20} + (2-2.5)^2\times\frac{6}{20} + (3-2.5)^2\times\frac{6}{20} + (4-2.5)^2\times\frac{4}{20}$	M1	Appropriate variance formula with *their* numerical probabilities using *their* $(E(X))^2$, accept unsimplified, with *their* k substituted. **Special ruling:** If denominator $20k$ used throughout, accept appropriate variance formula in terms of k.
	1·05	A1	AG, NFWW.
		4	

7) MARCH 2022_9709_52 Q1

Question	Answer	Marks	Guidance
(a)	X: −2, −1, 0, 1, 2 P(X): $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{5}{16}$, $\frac{2}{16}$ 0.0625, 0.1875, 0.3125, 0.3125, 0.125	B1	Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values.
		B1	3 correct probabilities linked with correct outcomes, may not be in table.
		B1	2 further correct probabilities linked with correct outcomes, may not be in table No repeated X values. SC if less than 3 correct probabilities seen, award **SCB1** Sum of *their* probabilities, $0 < p < 1$, of 4,5 or 6 X values = 1 (condone summing to 1 ± 0.01 or better).
		3	
(b)	$\left[\frac{1}{16}\times -2^2 + \frac{3}{16}\times -1^2\left(+\frac{5}{16}\times 0^2\right) + \frac{5}{16}\times 1^2 + \frac{2}{16}\times 2^2 - \left(\frac{1}{4}\right)^2\right]$ $\frac{1\times4+3\times1+5\times0+5\times1+2\times4}{16} - 0.25^2$	M1	Appropriate variance formula using $(E(X))^2$ value, accept unsimplified. FT *their* table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$. Condone 1 error providing all probabilities <1 and 0.25^2 used
	$\left[= \frac{5}{4} - \frac{1}{16} = \right]\frac{19}{16}$, 1.1875	A1	Condone 1.188 or 1.19 WWW
		2	

8) MARCH 2022_9709_52 Q2

Question	Answer	Marks	Guidance
(a)	[P(>2) = 1 – P(0,1,2) =] $1 - ({}^7C_0\ 0.18^0\ 0.82^7 + {}^7C_1\ 0.18^1\ 0.82^6 + {}^7C_2\ 0.18^2\ 0.82^5)$	M1	One term ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$
	= 1 – (0.249285 + 0.383048 + 0.252251) = 1 – 0.88458	A1	Correct unsimplified expression or better Condone omission of brackets if recovered
	0.115	B1	WWW. $0.115 \leqslant p < 0.1155$ not from wrong working
		3	
(b)	[P(at least 1 day of rain) = 1 – P(0) = $1-(0.82)^7$ =] 0.7507	B1	AWRT 0.751 seen
	[P(exactly 2 periods) =] $0.7507^2 \times (1-0.7507)\times 3$	M1	FT *their* $1-p^7$ or *their* 0.7507 if identified, not 0.18, 0.82 Accept $\times {}^3C_r$, r=1,2 or $\times {}^3P_1$ for ×3 Condone ×2
	0.421	A1	Accept $0.421 \leqslant p \leqslant 0.4215$ **SC B1** if 0/3 scored for final answer only $0.421 \leqslant p \leqslant 0.4215$
		3	

9) MARCH 2022_9709_52 Q6

(a)	[Probability of lemon = $\frac{3}{15}=\frac{1}{5}$] $\left[\left(\frac{4}{5}\right)^6\times\frac{1}{5}=\right]\frac{4096}{78125}$, 0.0524	B1	0.0524288 rounded to more than 3SF if final answer
		1	
(b)	$\left(1-\frac{1}{5}\right)^6$	M1	or $\left(\frac{4}{5}\right)^6$. FT *their* $\frac{1}{5}$ or correct. From final answer Condone $\left(\frac{4}{5}\right)^5$ or $\left(\frac{1}{5}\right)\times\left(\frac{4}{5}\right)^5+\left(\frac{4}{5}\right)^6$
	$\frac{4096}{15625}$, 0.262	A1	0.262144 rounded to more than 3SF
	Alternative method for question 6(b)		
	[1 – P(1,2,3,4,5,[6]) =] $1-\left(\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}+\left(\frac{4}{5}\right)^5\times\frac{1}{5}\right)$	M1	From final answer Condone omission of $\left(\frac{4}{5}\right)^5\times\frac{1}{5}$
	$\frac{4096}{15625}$, 0.262	A1	0.262144 rounded to more than 3SF
		2	

6(c)	$\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}$	M1	$\frac{a}{15}\times\frac{a-1}{14}\times\frac{a-2}{13}$, no additional terms
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{3}{15}\times\frac{2}{14}\times\frac{1}{13}+3\times\frac{3}{15}\times\frac{2}{14}\times\frac{7}{13}+3\times\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}+\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$	M1	[3Ls + 2Ls1S + 1L2Ss + 3Ss] Condone one numerator error. Condone no multiplications seen if tree diagram complete with probabilities on each branch, scenarios listed and attempt at evaluation
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$1-\left(\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}+3\times\frac{5}{15}\times\frac{4}{14}\times\frac{10}{13}+3\times\frac{5}{15}\times\frac{10}{14}\times\frac{9}{13}\right)$	M1	1 – P(3,2,1 oranges) Condone one numerator error.
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{^{10}C_3}{^{15}C_3}$	M1	
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
		2	

d)	$\frac{7}{15}\times\frac{5}{14}\times\frac{3}{13}\times3!$	M1	All probabilities of the form: $\frac{7}{a}\times\frac{5}{b}\times\frac{3}{c}$, $13 \leqslant a,b,c \leqslant 15$
		M1	$\frac{e}{f}\times\frac{g}{h}\times\frac{i}{j}\times3!$ e,f,g,h,i,j positive integers forming probabilities or 6 identical probability calculations or values added, no additional terms
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
	Alternative method for question 6(d)		
	$\frac{^3C_1\times{}^5C_1\times{}^7C_1}{^{15}C_3}$	M1	$\frac{^3C_1\times{}^5C_1\times{}^7C_1}{k}$, k integer > 1 Condone use of permutations
		M1	$\frac{^3C_a\times{}^5C_b\times{}^7C_c}{^{15}C_3}$, $0<a<3$, $0<b<5$, $0<c<7$, Condone use of permutations
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
		3	
(e)	$\frac{\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}+\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times3}{their(c)}$ $\left[=\frac{14}{65}\div\frac{24}{91}\right]$	B1	$\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times3$ seen (SSL, SLS, LSS) SC B1 $\frac{3}{65}\times3, \frac{126}{2730}\times3$ seen
		B1	$\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$ seen in numerator (SSS) SCB1 $\frac{210}{2730}, \frac{1}{13}$ seen in numerator
		M1	Fraction with *their* (c) or correct in denominator $\left(\frac{720}{2730}, \frac{24}{91}, 0.263736\right)$
	$=\frac{49}{60}$, 0·817	A1	Accept $0.81\dot{6}$
	Alternative method for question 6(e)		
	$\frac{^7C_2\times{}^3C_1+{}^7C_3}{^{10}C_3}$	B1	$^7C_2\times{}^3C_1$ seen (SSL, SLS, LSS) SCB1 21 × 3 seen or use of permutations
		B1	$^7C_3$ seen in numerator (SSS) SCB1 35 seen in numerator or use of permutations
		M1	Fraction with $^{10}C_3$ or consistent with *their* numerator of 6(e) in denominator
	$=\frac{49}{60}$, 0·817	A1	Accept $0.81\dot{6}$
		4	

10) MARCH 2023_9709_52 Q2

(a)	$0.6(0.5)^3+0.4(0.5)^3\times3$	B1	Either $0.6(0.5)^3+a$ or $b+0.4(0.5)^3\times(3 or {}^3C_1)$, $0<a, b<1$ seen.
		M1	$0.6(0.5)^3+0.4(0.5)^3\times d$ seen, $d=1, 3$. Condone $0.075+0.05\times d$, $d=1, 3$.
	= 0.225	A1	AG full supporting working required. Scenarios identified and linked to calculations.
		3	

(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X=x)$</td><td>0.05</td><td>0.225</td><td>0.375</td><td>0.275</td><td>0.075</td></tr></table>	B1	Either [P(2) =] 0.375, $\frac{3}{8}$ or [P(3) =] 0.275, $\frac{11}{40}$ seen. Condone not in table if identified.
		B1 FT	Both values in table. FT P(2) + P(3) = 0.650 .
		2	
(c)	Var(X) $= [1^2\times]0.225 + 2^2 \times \textit{their}\, 0.375 + 3^2 \times \textit{their}\, 0.275 + 4^2 \times 0.075 - 2.1^2$	M1	Appropriate variance formula from their probability distribution table with at least 4 terms, $0 < \textit{their}\ P(x) < 1$. Condone 4.41 for 2.1^2. Condone mean clearly recalculated inaccurately. Or $0.225 + 4 \times \textit{their}\, 0.375 + 9 \times \textit{their}\, 0.275 + 16 \times 0.075 - 2.1^2$ Condone 2.1^2 for 4.41.
	$\left[5.4 - 2.1^2\right] = 0.99[0]$	A1	If M0 awarded **SC B1** for 0.99[0] WWW.
		2	

11) MARCH 2023_9709_52 Q3

(a)	**Method 1 for Question 3(a)**		
	[P(X > 17) = P(18, 19, 20) =] $^{20}C_{18}\,(0.8)^{18}(0.2)^2 + {}^{20}C_{19}\,(0.8)^{19}(0.2)^1$ $+\ {}^{20}C_{20}\,(0.8)^{20}$	M1	One term $^{20}C_x\,(p)^x(1-p)^{20-x}$, $0 < p < 1, 0 < x < 20$.
	$= 0.13691 + 0.05765 + 0.01153$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.206	B1	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$.
	Method 2 for Question 3(a)		
	[P(X > 17) = 1 – P(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17) =] $1 - ({}^{20}C_0\,(0.8)^0(0.2)^{20} + {}^{20}C_1\,(0.8)^1(0.2)^{19}$	M1	One term $^{20}C_x\,(p)^x(1-p)^{20-x}$, $0 < p < 1, 0 < x < 20$.
	$+\ {}^{20}C_2\,(0.8)^2(0.2)^{18} + \ldots + {}^{20}C_{16}\,(0.8)^{16}(0.2)^4$ $+\ {}^{20}C_{17}\,(0.8)^{17}(0.2)^3)$ $= 1 - (1.048\times10^{-14} + 8.389\times10^{-13}$ $+ 3.188\times10^{11} + \ldots + 0.2182 + 0.2054)$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer. If answer correct, condone omission of any 15 of the 16 middle terms.
	0.206	B1	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$. Condone omission of brackets.
		3	
(b)	$\left[(0.8)^4(0.2) = \right]0.08192, \frac{256}{3125}$	B1	Accept $\frac{8192}{100000}$ OE.
		1	
(c)	$(0.8)^5(0.2)^2 \times 6$	M1	$(0.8)^5(0.2)^2 \times k$ *or* $(0.8)^5(0.2) \times k \times 0.2$, $2 \leqslant k \leqslant 7$.
	$= 0.0786, \frac{8144}{78125}$	A1	$0.0786 \leqslant p < 0.07865$, $\frac{786432}{10000000}$. If A0 awarded, **SC B1** for correct answer WWW.
		2	

12) JUNE 2020_9709_51 Q1

(a)	Prob of 4 (from 1,3, 3,1 or 2,2) = $\frac{3}{36}=\frac{1}{12}$ **AG**	**B1**
		1
(b)	Mean = $\frac{1}{\frac{1}{12}}$ = 12	**B1**
		1
(c)	$\left(\frac{11}{12}\right)^5 \times \frac{1}{12} = 0.0539$ or $\frac{161051}{2985984}$	**B1**
		1
(d)	$1-\left(\frac{11}{12}\right)^7$	**M1**
	0.456 or $\frac{16344637}{35831808}$	**A1**
		2

13) JUNE 2020_9709_51 Q3

(a)	see table below **(B1 for probability distribution table with correct outcome values)**	**B1**
	$P(0)=\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}=\frac{1}{56}$ $P(1)=\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3=\frac{15}{56}$ $P(2)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times3=\frac{30}{56}$ $P(3)=\frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\frac{10}{56}$ **(M1 for denominator 8×7×6)**	**M1**
	Any one probability correct (with correct outcome)	**A1**
	All probabilities correct	**A1**
		4
(b)	$1-P(8, 9, 10) = 1-\left[{}^{10}C_8\, 0.64^8 0.36^2 + {}^{10}C_9\, 0.64^9 0.36^1 + 0.64^{10}\right]$	**M1**
	$1-(0.164156+0.064852+0.11529)$	**M1**
	0.759	**A1**
		3

Table for (a):

x	0	1	2	3
Probability	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}$	$\frac{10}{56}$

14) JUNE 2020_9709_52 Q5

Question	Answer	Marks
(a)	(table below)	M1
	$\frac{7}{15}$ AG	A1
		2

	1	1	2	2	3
1	1	1	2	2	3
2	2	2	2	2	3
3	3	3	3	3	3

Question	Answer	Marks
(b)	(table below)	B1
	P(1) or P(2) correct	B1
	3[rd] probability correct, FT sum to 1	B1
		3

x	1	2	3
Probability	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{7}{15}$

Question	Answer	Marks
(c)	$E(X)=\frac{2+12+21}{15}=\frac{35}{15}=\frac{7}{3}$	B1
	$Var(X)=\frac{1^2\times2+2^2\times6+3^2\times7}{15}-\left(\frac{7}{3}\right)^2$	M1
	$\frac{22}{45}(0.489)$	A1
		3

15) JUNE 2020_9709_52 Q7(a)(b)

Question	Answer	Marks
(a)	$1-P(10, 11, 12)$ $=1-[{}^{12}C_{10}0.72^{10}0.28^2+{}^{12}C_{11}\,0.72^{11}0.28^1+0.72^{12}]$	M1
	$1-(0.19372+0.09057+0.01941)$	A1
	0.696	A1
		3
(b)	$0.28^3\times0.72=0.0158$	B1
		1

16) JUNE 2020_9709_53 Q2

Question	Answer	Marks
(a)	$0.22^3=0.0106$	B1
		1
(b)	$P(2, 3, 4)={}^{16}C_2\ 0.22^2 0.78^{14}+{}^{16}C_3 0.22^3 0.78^{13}+{}^{16}C_4 0.22^4 0.78^{12}$	M1
	$0.179205+0.235877+0.216221$	A1
	0.631	A1
		3

17) JUNE 2020_9709_53 Q4

(a)

–1	0	0	1
0	1	1	2
2	3	3	4

x	–1	0	1	2	3	4
Probability	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

(a)	Probability distribution table with correct scores with at least one probability	B1
	At least 4 probabilities correct	B1
	All probabilities correct	B1
		3
(b)	$E(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$	B1
	$Var(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$	M1
	$\frac{37}{18}$ $(= 2.06)$	A1
		3

18) JUNE 2020_9709_53 Q5(a)(b)(c)

(a)	$\frac{1}{\frac{1}{4}} = 4$	B1
		1
(b)	$\frac{9}{64}$ $(= 0.141)$	B1
		1
(c)	$P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$ (FT *their* probability/mean from part (a))	M1
	0.763	A1
		2

19) JUNE 2021_9709_51 Q6(a)

(a)	$1 - P(10, 11, 12) = 1 - ({}^{12}C_{10}0.6^{10}0.4^2 + {}^{12}C_{11}0.6^{11}0.4^1 + {}^{12}C_{12}0.6^{12}0.4^0)$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	$P(0,1,2,3,4,5,6,7,8,9) = {}^{12}C_0 0.6^0 0.4^{12} + {}^{12}C_1 0.6^1 0.4^{11} + \ldots\ldots\ldots\ldots {}^{12}C_9 0.6^9 0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	

20) JUNE 2021_9709_51 Q7

	Answer	Mark	Guidance
(a)	$P(X=3)=\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}$	M1	$\frac{m}{7}\times\frac{n}{6}\times\frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	
(b)	x: 1, 2, 3, 4, 5; p: $\frac{15}{35}$, $\frac{10}{35}$, $\frac{6}{35}$, $\frac{3}{35}$, $\frac{1}{35}$	B1	Table with x values and at least one probability. Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X = 3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X = 3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
(c)	$[E(X) = 1\times\frac{15}{35}+2\times\frac{10}{35}+3\times\frac{6}{35}+4\times\frac{3}{35}+5\times\frac{1}{35}]$ $E(X)=\frac{15+20+18+12+5}{35}\left[=\frac{70}{35}=2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1. May be implied by use in Variance, accept unsimplified expression.
	$\text{Var}(X)=\left[\frac{1^2\times15+2^2\times10+3^2\times6+4^2\times3+5^2\times1}{35}-2^2=\right]$ $\frac{15+40+54+48+25}{35}-2^2$	M1	Appropriate variance formula using *their* $(E(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[=\frac{182}{35}-4\right]=\frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	

x	1	2	3	4	5
p	$\frac{15}{35}$	$\frac{10}{35}$	$\frac{6}{35}$	$\frac{3}{35}$	$\frac{1}{35}$

21) JUNE 2021_9709_52 Q1

	Answer	Mark	Guidance
(a)	6	B1	WWW
		1	
(b)	$\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+\left(\frac{5}{6}\right)^6\frac{1}{6}$	M1	$p^3(1-p)+p^4(1-p)+p^5(1-p)+p^6(1-p)$, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
	Alternative method for Question 1(b)		
	$\left(\frac{5}{6}\right)^3-\left(\frac{5}{6}\right)^7$	M1	p^3-p^7, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
		2	
(c)	$1-\left(\frac{5}{6}\right)^9$	M1	$1-p^n$, $0<p<1$, $n=9, 10$
	0.806	A1	
	Alternative method for Question 1(c)		
	$\frac{1}{6}+\frac{1}{6}\left(\frac{5}{6}\right)+\frac{1}{6}\left(\frac{5}{6}\right)^2+\cdots+\frac{1}{6}\left(\frac{5}{6}\right)^8$	M1	$p+p(1-p)+p(1-p)^2+p(1-p)^3+p(1-p)^4+p(1-p)^5+p(1-p)^6+p(1-p)^7+p(1-p)^8(+p(1-p)^9)$, $0<p<1$. As per answer for minimum terms shown
	0.806	A1	
		2	

22) JUNE 2021_9709_52 Q4

		Mark	Guidance
(a)	<table><tr><td>X</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X)$</td><td>$\frac{1}{9}$</td><td>$\frac{2}{9}$</td><td>$\frac{1}{9}$</td><td>$\frac{3}{9}$</td><td>$\frac{2}{9}$</td></tr></table>	B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
		B1	2 correct probabilities linked with correct outcomes, may not be in table.
		B1	3 further correct probabilities linked with correct outcomes, may not be in table. **SC** if less than 2 correct probabilities seen, award **SCB1** for sum of *their* 4 or 5 probabilities in table = 1
		3	
(b)	$\left[E(X)=\frac{-1\times1+(0\times2)+1\times1+2\times3+3\times2}{9}=\right]$ $\frac{-1+1+6+6}{9}$	M1	May be implied by use in variance, accept unsimplified expression. FT *their* table if *their* 3 or more probabilities sum to 1 or 0.999
	[Var (X) =] $\left[\frac{-1^2\times1+(0^2\times2)+1^2\times1+2^2\times3+3^2\times2}{9}-(their\,E(X))^2\right]$ $\frac{1+0+1+12+18}{9}-(their\,E(X))^2$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table even if *their* 3 or more probabilities not summing to 1.
	$E(X)=\frac{4}{3}$ or 1.33 and $Var(X)=\frac{16}{9}$ or 1.78	A1	Answers for E(X) and Var(X) must be identified
		3	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1

23) JUNE 2021_9709_52 Q5

		Mark	Guidance
(a)	$[(0.7)^3=]0.343$	B1	Evaluated WWW
	Alternative method for Question 5(a)		
	$[(0.15)^3+{}^3C_1(0.15)^2(0.55)+{}^3C_2(0.15)(0.55)^2+(0.55)^3=]\ 0.343$	B1	Evaluated WWW
		1	
(b)	$1-(0.85^9+{}^9C_1\,0.15^1\,0.85^8+{}^9C_2\,0.15^2\,0.85^7)$ $[1-(0.231617+0.367862+0.259667)]$	M1	One term: ${}^9C_x\,p^x(1-p)^{9-x}$ for $0<x<9$, any $0<p<1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408\leqslant$ ans $\leqslant 0.141$, award at most accurate value.
	Alternative method for Question 5(b)		
	${}^9C_3\,0.15^3\,0.85^6+{}^9C_4\,0.15^4\,0.85^5+{}^9C_5\,0.15^5\,0.85^4+{}^9C_6\,0.15^6\,0.85^3+{}^9C_7\,0.15^7\,0.85^2+{}^9C_8\,0.15^8\,0.85+0.15^9$	M1	One term: ${}^9C_x\,p^x(1-p)^{9-x}$ for $0<x<9$, any $0<p<1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408\leqslant$ ans $\leqslant 0.141$, award at most accurate value.
		3	
:)	Mean $=[60\times0.15=]9$ Variance $=[60\times0.15\times0.85=]7.65$	B1	Correct mean and variance, allow unsimplified. ($2.765\leq\sigma\leq2.77$ imply correct variance)
	$[(X\geq12)=]P\left(Z>\frac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1-\Phi(0.9039)=1-0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

24) JUNE 2021_9709_53 Q2

$p + p + 0.1 + q + q = 1$	**B1**	Sum of probabilities = 1
$0.1 + 2q = 3(2p)$	**B1**	Use given information
Attempt to solve two correct equations in p and q	**M1**	**Either** use of Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	**A1**	CAO, both WWW
	4	

25) JUNE 2021_9709_53 Q4(b)

(b)	$\text{P}(18) = \left(\frac{1}{6}\right)^3 \left[= \frac{1}{216}\right]$	**B1**	
	$\text{P}(18 \text{ on 5th throw}) = \left(\frac{215}{216}\right)^4 \times \frac{1}{216}$	**M1**	$(1-p)^4 p$, $0 < their\ p < 1$
	0.00454	**A1**	
		3	

26) JUNE 2021_9709_53 Q7(b)(i)

(b)(i)	P(0, 1, 2) = ${}^{10}C_0\,(0.35)^0\,(0.65)^{10} + {}^{10}C_1\,(0.35)^1\,(0.65)^9 + {}^{10}C_2\,(0.35)^2\,(0.65)^8$	**M1**	One term: ${}^{10}C_x\, p^x (1-p)^{10-x}$ for $0 < x < 10$, any $0<p<1$
	$0.013463 + 0.072492 + 0.17565$	**A1**	Correct unsimplified expression, or better
	0.262	**A1**	
		3	

27) JUNE 2022_9709_51 Q4

(a)	$a = \text{P}(1 \text{ head}) = 0.7\times(0.5)^3 + \ 0.3\times(0.5)^3\times 3 = \frac{1}{5}$	**B1**	Clear statement of unevaluated correct calculation $= \frac{1}{5}$. AG
	$b = 0.7\times 0.5^3\times 3 + 0.3\times 0.5^3\times 3 = \frac{3}{8}$ $c = 0.7\times 0.5^3\times 3 + 0.3\times 0.5^3 = \frac{3}{10}$ $\left[or\ c = \frac{27}{40} - b\right]$	**M1**	Clear statement of unevaluated calculation for either b or c
		A1	For either b or c correct
		B1 FT	$their\ b + their\ c = \frac{27}{40}$
		4	
(b)	$\left[\text{E}(X) = \frac{3\times 0 + 16\times 1 + 30\times 2 + 24\times 3 + 7\times 4}{80} = \right] \frac{176}{80}$ or 2.2	**B1 FT**	Correct or accept unsimplified calculation using *their* values for b and c seen (sum of probabilities = 1)
		1	

:)	[P(0, 1, 2) =] $^{10}C_0\ 0.2^0 0.8^{10} + {}^{10}C_1\ 0.2^1 0.8^9 + {}^{10}C_2\ 0.2^2 0.8^8$	M1	One term $^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
	0.107374 + 0.268435 + 0.301989	A1	Correct expression, accept unsimplified leading to final answer
	0.678	B1	$0.677 < p \leqslant 0.678$
	Alternative method for question 4(c)		
	$1 - [{}^{10}C_{10}\ 0{\cdot}2^{10}0.8^0 + {}^{10}C_9\ 0{\cdot}2^9 0.8^1 + {}^{10}C_8\ 0{\cdot}2^8 0.8^2 + {}^{10}C_7\ 0{\cdot}2^7 0.8^3 + {}^{10}C_6\ 0{\cdot}2^6 0.8^4 + {}^{10}C_5\ 0{\cdot}2^5 0.8^5 + {}^{10}C_4\ 0{\cdot}2^4 0.8^6 + {}^{10}C_3\ 0{\cdot}2^3 0.8^7]$	M1	One term $^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
		A1	Correct expression, accept unsimplified
	0.678	B1	$0.677 < p \leq 0.678$
		4	
l)	$0.8^6 \times 0.2 + 0.8^7 \times 0.2 = 0.0524288 + 0.041943$	M1	$p^l \times (1-p) + p^m \times (1-p)$, $l = 6, 7$ $m = l + 1$, $0 < p < 1$
	0.0944	A1	$0.09437 \leqslant p \leqslant 0.0944$
		2	

28) JUNE 2022_9709_52 Q2

(a)		B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.

x	2	3	4	5	6
p	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
	0.02778	0.1111	0.2778	0.3333	0.25

	B1	3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
	B1	2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
	3	SC B1 for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.

2(b)	If method FT from *their* incorrect (a), expressions for E(X) and Var(X) must be seen at the stage shown in **bold** (or less simplified) in the scheme with all probabilities < 1.		
	$\left[E(X) = \frac{1\times2+4\times3+10\times4+12\times5+9\times6}{36} = \right]$ $\mathbf{\frac{2+12+40+60+54}{36}}$	M1	Accept unsimplified expression. May be calculated in variance. FT *their* table with 4 or more probabilities summing to $0.999 \leqslant$ total $\leqslant 1$ ($0 < p < 1$).
	$\left[\text{Var}(X) = \frac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36} - (\textit{their}\ E(X))^2 = \right]$ $\mathbf{\frac{1\times4+4\times9+10\times16+12\times25+9\times36}{36}} - \left(\textit{their}\ \frac{14}{3}\right)^2$ $\left[\frac{4+36+160+300+324}{36} - \left(\textit{their}\ \frac{14}{3}\right)^2\right]$	M1	Appropriate variance formula using *their* (E(X))^2 value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in **bold** (or less simplified) seen.
	$E(X) = \frac{168}{36}, \frac{14}{3}$, 4.67 $\text{Var}(X) = \frac{10}{9}, 1\frac{1}{9}$, 1.11, $\frac{1440}{1296}$	A1	Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned **SC B1** for identified correct final answers.
		3	

29) JUNE 2022_9709_53 Q3

(a)	$k = \frac{1}{18}$ $(4k + k + 4k + 9k = 18k = 1)$	B1	SOI
		M1	Table with correct x values and at least one probability accurate using *their* k. Values need not be in order, lines may not be drawn, may be vertical, x and P(X=x) may be omitted. Condone any additional X values if probability stated as 0.
		A1	Remaining probabilities correct.
		3	

x	–2	1	2	3
P(X=x)	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$

(b)	$\left[E(X)=\dfrac{4\times-2+1\times1+4\times2+9\times3}{18}=\right]$ $\dfrac{-8+1+8+27}{18}$	M1	$-8k+k+8k+27k$ May be implied by use in Variance. Accept unsimplified expression. FT *their* table if probabilities sum to 1 or 0.999. **SC B1** $28k$.
	$\left[\text{Var}(X)=\dfrac{4\times(-2)^2+1\times1^2+4\times2^2+9\times3^2}{18}-(\textit{their}\ E(X))^2=\right]$ $=\dfrac{16+1+16+81}{18}-\left(\textit{their}\ \dfrac{28}{18}\right)^2$	M1	$16k+k+16k+81k-$ (*their* mean)^2 FT *their* table even if probabilities not summing to 1. Note: If table is correct, $\dfrac{114}{18}-(\textit{their}\ E(X))^2$ M1. **SC B1** $114k-(\textit{their}\ \text{mean})^2$.
	$E(X)=\dfrac{14}{9}$, $1\dfrac{5}{9}$, 1.56, $\text{Var}(X)=\dfrac{317}{81}$, $3\dfrac{74}{81}$, 3.91	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $3.91\leqslant \text{Var}(X)\leqslant 3.914$
		3	

30) JUNE 2022_9709_53 Q4

(a)	$\left[\left(\dfrac{5}{6}\right)^7\times\dfrac{1}{6}=\right]$ 0.0465, $\dfrac{78125}{1679616}$	B1	$0.0465\leqslant p<0.04652$
		1	
(b)	$P(X<6)=1-\left(\dfrac{5}{6}\right)^5$ or $\dfrac{1}{6}+\left(\dfrac{5}{6}\right)\left(\dfrac{1}{6}\right)+\left(\dfrac{5}{6}\right)^2\left(\dfrac{1}{6}\right)+\left(\dfrac{5}{6}\right)^3\left(\dfrac{1}{6}\right)+\left(\dfrac{5}{6}\right)^4\left(\dfrac{1}{6}\right)$	M1	$1-p^n$, $0<p<1$, $n=4, 5, 6$ or sum of 4, 5 or 6 terms $p\times(1-p)^n$ for $n=0,1,2,3,4(5)$.
	0.598, $\dfrac{4651}{7776}$	A1	
		2	
(c)	[Probability of total less than 4 is] $\dfrac{3}{36}$ or $\dfrac{1}{12}$	B1	SOI
	$[1-P(0, 1, 2)]$ $=1-\left({}^{10}C_0\left(\dfrac{1}{12}\right)^0\left(\dfrac{11}{12}\right)^{10}+{}^{10}C_1\left(\dfrac{1}{12}\right)^1\left(\dfrac{11}{12}\right)^9+{}^{10}C_2\left(\dfrac{1}{12}\right)^2\left(\dfrac{11}{12}\right)^8\right)$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0<x<10$, $0<p<1$.
	$1-(0.418904+0.380822+0.155791)$	A1 FT	Correct expression. Accept unsimplified.
	0.0445	A1	$0.04448\leqslant p\leqslant 0.0445$
		4	

31) JUNE 2023_9709_51 Q6

(a)	$[P(X=3)=]\ \frac{3}{4}\times\left(\frac{1}{4}\right)^3\times 4$	M1	$\frac{3}{4}\times\left(\frac{1}{4}\right)^3\times q$; q a positive integer (1 may be implied).
	$=\frac{3}{64}$	A1	AG.
		2	
(b)	see table below	B1	Either $P(1)=\frac{27}{64}, 0.421875$ or $P(2)=\frac{27}{128}, 0.2109375$ correct to at least 3SF. Condone not in table.
		B1 FT	Both values in table. FT $P(1)+P(2)=\frac{81}{128}, 0.6328125$.
		2	
(c)	$[E(X)=]\ [0\times\frac{81}{256}]+1\times \textit{their}\frac{27}{64}+2\times \textit{their}\frac{27}{128}+3\times\frac{12}{256}+4\times\frac{1}{256}$	M1	Correct method from *their* probability distribution table with at least 4 terms, $0<\textit{their}\ P(x)<1$, accept partially evaluated. $=0+\frac{27}{64}+\frac{54}{128}+\frac{36}{256}+\frac{4}{256}$
	$=1$	A1	
		2	
d)	Mean $=96\times\frac{67}{256}=25.125$ Var $=96\times\frac{67}{256}\times\frac{189}{256}=18.549$	B1	25.125, $25\frac{1}{8}$ and 18.5493… to at least 3SF seen, allow unsimplified ($4.3068\leqslant\sigma\leqslant4.307$ implies correct variance).
	$P(X<20)=P(Z<\frac{19.5-25.125}{\sqrt{18.549}})$	M1	Substituting *their* μ and σ into $\pm$ standardisation formula (any number for 19.5). Condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in *their* standardisation formula. Note: $\frac{\pm5.625}{\sqrt{18.549}}$ seen gains M2 BOD.
	$[=P(Z<-1.306)\ =1-\Phi(1.306)=]\ 1-0.9042=$	M1	Appropriate area Φ, from final process. Must be a probability.
	0.0958	A1	$0.0957\leqslant p\leqslant0.0958$. **SC B1** for $0.0957\leqslant p\leqslant0.0958$ if B1M0M0M1 scored.
		5	

Table for (b):

x	0	1	2	3	4
$P(X=x)$	$\frac{81}{256}$	$\frac{27}{64}$	$\frac{27}{128}$	$\frac{3}{64}$	$\frac{1}{256}$

32) JUNE 2023_9709_51 Q7

a)	**Method 1**		
	$[P(X<6)=P(X\leqslant5)=]\ 1-0.8^5$	M1	$1-0.8^r$, $r=5, 6$.
	$=0.672$	A1	
	Method 2		
	$[P(X<6)=P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)=]$ $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	M1	Condone an extra term $(\frac{4}{5})^5\times\frac{1}{5}$. First, last and one of the 3 middle terms implies M1.
	$=0.672$	A1	
		2	

(b)	**Method 1**		
	$[1 - P(0, 1, 2)]$ $= 1 - ({}^{12}C_0\,(0.8)^{12} + {}^{12}C_1\,(0.2)(0.8)^{11} + {}^{12}C_2\,(0.2)^2\,(0.8)^{10})$ $[= 1 - (0.06872 + 0.20615 + 0.28347)]$	M1	One term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. Correct unsimplified expression or better.
	$= 0.442$	B1	$0.411 < p \leqslant 0.442$ WWW.
	Method 2		
	$[P(3,4,5,6,7,8,9,10,11,12) =]$ ${}^{12}C_3\,(0.2)^3\,(0.8)^9 + {}^{12}C_4\,(0.2)^4\,(0.8)^8 + \ldots + {}^{12}C_{11}\,(0.2)^{11}\,(0.8)^1 + {}^{12}C_{12}\,(0.2)^{12}$ $[= 0.23622 + 0.13288 + \ldots + 1.966 \times 10^{-7} + 4.096 \times 10^{-9}]$	M1	One term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, leading to final answer. Accept first, last and 8 of the middle terms.
	$=0.442$	B1	$0.411 < p \leqslant 0.442$.
		3	
(c)	$(0.2)^5 \times 5!$	M1	$(0.2)^5 \times s$, s a positive integer. 1 may be implied.
		M1	$t \times 5!$ where $0 < t < 1$.
	$= 0.0384, \dfrac{24}{625}$	A1	
	Alternative Method for Question 7(c)		
	$\dfrac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]}{({}^5C_1)^5}$	M1	$({}^5C_1)^5$ or 5^5 as denominator.
		M1	${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]$ or 5! as numerator.
	$= 0.0384, \dfrac{24}{625}$	A1	
		3	

33) JUNE 2023_9709_52 Q1

1(a)	$[3k + 3k + 8k = 1, so]\, k = \dfrac{1}{14}$	B1	
	<table><tr><td>x</td><td>–2</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$\frac{3}{14}$, 0.214</td><td>$\frac{3}{14}$, 0.214</td><td>$\frac{8}{14}$, 0.571</td></tr></table>	B1 FT	Table with correct values of x, and at least one correct probability linked with outcome. FT *their k*. Condone any additional X values if probability stated as 0.
		B1 FT	The outcomes in the table must be –2, 2 and 3. 2 further correct probabilities in table **or** 3 correct probabilities not in table linked to outcomes, **or** 3 correct FT probabilities in table using *their k*, **or** 3 incorrect probabilities summing to 1 in table if k not stated.
			If k not calculated, **SC B1** for the below. <table><tr><td>x</td><td>–2</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$3k$</td><td>$3k$</td><td>$8k$</td></tr></table>
		3	

(b)	$\left[E(X) = -2\times\frac{3}{14}+2\times\frac{3}{14}+3\times\frac{8}{14}=\right]$ $-\frac{6}{14}+\frac{6}{14}+\frac{24}{14}$	M1	Accept unsimplified expression. May be calculated in variance. FT *their* table with 3 probabilities summing to 0.999 ⩽ *total* ⩽ 1 ($0 < p < 1$) or in terms of *k*.
	$\left[Var(X) = (-2)^2\times\frac{3}{14}+2^2\times\frac{3}{14}+3^2\times\frac{8}{14}-(\textit{their}\ E(X))^2=\right]$ $4\times\frac{3}{14}+4\times\frac{3}{14}+9\times\frac{8}{14}-\left(\textit{their}\ \frac{12}{7}\right)^2$ $\left[\frac{12+12+72}{14}-\left(\textit{their}\ \frac{12}{7}\right)^2\right]$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1, or in terms of *k* with an expression no more evaluated than shown.
	$E(X) = \frac{12}{7}$, 1.71, $1\frac{5}{7}$ $Var(X) = \frac{192}{49}$, 3.92, $3\frac{45}{49}$	A1	Answers for $E(X)$ and $Var(X)$ must be identified. $E(X)$ may be identified by correct use in Variance (condone E, V, μ, σ^2, etc.). If A0 earned, **SC B1** for identified correct final answers.
		3	

34) JUNE 2023_9709_52 Q2

(a)	[P(no rain) = $0.6\times(0.8)^3$ =] 0.3072, $\frac{192}{625}$	B1	Exact value required
		1	
(b)	$0.6\times0.8\times0.2$	M1	$a\times b\times c$ where $a, b = 0.6, 0.8, c = 0.2, 0.4, 0.7$. Condone including Wednesday with both 0.3 and 0.7 used.
	$= 0.096[0]$, $\frac{12}{125}$	A1	
		2	
(c)	$P(RDDD) = 0.4\times0.3\times0.8\times0.8 = 0.0768$, $\frac{48}{625}$ $P(DRDD) = 0.6\times0.2\times0.3\times0.8 = 0.0288$, $\frac{18}{625}$ $P(DDRD) = 0.6\times0.8\times0.2\times0.3 = 0.0288$, $\frac{18}{625}$ $P(DDDR) = 0.6\times0.8\times0.8\times0.2 = 0.0768$, $\frac{48}{625}$	B1	Correct probability for one clearly identified outcome evaluated accept unsimplified. A correct unsimplified expression is not sufficient.
		M1	Add 4 probability values, $0 < p < 1$, for appropriate identified scenarios. Accept unsimplified. Ways of identifying scenarios for this mark: Stating the days. All the unsimplified probability calculations exactly as stated in the mark scheme. Identifying the correct branches on a tree diagram and linking with the values. No repeated scenarios. No incorrect scenarios.
	0.2112, $\frac{132}{625}$	A1	Accept 0.211 If 0/3 scored **SC B1** for 0.2112, $\frac{132}{625}$.
		3	

35) JUNE 2023_9709_52 Q4

(a)	[$P(X = 4) = (0.8)^3(0.2)$ =] 0.1024, $\frac{64}{625}$	B1	Condone 0.102 .
		1	

(b)	$[\mathrm{P}(X<6)=]\,1-0.8^5$	**M1**	$1-0.8^d$, $d=5, 6$.
	$=0.672,\ \dfrac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\dfrac{2101}{3125}$ or 0.67232 only.
	Alternative Method for Question 4(b)		
	$[\mathrm{P}(X<6)=]\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	**M1**	If answer correct, condone omission of 2 from 3 middle terms. Allow M1 for $\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$
	$=0.672,\ \dfrac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\dfrac{2101}{3125}$ or 0.67232 only.
		2	
(c)	$\left[\mathrm{P}(X>0 \mid X\neq 2)=\dfrac{\mathrm{P}(X>0\cap X\neq 2)}{\mathrm{P}(X\neq 2)}=\right]$ $=\dfrac{14}{25}\Big/\dfrac{19}{25}$ $=\dfrac{14}{19},\ 0.737$	**M1**	$[\mathrm{P}(X>0\cap X\neq 2)=]\ \dfrac{14}{25}$, 0.56[0] seen as numerator or denominator of conditional probability fraction.
		M1	$[\mathrm{P}(X\neq 2)=]\ \dfrac{19}{25}$, 0.76[0] seen as denominator of conditional probability fraction.
		A1	Final answer $=\dfrac{14}{19}$, 0.7368421… to at least 3SF. If A0, **SC B1** for correct final answer www.
	Alternative Method for Question 4(c)		
	$\begin{array}{c\|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{array}$ $\left[\mathrm{P}(X>0 \mid X\neq 2)=\dfrac{\text{Number of outcome}(X>0\cap X\neq 2)}{\text{Number of outcomes } X\neq 2}=\right]$ $\dfrac{14}{19},\ 0.737$	**M1**	$[\text{Number of outcome}(X>0\cap X\neq 2)=]$ 14 seen as numerator or denominator of conditional probability fraction.
		M1	$[\text{Number of outcome}(X\neq 2)=]$ 19 seen as denominator of conditional probability fraction.
		A1	Final answer $=\dfrac{14}{19}$, 0.7368421… to at least 3SF.
		3	
(d)	$[\mathrm{P}(X>2)=1-\mathrm{P}(0, 1, 2)$ with $p=\dfrac{6}{25}]$	**M1**	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0<p<1, 0<x<9$.
	$1-\left({}^9C_0\left(\frac{19}{25}\right)^9+{}^9C_1\left(\frac{6}{25}\right)^1\left(\frac{19}{25}\right)^8+{}^9C_2\left(\frac{6}{25}\right)^2\left(\frac{19}{25}\right)^7\right)$ $[1-(0.08459+0.2404+0.3037)]$	**A1**	$1-\left({}^9C_0\,(1-p)^9+{}^9C_1\,(p)^1(1-p)^8+{}^9C_2\,(p)^2(1-p)^7\right)$, $0<p<1$. Correct expression from *their* p, accept unsimplified, no terms omitted leading to final answer. Condone omission of last bracket only.
	0.371	**B1**	$0.371\leqslant p<0.3715$.
	Alternative Method for Question 4(d)		
	$[\mathrm{P}(X>2)=\mathrm{P}(3,4,5,6,7,8,9)$ with $p=\dfrac{6}{25}]$	**M1**	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0<p<1, 0<x<9$.
	${}^9C_3\left(\frac{6}{25}\right)^3\left(\frac{19}{25}\right)^6+{}^9C_4\left(\frac{6}{25}\right)^4\left(\frac{19}{25}\right)^5+\ldots+{}^9C_8\left(\frac{6}{25}\right)^8\left(\frac{19}{25}\right)^1+{}^9C_9\left(\frac{6}{25}\right)^9$ $[0.2238+0.1060+\ldots+7.529\times10^{-5}+2.642\times10^{-6}]$	**A1**	${}^9C_3\,(p)^3(1-p)^6+{}^9C_4\,(p)^4(1-p)^5+\ldots+{}^9C_8\,(p)^8(1-p)^1+{}^9C_9\,(p)^9$, $0<p<1$. Correct expression from *their* p, accept unsimplified, no terms omitted leading to final answer.
	0.371	**B1**	$0.371\leqslant p<0.3715$.
		3	

36) JUNE 2023_9709_53 Q1

a)	$\left[P(HH)=\frac{1}{4}\right]$ [E(X) =] 4	B1	
		1	
b)	$\left[P(X=5)=\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right)=\right]$ 0.0791	B1	$\frac{81}{1024}$
		1	
c)	$[P(X<7)=]\,1-\left(\frac{3}{4}\right)^6$ or $\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\frac{3^2}{4}\times\frac{1}{4}+\ldots+\frac{3^5}{4}\times\frac{1}{4}$	M1	$1-p^n$, $0<p<1$, $n=6, 7$ or $p+p(1-p)+p(1-p)^2+\ldots+p(1-p)^n$, where $n=4, 5$.
	$=\frac{3367}{4096}$, 0.822	A1	Accept 0.82202148… to at least 3SF.
		2	

37) JUNE 2023_9709_53 Q3

(a)	$[P(X=4)=3P(X=2)]$ $4k(4+a)=3\times 2k(2+a)$ $16k+4ak=12k+6ak$	M1	Using $P(X=4)=3P(X=2)$ to form an equation in a and k.
	$a=2$	A1	If M0 scored, SC **B1** for $a=2$ www.
	$3k+8k+15k+24k=1$	M1	Using sum of probabilities = 1 to form an equation in k: $k(1+a)+2k(2+a)+3k(3+a)+4k(4+a)=1$.
	$k=\frac{1}{50}$	A1	If M0 scored, SC **B1** for $k=\frac{1}{50}$ www.
		4	
(b)	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$\frac{3}{50}$, 0.06</td><td>$\frac{8}{50}$, 0.16</td><td>$\frac{15}{50}$, 0.3</td><td>$\frac{24}{50}$, 0.48</td></tr></table>	B1 FT	<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$k(1+a)$</td><td>$2k(2+a)$</td><td>$3k(3+a)$</td><td>$4k(4+a)$</td></tr></table> $0<p<1$ for all outcomes, must be numerical.
		1	
(c)	$\text{Var}(X)=\frac{3}{50}\times 1+\frac{8}{50}\times 2^2+\frac{15}{50}\times 3^2+\frac{24}{50}\times 4^2-3.2^2$	M1	Correct formula for variance method from their probability distribution table, $0 \leqslant$ *their* $P(x) \leqslant 1$. Accept $\frac{3+32+135+384}{50}-\frac{256}{25}$.
	$[=11.08-3.2^2=]\,0.84[0]$, $\frac{21}{25}$	A1	If M0 score SC **B1** for 0.84 www.
		2	

38) OCT 2020_9709_51 Q3

(a)	$P(X>6)=0.75^6$	M1	p^n, $n=6, 7$ $0<p<1$
	0.178, $\frac{729}{4096}$	A1	0·17797…
		2	

(b)	$1 - P(0, 1, 2) = 1 - (0.75^{10} + {}^{10}C_1\ 0.25^1 0.75^9 + {}^{10}C_2 0.25^2 0.75^8)$	M1	Binomial term of form ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0, 10$
	$1 - (0{\cdot}0563135 + 0{\cdot}1877117 + 0{\cdot}2815676)$	A1	Correct unsimplified expression
	$0{\cdot}474$	A1	$0{\cdot}474 \leqslant p \leqslant 0{\cdot}4744$
		3	

39) OCT 2020_9709_51 Q4

(a)

y	1	2	3	4
prob	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{3}{16}$	$\frac{1}{16}$

B1

	1	2	3	4
1	1	1	2	3
2	1	2	1	2
3	2	1	3	1
4	3	2	1	4

Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated'

Mark	Guidance
B1	One probability (linked with correct score) correct
B1	2 more probs (linked with correct scores) correct
B1 FT	4th prob correct, FT sum of 3 or 4 terms = 1
4	

(b)	$P(2 \mid \text{even}) = \dfrac{\frac{5}{16}}{\frac{6}{16}}$	M1	$\dfrac{\textit{their}\ P(2)}{\textit{their}\ P(2) + \textit{their}\ P(4)}$ seen or correct outcome space.
	$\frac{5}{6}$ or $0{\cdot}833$	A1	
		2	

40) OCT 2020_9709_52 Q1

(a)	$1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}$	M1	$1-p^n$ $n = 5,6$ or $p + pq + pq^2 + pq^3 + pq^4 (+ pq^5)$ $0 < p < 1$, $p + q = 1$,
	$0{\cdot}598$, $\frac{4651}{7776}$	A1	
		2	
(b)	$(1 - P(0, 1, 2))$ $1 - \left(\left(\frac{5}{6}\right)^{10} + {}^{10}C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9 + {}^{10}C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8\right)$	M1	${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0,10$
	$1 - (0{\cdot}1615056 + 0{\cdot}3230111 + 0{\cdot}290710)$	A1	Correct expression, accept unsimplified, condone omission of final bracket
	$0{\cdot}225$	A1	$0{\cdot}2247 < p \leq 0{\cdot}225$, WWW
		3	

41) OCT 2020_9709_52 Q2

(a)	P(1 red) = $\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3$	M1	$\frac{a}{8}\times\frac{b}{7}\times\frac{c}{6}\times k$ or $\frac{5}{d}\times\frac{3}{e}\times\frac{2}{f}\times3$, $1\leqslant a,b,c\leqslant5$, $d,e,f\leqslant8$, a, b, c, d, e, f, k all integers. $1<k\leqslant3$,
	$\frac{15}{56}$	A1	AG, WWW
	Alternative method for question 2(a)		
	$\frac{^5C_1\times{}^3C_2}{^8C_3}$	M1	$\frac{^aC_1\times{}^bC_2}{^8C_3}$ or $\frac{^5C_d\times{}^3C_e}{^8C_3}$ or $\frac{^5C_d\times{}^3C_e\,(or\ ^aC_1\times{}^bC_2)}{^5C_3\times{}^3C_0+{}^5C_2\times{}^3C_1+{}^5C_1\times{}^3C_2+{}^5C_0\times{}^3C_3}$, $a+b=8$, $d+e=3$
	$\frac{15}{56}$	A1	AG, WWW, $\frac{15}{56}$ must be seen
		2	
(b)	(table below)	B1	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		B1	2 of P(0), P(2) and P(3) correct
		B1 FT	4th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct
		3	
(c)	Var(X) = $\frac{(0^2\times1)+1^2\times15+2^2\times30+3^2\times10}{56}-\left(\frac{15}{8}\right)^2$ $=\frac{15}{56}+\frac{120}{56}+\frac{90}{56}-\left(\frac{15}{8}\right)^2$	M1	Substitute *their* attempts at scores in correct variance formula, must have '– mean^2' (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\frac{225}{448}$, 0·502	A1	
		2	

Table for (b):

x	0	1	2	3
Prob.	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{30}{56}=\frac{15}{28}$	$\frac{10}{56}=\frac{5}{28}$
	0·0179	0·268	0·536	0·179

42) OCT 2020_9709_53 Q2

(a)	$\left(\frac{5}{6}\right)^8$	M1	p^8, $0<p<1$, no x, + or -
	0.233	A1	
		2	
(b)	36	B1	
		1	
(c)	$P(X=10)+P(X=11)=\left(\frac{35}{36}\right)^9\frac{1}{36}+\left(\frac{35}{36}\right)^{10}\frac{1}{36}$	M1	OE, unsimplified expression in form $p^9q+p^{10}q$, $p+q=1$, no ×
	0.0425	A1	
		2	

43) OCT 2021_9709_51 Q1

(a)	$\left(\frac{3}{4}\right)^6 \frac{1}{4}$	M1	$(1-p)^6 p,\ 0<p<1$
	$0.0445,\ \frac{729}{16384}$	A1	
		2	
(b)	$\left(\frac{3}{4}\right)^9$	M1	$\left(\frac{3}{4}\right)^n$ *or* $p^n,\ 0<p<1, n=8,9,10$
	$0.0751,\ \frac{19683}{262144}$	A1	
		2	

44) OCT 2021_9709_51 Q4

(a)	(table below)	B1	(table below) Table with x values and at least one probability substituted, $0<p<1$. Condone any additional x values if probability stated as 0.
		B1	2 correct identified probabilities.
		B1	All probabilities correct (accept to 3sf). SC if less than 2 correct probabilities: SC B1 4 or 5 probabilities summing to one.
		3	
(b)	$E(X) = -\frac{1}{12}+\frac{4}{12}+\frac{6}{12}+\frac{6}{12}\left[=\frac{15}{12}\right]$	M1	May be implied by use in Variance, accept unsimplified expression. Probabilities must sum to 1 ± 0.001.
	$\text{Var}(X) = \frac{1}{12}+0+\frac{4}{12}+\frac{12}{12}+\frac{18}{12}-\left(\frac{15}{12}\right)^2$	M1	Appropriate variance formula using *their* $(E(X))^2$. FT accept probabilities not summing to 1. Condone $\frac{35}{12}-\left(\frac{15}{12}\right)^2$ *or* $\frac{35}{12}-\frac{25}{9}$ from correct table.
	$\left[\frac{35}{12}-\frac{25}{16}=\right]\frac{65}{48},\ 1.35$	A1	WWW
		3	

Answer table for (a):

x	-1	0	1	2	3
p	$\frac{1}{12}=0.0833$	$\frac{2}{12}=0.167$	$\frac{4}{12}=0.333$	$\frac{3}{12}=0.25$	$\frac{2}{12}=0.167$

Guidance table for (a):

	0	1	2	2
−1	−1	0	1	1
0	0	1	2	2
1	1	2	3	3

45) OCT 2021_9709_52 Q3

(a)	For one yellow: YGG + GYG +GGY $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times 3$	M1	$\frac{a}{9}\times\frac{b}{8}\times\frac{c}{7}$, $0 < a,b,c$ integers $\leqslant 5$, for one arrangement.
		M1	*Their* three-factor probability × 3, ${}^3C_1$, ${}^3C_2$ or ${}^3P_1$ (or repeated adding) no additional terms.
	$\left[\frac{180}{504}=\right]\frac{5}{14}$	A1	AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.
		3	
	Alternative method for question 3(a)		
	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_3}$	M1	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_r}$, $r = 2, 3, 4$
		M1	$\frac{{}^5C_s\times{}^4C_t}{{}^9C_3}$, $s + t = 3$
	$\left[\frac{30}{84}=\right]\frac{5}{14}$	A1	AG. Convincingly shown, WWW.
		3	
(b)	see table below	B1	Table with correct X values and one correct probability inserted appropriately. Condone any additional X values if probability stated as 0.
		B1	Second identified correct probability, may not be in table.
		B1	All probabilities identified and correct . **SC** if less than 2 correct probabilities or X value(s) omitted: **SC B1** 3 or 4 probabilities summing to one.
		3	
(c)	$[E(X) =]\ \frac{840}{504}, \frac{5}{3}, 1.67$	B1	OE Must be evaluated. **SC B1 FT** correct unsimplified expression from incorrect **3(b)** using at least 3 probabilities, $0 < p < 1$.
		1	

Table for (b):

X	0	1	2	3
$P(X)$	$\frac{24}{504}$ $\left[=\frac{1}{21}, 0.0476\right]$	$\frac{180}{504}$ $\left[=\frac{5}{14}, 0.357\right]$	$\frac{240}{504}$ $\left[=\frac{10}{21}, 0.476\right]$	$\frac{60}{504}$ $\left[=\frac{5}{42}, 0.119\right]$

46) OCT 2021_9709_52 Q5

(a)	$[P(0, 1, 2) =]\ {}^{10}C_0 0.16^0 0.84^{10} + {}^{10}C_1 0.16^1 0.84^9 + {}^{10}C_2 0.16^2 0.84^8$ $[= 0.17490 + 0.333145 + 0.28555]$	M1	One term: ${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	A1	$0.7935 < p \leqslant 0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
(b)	$(0.84)^7\, 0.16$	M1	$(1-p)^7 p$, $0 < p < 1$
	0.0472	A1	0.0472144 to at least 3sf.
		2	
(c)	$4 \times 0.0472 \times (1 - 0.0472)^3$	M1	$4 \times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	A1	$0.163 \leqslant p \leqslant 0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

47) OCT 2021_9709_53 Q6

(a)	$p+q+0.65=1$	B1	Sum of probabilities = 1.
	$p+2q+0.15=0.55$	B1	Use given information.
	Solve 2 linear equations	M1	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p=0.3, \frac{3}{10}$, $q=0.05, \frac{1}{20}$	A1	CAO, both WWW If M0 with correct answers SC B1.
		4	
(b)	Var (X) = *their* $0.3+4\times$ *their* $0.05+9\times0.05-0.55^2$	M1	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	$0.6475\ \left[\frac{259}{400}\right]$	A1	CAO (must be exact).
		2	
(c)	$1 - P(0, 1, 2) = 1 - ({}^{12}C_0\, 0.3^0\, 0.7^{12} + {}^{12}C_1\, 0.3^1\, 0.7^{11} + {}^{12}C_2\, 0.3^2\, 0.7^{10})$	M1	One correct term: ${}^{12}C_x p^x (1-p)^{12-x}$ for $0 < x < 12$, $0 < p < 1$.
	$1 - (0.01384 + 0.07118 + 0.16779)$	A1FT	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to FT *their p* from 6(a) or correct.
	0.747	A1	
		3	
(d)	$(0.95)^8\times0.05 = 0.0332$ or $0.95^8 - 0.95^9 = 0.0332$	B1	Evaluated.
		1	

48) OCT 2022_9709_51 Q1

$0.12+p+q+0.16+0.3=1$	B1	Sum of probabilities = 1 $p+q = 0.42$ OE.
$-0.24-p+0.5q+0.16+0.6=0.28$	B1	Form equation using $E(X) = 0.28$ $-p + 0.5q = -0.24$ OE. Accept unsimplified.
Attempt to solve *their* two equations in p and q	M1	**Either** Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** Elimination method by writing both equations in the same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
$q=0.12$, $p=0.3$	A1	CAO, both WWW. If M0 awarded SC B1 for both correct WWW.
	4	

49) OCT 2022_9709_52 Q3

(a)	[P(17 or 18) =] $\frac{4}{216}=\frac{1}{54}$,0.0185(185…)	B1	May be seen used in calculation.
	$P(X=6)=\left(\frac{53}{54}\right)^5\cdot\frac{1}{54}$	M1	$p(1-p)^5, 0<p<1$
	0.0169	A1	$0.01686<p\leqslant 0.0169$ If A0 scored **SC B1** for $0.01686<p\leqslant 0.0169$
		3	
(b)	[P($X<8$) =] $1-\left(\frac{53}{54}\right)^7$	M1	$1-\left(\textit{their}\left(\frac{53}{54}\text{or }0.98148\right)\text{or correct}\right)^r$, $r=7,8$ $0<\textit{their}\ p<1$
	0.123	A1	$0.1225\leqslant p\leqslant 0.123$
	Alternative method for Question 3(b)		
	[P($X<8$) =] $\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^2\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^3\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^4\left(\frac{1}{54}\right)+$ $\left(\frac{53}{54}\right)^5\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^6\left(\frac{1}{54}\right)$	M1	$q+pq+p^2q+p^3q+p^4q+p^5q\left[+p^6q\right]$, $p+q=1, 0<p,q<1, q$ $=\textit{their}\ \frac{53}{54}$
	0.123	A1	$0.1225\leqslant p\leqslant 0.123$
		2	

50) OCT 2022_9709_53 Q4

(a)	**Method 1: Scenarios identified**		
	[no of ways for score of 2 are] 222, 211, 212, 221, 122, 112, 121 [Total options = 64]	B1	7 correct scenarios identified, no incorrect.
	[So P(X = 2) =] $\frac{7}{4\times4\times4} = \frac{7}{64}$	M1	$\frac{a}{4\times4\times4}$, a = *their* number of correct identified scenarios > 4
		A1	Approach identified, WWW.
	Method 2: P(2 on all spinners) + P(2 on two spinners and 1 on one spinner) + P(2 on one spinner and 1 on two spinners)		
	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + {}^3C_1\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$	B1	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(or\ {}^3C_1\right)\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + d$, $0 < d < 1$
		M1	$\left(\frac{1}{4}\right)^3 + e\left(\frac{1}{4}\right)^3 + f\left(\frac{1}{4}\right)^3$ $1 < e < 5$ and $1 < f < 5$
	[So P(X = 2) =] = $\frac{7}{64}$	A1	Approach identified, WWW.
	Method 3: P(1 or 2 on each spinner) – P(1 on all spinners)		
	$\left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3$	B1	$\left(\frac{1}{2}\right)^3 - b$ seen, $0 < b < 1$
		M1	$\left(\frac{1}{2}\right)^3 - c^3$, $0 < c < ½$
	[So P(X = 2) =] $\frac{7}{64}$	A1	Approach identified, WWW.
		3	
4(b)	P(X = 1) = $\frac{1}{64}$	B1	P(X = 1) or P(X = 4) correct. Condone answers not in probability distribution table if clearly identified.
	P(X = 4) = $\left[1-\frac{1}{64}-\frac{7}{64}-\frac{19}{64}=\right]\frac{37}{64}$	B1 FT	All 4 probabilities summing to 1.
		2	
4(c)	P(Y = 6) = $\left[\left(\frac{3}{4}\right)^5\times\frac{1}{4}=\right]0.0593, \frac{243}{4096}$	B1	Accept 0.059326… to 4 or more SF.
		1	
(d)	$\left(\frac{3}{4}\right)^4$	M1	$\left(\frac{3}{4}\right)^g$, $g = 4, 5$ or p^4 where $0 < p < 1$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406…to 4 or more SF.
	Alternative method for Question 4(d)		
	P(Y >4) = 1 – P(Y ⩽4) = $1-\left(\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\left(\frac{3}{4}\right)^2\times\frac{1}{4}+\left(\frac{3}{4}\right)^3\times\frac{1}{4}\right)$ $\left[=1-\frac{175}{256}\right]$	M1	Correct or $1-\left(\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\left(\frac{3}{4}\right)^2\times\frac{1}{4}+\left(\frac{3}{4}\right)^3\times\frac{1}{4}+\left(\frac{3}{4}\right)^4\right)$ or $1-\left(p+qp+q^2p+q^3p\right)$ where $0 < p < 1$ and $q = 1-p$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406…to 4 or more SF.
		2	