PROBABILITY AND STATISTICS -1

9709

(March, June and November series 2020 – 2023 With marking scheme)

Permutation and Combination

EXERCISE -1

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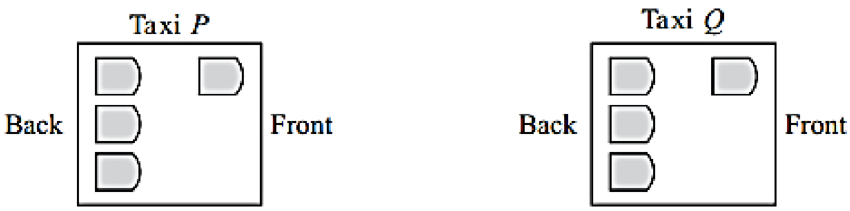
1) SP-2020_9709_5Q6

A group of 8 friends travels to the airport in two taxis, P and Q. Each taxi can take 4 passengers.

(a) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi.

Find the number of different ways in which this can be done. [3]



Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(b) Find the number of different seating arrangements that are now possible for the 8 friends. [4]

2) MARCH 2020_9709_52 Q1

The 40 members of a club include Ranuf and Saed. All 40 members will travel to a concert. 35 members will travel in a coach and the other 5 will travel in a car. Ranuf will be in the coach and Saed will be in the car.

In how many ways can the members who will travel in the coach be chosen? [3]

3) MARCH 2020_9709_52 Q4

Richard has 3 blue candles, 2 red candles and 6 green candles. The candles are identical apart from their colours. He arranges the 11 candles in a line.

(a) Find the number of different arrangements of the 11 candles if there is a red candle at each end. [2]

(b) Find the number of different arrangements of the 11 candles if all the blue candles are together and the red candles are not together. [4]

4) MARCH 2021_9709_52 Q6

(a) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR. [2]

(b) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR in which there is an R at the beginning and an R at the end, and the two As are not together. [4]

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contain both Rs and at least one A and at least one L. [4]

5) MARCH 2022_9709_52 Q5

A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways can a team of 5 people be chosen from the group if exactly one adult is included? [2]

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? [4]

The same group of 12 people stand in a line.

(c) How many different arrangements are there in which the 3 boys stand together and an adult is at each end of the line? [4]

6) MARCH 2023_9709_52 Q7

(a) Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three Es are together and the two Ds are **not** next to each other. [4]

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two Ds. [5]

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. [3]

7) JUNE 2020_9709_51 Q2

(a) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

8) JUNE 2020_9709_51 Q4

In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.

How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists? [4]

9) JUNE 2020_9709_52 Q6

(a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

10) JUNE 2020_9709_53 Q7

(a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

11) JUNE 2021_9709_51 Q1

A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.

How many different selections of 5 marbles contain at least 4 marbles of the same colour? [4]

12) JUNE 2021_9709_51 Q3

(a) How many different arrangements are there of the 8 letters in the word RELEASED? [1]

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

13) JUNE 2021_9709_52 Q6

(a) Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

14) JUNE 2021_9709_53 Q6

(a) How many different arrangements are there of the 11 letters in the word REQUIREMENT? [2]

(b) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which the two Rs are together and the three Es are together? [1]

(c) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs? [3]

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and at least one R? [4]

15) JUNE 2022_9709_51 Q1

(a) Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

16) JUNE 2022_9709_51 Q2

There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee? [2]

(b) In how many different ways can the committee be selected if Mrs Lan must be on the committee and there must be more women than men on the committee? [4]

17) JUNE 2022_9709_52 Q6

(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

18) JUNE 2023_9709_51 Q2

(a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

19) JUNE 2023_9709_51 Q3

(a) Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

(c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

20) JUNE 2023_9709_52 Q6

In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

(c) How many different arrangements are there of the 9 people if none of the cyclists stand next to each other? [4]

21) JUNE 2023_9709_53 Q7

(a) Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

22) OCT 2020_9709_51 Q7

(a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

23) OCT2020_9709_52 Q6

Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many ways can the 9 people be divided into a group of 6 and a group of 3? [2]

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of the Baker children. [3]

All 9 people stand in a line.

(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. [3]

(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. [3]

24) OCT 2020_9709_53 Q3

A committee of 6 people is to be chosen from 9 women and 5 men.

(a) Find the number of ways in which the 6 people can be chosen if there must be more women than men on the committee. [3]

The 9 women and 5 men include a sister and brother.

(b) Find the number of ways in which the committee can be chosen if the sister and brother cannot both be on the committee. [3]

25) OCT 2020_9709_53 Q5

The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

26) OCT 2021_9709_51 Q5

Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? [1]

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? [3]

For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? [2]

(d) For a random division into a group of 5 and a group of 4, find the probability that Raman and Sanjay are in the same group as each other. [4]

27) OCT 2021_9709_52 Q2

A group of 6 people is to be chosen from 4 men and 11 women.

(a) In how many different ways can a group of 6 be chosen if it must contain exactly 1 man? [2]

Two of the 11 women are sisters Jane and Kate.

(b) In how many different ways can a group of 6 be chosen if Jane and Kate cannot both be in the group? [3]

28) OCT 2021_9709_52 Q4

(a) In how many different ways can the 9 letters of the word TELESCOPE be arranged? [2]

(b) In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the T and the C? [4]

29) OCT 2021_9709_53 Q1

The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is only room for 20 people to attend.

In how many ways can the 20 people be chosen from the 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [2]

30) OCT 2021_9709_53 Q5

A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from {A, B, C, D, E} and the digits are chosen from {1, 2, 3, 4, 5, 6, 7}. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed? [2]

(b) Find the number of different codes that include the letter A or the digit 5 or both. [3]

A security code is formed at random.

(c) Find the probability that the code is DE followed by a number between 4500 and 5000. [3]

31) OCT 2022_9709_51 Q6

A Social Club has 15 members, of whom 8 are men and 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. [4]

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? [3]

32) OCT 2022_9709_52 Q7

(a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

33) OCT 2022_9709_53 Q6

(a) Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

Five letters are selected at random from the 9 letters in the word ACTIVATED.

(c) Find the probability that the selection does **not** contain more Ts than As. [5]

MARKING SCHEME

1) SP-2020_9709_5Q6

(a)	[Two in same taxi:] $^6C_2 \times {}^4C_4 \times 2$ or $^6C_2 + {}^6C_4$	1	M1	$^6C_4$ or $^6C_2$ OE seen anywhere
		1	M1	'something' × 2 only or adding 2 equal terms
	= 30	1	A1	
		3		
(b)	[Mark, Jon and Sarah in taxi P:] $(^5C_1 \times 2 \times 2) \times {}^4P_4$	1	M1	$^5P_1$, $^5C_1$ or 5 seen anywhere
		1	M1	Multiply by 2 or 4 OE
		1	M1	Multiply by $^4P_4$ OE, e.g. 4! or 4 × $^3P_3$ or can be part of 5!
	= 480	1	A1	
		4		

2) MARCH 2020_9709_52 Q1

$^{38}C_r$ or $^nC_{34}$	M1	Either expression seen OE, no other terms, condone x1
$^{38}C_{34}$	A1	Correct unsimplified OE
73815	A1	If M0, SCB1 $^{38}C_{34}$ x k, k an integer
	3	

3) MARCH 2020_9709_52 Q4

(a)	R ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ R $\frac{9!}{3!6!}$	M1	9! Alone on numerator, 3! × k or 6! × k on denominator
	= 84	A1	
		2	
(b)	^ (B B B) ^ ^ ^ ^ ^	M1	$\frac{7!}{6!} \times k$ or $7k$ seen, k an integer > 0
	$\frac{7!}{6!} \times \frac{8 \times 7}{2}$	M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n=7, 8 or 9, m an integer > 0
		M1	n = 8 used in above expression
	= 196	A1	
	Alternative for question 4(b)		
	[Arrangements, blues together – Arrangements with blues together and reds together =] $\frac{9!}{2!6!} - \frac{8!}{6!}$	M1	9! Seen alone or as numerator with subtraction
	= [252 – 56]	M1	8! Seen alone or as numerator in a second term and no other terms
		M1	All terms divided by 6! x k, k an integer
	= 196	A1	
		4	

4) MARCH 2021_9709_52 Q6

(a)	$\frac{11!}{2!2!2!}$	M1	11! alone as numerator. 2! × m! × n! on denominator, m = 1, 2, n = 1, 2. no additional terms, no additional operations.
	4989600	A1	Exact answer only.
		2	

b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$	**B1**	$\frac{7!}{2!} \times k$ seen, k an integer > 1.
	Number of ways of placing As in non-adjacent places = ${}^8C_2$ $\frac{7!}{2!} \times {}^8C_2$	**M1**	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 7, 8 or 9, m an integer > 1.
		M1	$\frac{7!}{p!} \times {}^8C_2$ or $\frac{7!}{p!} \times {}^8P_2$, p integer ⩾ 1, condone 2520×28.
	= 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end = $\frac{9!}{2!2!}$	**M1**	$\frac{9!}{2!m!} - k$, 90720 > k integer > 1, m = 1, 2.
	Arrangements with R at ends and As together = $\frac{8!}{2!}$	**B1**	$s - \frac{8!}{2!}$, s an integer >1
	With As not together = $\frac{9!}{2!2!} - \frac{8!}{2!}$	**M1**	$\frac{9!}{p} - \frac{8!}{q}$, p, q integers ⩾ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	
(c)	**Method 1**		
	R R A L _ _ ${}^5C_2$ = 10 R R A L L̄ _ ${}^5C_1$ = 5 R R A A L _ ${}^5C_1$ = 5 R R A A L L̄ = 1	**M1**	${}^5C_x$ seen alone or ${}^5C_x \times k$, 2⩾ k ⩾ 1, k an integer, 0 < x < 5 linked to an appropriate scenario.
		A1	${}^5C_2 \times k$, k = 1 oe or ${}^5C_1 \times m$, m = 1,2 oe alone. **SC** if ${}^5C_x$ not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. ${}^2C_w \times {}^2C_x \times {}^2C_y \times {}^5C_z$, w+x+y+z=6 identifies w Rs, x As and y Ls.
	[Total =] 21	**A1**	WWW, only dependent on 2nd M mark. Note: ${}^5C_2 + {}^5C_1 + {}^5C_1 + 1 = 21$ is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = ${}^7C_2$	**M1**	${}^7C_x$ seen alone or ${}^7C_x \times k$, 2⩾ k⩾ 1, k an integer, 0<x<7. Condone ${}^7P_x$ or ${}^7P_x \times k$, 2⩾ k⩾ 1, k an integer, 0<x<7.
		M1	${}^7C_2 \times k$, 2⩾ k⩾ 1oe
		A1	${}^7C_2 \times k$, k = 1oe no other terms.
	[Total =] 21	**A1**	Value stated.
		4	

5) MARCH 2022_9709_52 Q5

(a)	${}^5C_1 \times {}^7C_4$	**M1**	${}^7C_4 \times k$, k integer ⩾ 1 Condone ${}^5P_1$ for M1 only
	175	**A1**	
		2	

(b)	2B 1G 2A $^3C_2 \times {^4C_1} \times {^5C_2} = 120$ 2B 2G 1A $^3C_2 \times {^4C_2} \times {^5C_1} = 90$ 2B 3G $^3C_2 \times {^4C_3} = 12$ 3B 1G 1A $^3C_3 \times {^4C_1} \times {^5C_1} = 20$ 3B 2G $^3C_3 \times {^4C_2} = 6$	M1	$^3C_x \times {^4C_y} \times {^5C_z}$, $x + y + z = 5$, x,y,z integers $\geqslant 1$ Condone use of permutations for this mark
		B1	2 appropriate identified outcomes correct, allow unsimplified
		M1	Summing *their* values for 4 or 5 correct identified scenarios only (no repeats or additional scenarios), condone identification by unsimplified expressions
	[Total =] 248	A1	Note: Only dependent upon M marks
		4	
(c)	$8! \times 3! \times {^5P_2}$	M1	$8! \times m$, m an integer $\geqslant 1$ Accept $8 \times 7!$ for 8!
		M1	$3! \times n$, n an integer > 1
		M1	$p \times {^5P_2}$, $p \times {^5C_2} \times 2$, $p \times 20$, p an integer > 1 If extra terms present, maximum 2/3 M marks available
	4 838 400	A1	Exact value required
		4	

6) MARCH 2023_9709_52 Q7

(a)	**Method 1: Arrangements with 3 Es together – arrangements with 3 Es together and 2 Ds together**		
	$\frac{7!}{2!} - 6!$	B1	$\frac{7!}{2!} - e$, e a positive integer (including 0).
		M1	$f - 6!$, $f > 6!$
		M1	$\frac{7!}{a!b!} - \frac{6!}{c!d!}$, $a,c = 1, 2$ and $b,d = 1, 3$.
	1800	A1	
	Method 2: Identified scenarios ^ EEE ^ ^ ^		
	$5! \times \frac{6 \times 5}{2}$	B1	$5! \times j$, j a positive integer ($j = 1$ may be implied).
		M1	$\frac{k!}{m!} \times \frac{6 \times 5}{2}$, $\frac{k!}{m!} \times {^6C_2}$, $\frac{k!}{m!} \times \frac{^6P_2}{2}$ or $k! \times \frac{7 \times 6}{n}$, k a positive integer ($k = 1$ may be implied), $m = 1, 2$ $n = 1, 2, 3$.
		M1	$k! \times \frac{m \times (m-1)}{n}$ k a positive integer > 1, $m = 10, 9, 8, 7, 6$ and $n = 1, 2$.
	1800	A1	
		4	

b)	**First 2 marks: Method 1 – Number of arrangements with 2 Ds in one position with 4 letters in between – repeats allowed**		
	$7! \times 4 \times 2$	M1	$7! \times s$, s = positive integer > 1.
		M1	$t! \times 4 \times 2$, $t = 8, 7, 6$. Condone $t! \times 8$.
	First 2 marks: Method 2 – Picking 2Ds, arranging 4 letters from remaining letters between and then arranging terms		
	$^7P_4 \times 4! \times 2!$	M1	$^7P_4 \times a! \times b!$, $1 \leqslant a \leqslant 6$ and $b = 1, 2, 3$.
		M1	$^7P_c \times 4! \times 2!$, $c = 3, 4, 5$.
	First 2 marks: Method 3 – Identified scenarios involving Es between Ds		
	D ^ ^ ^ ^ D E E E = $^4C_4 \times 4! \times 4! \times 2! = 1152$ D E ^ ^ ^ D E E ^ = $^4C_3 \times 4! \times 4! \times 3 \times 2! = 13824$ D E E ^ ^ D E ^ ^ = $^4C_2 \times 4! \times 4! \times 3 \times 2! = 20736$ D E E E ^ D ^ ^ ^ = $^4C_1 \times 4! \times 4! \times 2! = 4608$	M1	1 identified scenario value correct.
		M1	4 appropriate scenarios added, no incorrect.

b)	**Final 3 marks for Methods 1, 2 and 3**		
	40320	**A1**	If A0 scored, **SC B1** for 40320 WWW.
	[Total number of arrangements =] $[9! =]$ 362880	**B1**	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\frac{40320}{362880} = \frac{1}{9}$	**B1FT**	$\frac{\textit{their}\ 40320}{\textit{their}\ 362880}$, accept unsimplified. B1FT if *their* 40320 and *their* 362880 supported by work in this part. Condone *their* 362880 supported by calculation in **7(a)**.
		5	
c)	Scenarios D E _ _ _ $\quad {}^4C_3 \quad$ 4 D E E _ _ $\quad {}^4C_2 \quad$ 6 D E E E _ $\quad {}^4C_1 \quad$ 4 D D E _ _ $\quad {}^4C_2 \quad$ 6 D D E E _ $\quad {}^4C_1 \quad$ 4 D D E E E $\quad [{}^4C_0] \quad$ 1	**B1**	1 correct unsimplified outcome/value for one identified scenario excluding DDEEE. Note: ${}^4C_1$ cannot be used for ${}^4C_3$.
		M1	Add values of 6 appropriate scenarios, no additional, incorrect or repeated scenarios. Accept unsimplified.
	[Total =] 25	**A1**	
		3	

7) JUNE 2020_9709_51 Q2

(a)	6!	**M1**
	720	**A1**
		2
(b)	Total number: $\frac{9!}{3!2!}(30240)$	**M1**
	Number with Ls together = $\frac{8!}{3!}(6720)$	**M1**
	Number with Ls not together = $\frac{9!}{3!2!} - \frac{8!}{3!}$ $= 30\,240 - 6720$	**M1**
	23 520	**A1**
	Alternative method for question 2(b)	
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	
	$7! \times k$ in numerator, k integer ≥ 1	**M1**
	$8 \times 7 \times m$ in numerator or 8C2 $\times$ m, m integer ≥ 1	**M1**
	3! in denominator	**M1**
	23 520	**A1**
		4

8) JUNE 2020_9709_51 Q4

Scenarios: 2P 3V 2G $\quad {}^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $\quad {}^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $\quad {}^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $\quad {}^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (M1 for ${}^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	M1
Two unsimplified products correct	B1
Summing the number of ways for 3 or 4 correct scenarios	M1
Total: 13 720	A1
	4

9) JUNE 2020_9709_52 Q6

(a)	$\frac{8!}{3!}$	M1
	6720	A1
		2

(b)	Total number = $\frac{10!}{2!3!}$ (302400) (A)	B1
	With Es together = $\frac{9!}{3!}$ (60480) (B)	B1
	Es not together = *their* (A) – *their* (B)	M1
	241920	A1
	Alternative method for question 6(b)	
	_^_^_^_^_^_^_^_^_^_ $\frac{8!}{3!} \times \frac{9 \times 8}{2}$	
	$8! \times k$ in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	B1
	$3! \times m$ in denominator, m integer $\geqslant 1$	B1
	Their $\frac{8!}{3!}$ Multiplied by ${}^9C_2$ (OE) only (no additional terms)	M1
	241920	A1
		4
6(c)	Scenarios: E M M M $\quad {}^5C_0 = 1$ E M M _ $\quad {}^5C_1 = 5$ E M _ _ $\quad {}^5C_2 = 10$	M1
	Summing the number of ways for 2 or 3 correct scenarios	M1
	Total = 16	A1
		3

10) JUNE 2020_9709_53 Q7

7(a)	$\frac{9!}{2!2!} = 90\,720$	B1
		1
7(b)	$\frac{6!}{2!}$	M1
	360	A1
		2

(c)	2 Es together = $\frac{8!}{2!}(=20160)$	M1
	Es not together = 90720 – 20160 = 70560	M1
	Probability = $\frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
	Alternative method for question 7(c)	
	_^_^_^_^_^_^_^_ $\frac{7!}{2!}\times\frac{8\times7}{2}=70560$	
	$7!\times k$ in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	M1
	Multiplying by $^8C_2$ OE	M1
	Probability = $\frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
		4

11) JUNE 2021_9709_51 Q1

RRRRB $^8C_4\times{}^4C_1=280$ BBBBR $^8C_1\times{}^4C_4=8$ RRRRR $^8C_5=56$	M1	$^8C_x\times{}^4C_y$ with $x+y=5$. x, y both integers, $1\leqslant x\leqslant 5$, $0\leqslant y\leqslant 4$ condone $^8C_1\times 1$
	A1	Two correct outcomes evaluated
	M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
[Total =] 344	A1	WWW, only dependent on 2nd M mark
	4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

12) JUNE 2021_9709_51 Q3

(a)	$\left[\frac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
(b)	_ _ _ L E D _ _ : With LED together: $\frac{6!}{2!}$	M1	$\frac{6!}{k}$ or $\frac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\frac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\frac{6!}{3!}$ [= 120]	*M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 6, 7 or 8, m an integer > 0
	[Probability =] $\frac{their\ 5040}{their\ 6720}$	DM1	Denominator = *their* (a) or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\frac{7! \times 2}{3!}$ [= 5040]	*M1	*Their* 6720 – k, k a positive integer
		*M1	$(m -)\frac{7! \times k}{3!}, k = 1, 2$
	[Probability =] $\frac{their\ 5040}{their\ 6720}$	DM1	With denominator = *their* (a) or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using '1 – Probability of arrangements with A and D together': $\frac{7! \times 2}{3!}$ [= 1680]	*M1	$\frac{7! \times k}{3!}, k = 1, 2$
	[Probability =] $\frac{their\ 1680}{their\ 6720}$	*M1	With denominator = *their* (a) or correct
	$1 - \frac{their\ 1680}{their\ 6720}$	DM1	$1 - m$, $0 < m < 1$, dependent on at least one M mark already gained
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
		4	

13) JUNE 2021_9709_52 Q6

i(a)	$\frac{8!}{2!3!}$	M1	$\frac{8!}{k! \times m!}$ k = 1 or 2, m = 1 or 3, not $k = m = 1$ no additional terms
	3360	A1	
		2	

(b)	**Method 1** Arrangements Rs at ends – Arrangements Rs at ends and Os together		
	[Os not together =] $\frac{6!}{3!} - 4!$	M1	$\frac{6!}{k!} - m$, $1 \leqslant k \leqslant 3$, m an integer, condone $2\times\left(\frac{6!}{k!}\right) - m$.
		M1	$w - 4!$ or $w - 24$, w an integer Condone $w - 2 \times 4!$
	96	A1	
	Method 2 identified scenarios R _ _ _ R, Arrangement No Os together + 2Os and a single O		
	${}^4C_3 \times 3! + {}^4C_2 \times 2 \times 3!$	M1	${}^4C_3 \times 3! + r$ or $4 \times 3! + r$ or ${}^4P_3 \times 3! + r$, r an integer. Condone $2 \times {}^4C_3 \times 3! + r$. $2 \times 4 \times 3! + r$ or $2 \times {}^4P_3 \times 3! + r$.
		M1	$q + {}^4C_2 \times 3! \times k$ or $q + {}^4P_2 \times 3! \times k$, $k = 1,2$, q an integer
	[24 + 72 =] 96	A1	
		3	
(c)	**Method 1** Identified scenarios		
	OORR $\;{}^3C_2 \times {}^2C_2 \times \left[{}^3C_0\right] = 3 \times 1 = 3$ ORR_ $\;{}^3C_1 \times {}^2C_2 \times {}^3C_1 = 3 \times 1 \times 3 = 9$ OOR_ $\;{}^3C_2 \times {}^2C_1 \times {}^3C_1 = 3 \times 2 \times 3 = 18$ OR_ _ $\;{}^3C_1 \times {}^2C_1 \times {}^3C_2 = 3 \times 2 \times 3 = 18$ OOOR $\;{}^3C_3 \times {}^2C_1 \times \left[{}^3C_0\right] = 1 \times 2 = 2$	B1	Outcomes for 2 identifiable scenarios correct, accept unsimplified.
		M1	Add 4 or 5 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{{}^8C_4}$	M1	$\frac{\textit{their}\text{'}50\text{'}}{{}^8C_4}$, accept numerator unevaluated

(c) cont'd	$\frac{50}{70}$ or 0.714	A1	
	Method 2 Identified outcomes		
	ORTM $\;{}^3C_1 \times {}^2C_1 = 6$ ORTW $\;{}^3C_1 \times {}^2C_1 = 6$ ORMW $\;{}^3C_1 \times {}^2C_1 = 6$ ORRM $\;{}^3C_1 \times {}^2C_2 = 3$ ORRW $\;{}^3C_1 \times {}^2C_2 = 3$ ORRT $\;{}^3C_1 \times {}^2C_2 = 3$ OROR $\;{}^3C_2 \times {}^2C_2 = 3$ OROT $\;{}^3C_2 \times {}^2C_1 = 6$ OROM $\;{}^3C_2 \times {}^2C_1 = 6$ OROW $\;{}^3C_2 \times {}^2C_1 = 6$ OROO $\;{}^3C_3 \times {}^2C_1 = 2$	B1	Outcomes for 5 identifiable scenarios correct, accept unsimplified.
		M1	Add 9, 10 or 11 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{{}^8C_4}$	M1	$\frac{\textit{their}\text{'}50\text{'}}{{}^8C_4}$, accept numerator unevaluated.
	$\frac{50}{70}$ or 0.714	A1	
		5	

14) JUNE 2021_9709_53 Q6

Question	Answer	Marks	Guidance
(a)	$\frac{11!}{2!3!}$	M1	11! alone on numerator – must be a fraction. $k! \times m!$ on denominator, $k = 1, 2$, $m = 1, 3$, 1 can be implied but cannot both = 1. No additional terms
	3326400	A1	Exact value only
		2	
(b)	8! = 40320	B1	Evaluate, exact value only
		1	
(c)	$\frac{9!}{3!} \times 7$	M1	$\frac{9!}{3!} \times k$ seen, k an integer > 0, no +, – or ÷
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	${}^9C_3 \times 7! \left(\times \frac{3!}{3!}\right)$	M1	$9C3 \times k$ seen, k an integer > 0, no + or –
		M1	$7! \times k$ seen, , k an integer > 0, no + or –
	423360	A1	Exact value only but there must be evidence of $\times\frac{3!}{3!}$
(c)	**Alternative method for Question 6(c)**		
	$3 \times 7 \times \frac{8!}{2!}$	M1	$3 \times \frac{8!}{2!} \times k$ seen, k an integer > 0, no + or –
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	$7 \times \frac{2}{11} \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8} \times \frac{1}{7} \times$ total no. of arrangements	M1	Product of correct five fractions × k seen, k an integer > 0, no + or –
		M1	7×'total no of arrangements' ×k seen, k an integer > 0, no + or –
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	No E between the Rs – $\frac{{}^6C_3 \times 3! \times 7!}{3!} = 100800$ 1E between the Rs – $\frac{{}^6C_2 \times 3! \times 7!}{2!} = 226800$ 2Es between the Rs – ${}^6C_1 \times 3! \times 7! = 90720$ 3Es between the Rs – 7! = 5040	M1	Finding the correct number of ways for no, 1 or 2 Es between the Rs, accept unsimplified.
		M1	Adding the number of ways for 3 or 4 correct scenarios
	$[\text{Total} = 7! \times (20 + 45 + 18 + 1) = 7! \times 84 =]\, 423360$	A1	CAO
		3	

(d)	E E R _ _ $^6C_2 = 15$ E E R $\bar{R}$ _ $^6C_1 = 6$ E E E R _ $^6C_1 = 6$ E E E R $\bar{R}$ $^6C_0 = 1$	M1	Identifying four correct scenarios only.
		B1	Correct number of selections unsimplified for 2 or more scenario.
		M1	Adding the number of selections for 3 or 4 identified correct scenarios only, accept unsimplified. $^3C_x \times {}^2C_y \times {}^6C_z$, $x+y+z=5$ correctly identifies x Es and y Rs
	[Total =] 28	A1	WWW, only dependent upon 2nd M mark.
	Alternative method for Question 6(d) – Fixing EER first. No other scenarios can be present anywhere in solution.		
	E E R ^ ^ = $^8C_2$	M1	^8C_x seen alone or $^8C_x \times k$, , $k = 1$ or 2, $0<x<8$ Condone ^8P_x or $^8P_x \times k$, $k = 1$ or 2, $0<x<8$
		B1	$^8C_2 \times k$, $k = 1$ or 2 OE
		M1	$^8C_2 \times k$, $k = 1$ OE and no other terms
	[Total =] 28	A1	Value stated
		4	

15) JUNE 2022_9709_51 Q1

(a)	5!	M1	$k!$ where $k = 5, 6$ or 7 Condone × 1 OE
	120	A1	
		2	
(b)	[Total no of ways =] $\frac{8!}{2!3!}$ [= 3360]	M1	$\frac{8!}{a!b!}$, $a = 1,2$ $b = 1,3$ $a \neq b$
	[With 3Es together =] $\frac{6!}{2!}$ [= 360]	M1	$\frac{6!}{c!}$, $c = 1,2$ seen in an addition/subtraction
	[With 3Es not together] = 3360 – 360	M1	$\frac{8!}{d!e!} - \frac{6!}{f!}$ where $d, f = 1,2$ & $e = 1,3$
	3000	A1	
		4	

16) JUNE 2022_9709_51 Q2

(a)	$^{12}C_4 \times 2$	M1	$^gC_4 \times h$ $g = 12, 13$, $h = 1,2$
	990	A1	
	Alternative method for question 2(a)		
	[total – both on – neither on] $^{14}C_5 - ({}^{12}C_3 + {}^{12}C_5) = [2002 - 220 - 792]$	M1	$^kC_5 - ({}^aC_3 + {}^aC_5)$ $a = 12, 13$ and $k = 13, 14$
	990	A1	
		2	
(b)	[Mrs Lan plus] 2W 2M $^7C_2 \times {}^6C_2$ = 315 3W 1M $^7C_3 \times {}^6C_1$ = 210 4W $^7C_4$ = 35	M1	$^7C_r \times {}^6C_{4-r}$ for $r = 2, 3$ or 4
		B1	Outcome for one identifiable scenario correct, accept unevaluated
		M1	Add outcomes for 3 identifiable correct scenarios Note: if scenarios not labelled, they may be identified by seeing $^7C_r \times {}^6C_s$ $r + s = 4$ to imply r women and s men for both **B & M** marks only
	[Total =] 560	A1	
		4	

17) JUNE 2022_9709_52 Q6

(a)	$\left[\frac{9!}{2!2!}=\right]$ 90 720	B1	
		1	
(b)	**Method 1** Arrangements Cs at ends – Arrangements Cs at ends and Os together		
	[Os not together =] $\frac{7!}{2!}-6!$ [= 2520 – 720]	M1	$\frac{w!}{2!}-y$, $w = 6, 7$ y an integer. Condone $2\times\left(\frac{w!}{2!}\right)-y$.
		M1	$a - 6!$ or $a - 720$, a an integer resulting in a positive answer.
	1800	A1	
	Method 2 identified scenarios R ^ ^ ^ R		
	[Os not together =] $5!\times\frac{6\times5}{2!}=$	M1	$5!\times b$, b integer >1.
		M1	$c\times\left(\frac{6\times5}{2!}\text{or}\,{}^6C_2\,\text{or}\,\frac{{}^6P_2}{2!}\text{or}\,15\right)$, c integer > 1.
	1800	A1	
		3	

18) JUNE 2023_9709_51 Q2

(a)	${}^6C_3\times{}^8C_3$	M1	${}^6C_3\times b$ or $c\times{}^8C_3$ seen. b, c integers $\geqslant 1$ (1 may be implied).
	1120	A1	
		2	
(b)	**Method 1**		
	0 brothers [${}^3C_0$] × ${}^{11}C_6$ 462 1 brother ${}^3C_1\times{}^{11}C_5$ 1386 2 brothers ${}^3C_2\times{}^{11}C_4$ 990	B1	${}^3C_x\times{}^{11}C_{6-x}$, with x = 1 or 2 seen.
		M1	Add values of 3 correct scenarios, (may be identified by the appropriate calculations) no incorrect/repeated scenarios, condone use of permutations.
	2838	A1	Only dependent on the M mark. SC B1 for the correct calculation or 2838 seen WWW.
	Method 2		
	${}^{14}C_6-{}^{11}C_3$ 3003 – 165	B1	${}^{14}C_6-d$, where d a positive integer.
		M1	$e-{}^{11}C_3$, where e is a positive integer >165.
	= 2838	A1	
		3	

19) JUNE 2023_9709_51 Q3

a)	$\left[\frac{8!}{2!3!}=\right]$ 3360	B1	
		1	
b)	$\frac{6!}{2!2!}$	M1	$\frac{6!}{2!\,f!}$; f = 1, 2, 3.
	180	A1	
		2	
c)	$\left[P(OOO\mid CC)=\frac{P(OOO\cap CC)}{P(CC)}=\right]$ $\frac{5!}{\frac{7!}{3!}}$	M1	$\frac{5!}{g}$ g a positive integer, $g \neq 3360, 1$. Condone numerator of $\frac{5!}{3360g}$.
		M1	$\frac{h}{\frac{7!}{3!}}$ or $\frac{h}{\frac{8!}{3!}}$, where h is a positive integer. Condone division by 3360 in denominator.
	$=\frac{120}{840}, \frac{1}{7}, 0.143$	A1	0.1428571… to at least 3SF. If M0 scored SC **B1** for $\frac{1}{7}$ WWW.
		3	

20) JUNE 2023_9709_52 Q6

(a)	S + 4C + 2R $^6C_1 \times {}^8C_4 \times {}^{11}C_2$ [= 6 × 70 × 55] = 23 100 S + 5C + 1R $^6C_1 \times {}^8C_5 \times {}^{11}C_1$ [= 6 × 56 × 11] = 3696 S + 6C [+ 0R] $^6C_1 \times {}^8C_6$ [$\times {}^{11}C_0$][= 6 × 28] = 168	M1	$^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ seen.
		B1	Correct outcome/value for 1 identified scenario, accept unsimplified, www.
		M1	Add values of 3 correct scenarios. No incorrect scenarios, no repeated scenarios. Condone $^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ to identify S, C, R.
	[Total =] 26964	A1	cao
		4	
(b)	2! × 3! × 4! × 6	M1	2! × 3! × 4! × k, k an integer > 0. 1 can be implied.
	=1728	A1	If A0 scored SC **B1** for 1728 www.
		2	

Question	Answer	Marks	Guidance
(c)	**Method 1**		
	$6! \times 7 \times 6 \times 5$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 7 \times n \times r$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant n, r \leqslant 6$, $n \neq r$.
		M1	$\frac{m!}{a! \times b!} \times 7 \times 6 \times 5$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 2		
	$6! \times {}^7P_3$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_q$, or $\frac{m!}{a! \times b!} \times {}^7C_q \times q!$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$; $1 \leq q \leq 6$.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_3$, or $\frac{m!}{a! \times b!} \times {}^7C_3 \times 3!$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
c)	**Method 3**		
	$6! \times 35 \times 3!$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 35 \times q!$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant q \leqslant 3$.
		M1	$\frac{m!}{a! \times b!} \times 35 \times 6$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 4		
	$9! - 7!3! - {}^3P_2 \times 6! \times 7 \times 6$ Or $9! - 7!3! - 3! \times 7! \times 6$ [= 362 880 – 30 240 – 181 440]	**M1**	$9! - 7!r! - q$, r an integer > 1, q an integer $\leqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - q$; $s = 8, 9$; $a = 1, 2$; $b = 1, 3$; $c = 1, 4$; q an integer $\geqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - {}^3P_2 \times 6! \times 6 \times 7$, $6 \leqslant s \leqslant 9$, or $\frac{s!}{a! \times b! \times c!} - 7!3! - 3! \times 7! \times 6$, $6 \leqslant s \leqslant 9$. $a = 1, 2$ $b = 1, 3$ $c = 1, 4$. 1 may be implied.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
		4	

21) JUNE 2023_9709_53 Q7

(a)	**Method 1: Total number of arrangements – number of arrangements with Cs together**		
	$\frac{10!}{2!4!} - \frac{9!}{4!}$ [75600-15120]	**M1**	$\frac{10!}{a!b!} - c$, $a \neq b$, $a = 1, 2$, $b = 1, 4$, with c being a positive integer.
		M1	$d - \frac{e!}{4!}$, $e = 8, 9, 10$, with d being a positive integer.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
	Method 2: Arrangements ^ ^ C ^ C ^ ^ ^ ^ ^		
	$\frac{8!}{4!} \times \frac{9 \times 8}{2}$	**M1**	$\frac{8!}{4!} \times f$ seen, with f being a positive integer.
		M1	$g \times \frac{9 \times 8}{h}$, with g being a positive integer, $h = 1, 2$. $g \times {}^9C_2$ and $g \times {}^9P_2$ are acceptable.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
		3	
(b)	AC^^^C^^^A $\frac{6!}{2!} \times 4$	**M1**	$\frac{6!}{2!} \times s$, with s being a positive integer.
		M1	$\frac{t!}{r!} \times 4$, $r = 1, 2, 3$ and $t = 8, 7, 6$.
	1440	**A1**	
	Alternative Method for Question 7(b)		
	$\frac{4 \times {}^6P_3 \times 3!}{2!}$	**M1**	$\frac{{}^6P_3}{2!} \times k$, with k being a positive integer.
		M1	$4 \times 3! \times \frac{{}^6P_m}{n!}$, $m = 2, 3$ and $n = 1, 2, 3$.
	1440	**A1**	
		3	
(c)	Scenarios AA _ _ _ $\quad {}^5C_3 = 10$ AAA _ _ $\quad {}^5C_2 = 10$ AAAA _ $\quad {}^5C_1 = 5$	**B1**	Correct number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www.
		M1	Add 3 values for 2, 3 and 4 As, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
	Alternative Method 2 for Question 7(c)		
	Scenarios: AAC _ _ $\quad {}^4C_2 = 6$ AA _ _ _ $\quad {}^4C_3 = 4$ AAAC _ $\quad {}^4C_1 = 4$ AAA _ _ $\quad {}^4C_2 = 6$ AAAAC $\quad 1$ AAAA _ $\quad 4$	**B1**	Correct total number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www (e.g., both values for AAC^^ and AA^^^ shown would be fine for 2As).
		M1	Add 6 values of appropriate scenarios only, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
		3	

22) OCT 2020_9709_51 Q7

(a)	$\frac{8!}{2!}$	**M1**	$\frac{8!}{k} = \frac{7! \times 8}{k}$, where $k \in \mathbb{N}$, $\frac{a!}{2(!)}$, where $a \in \mathbb{N}$
	20160	**A1**	
		2	

(b)	Total number of ways: $\frac{10!}{2!3!}$ $(=302\,400)$ (A)	B1	Accept unsimplified
	With Ps together: $\frac{9!}{3!}$ (= 60 480) (B)	B1	Accept unsimplified
	With Ps not together: 302 400 – 60 480	M1	$\frac{10!}{m}-\frac{9!}{n}$, m, n integers or (A) – (B) if clearly identified
	241 920	A1	
	Alternative method for question 7(b)		
	$\frac{8!}{3!}$	B1	$k\times 8!$ in numerator, k a positive integer, no ±
		B1	$m\times 3!$ in denominator, m a positive integer, no ±
	$\times\frac{9\times 8}{2}$	M1	*Their* $\frac{8!}{3!}$ multiplied by $^9C_2$ or $^9P_2$ no additional terms
	241 920	A1	Exact value, WWW
		4	
(c)	Probability = $\frac{\text{Number of ways Es at beginning and end}}{\text{Total number of ways}}$ Probability = $\frac{\frac{8!}{2!}}{\frac{10!}{2!\times 3!}}=\frac{20160}{302\,400}$	M1	$\frac{\left(\frac{8!}{k!}\right)}{\frac{10!}{k!l!}}$ $1\leqslant k, l\in \mathbb{N}\leqslant 3$, FT denominator from 7(b) or correct
	$\frac{1}{15}$, 0·0667	A1	
	Alternative method for question 7(c)		
	Probability = $\frac{3}{10}\times\frac{2}{9}$	M1	$\frac{a}{10}\times\frac{a-1}{9} a=3,2$
	$\frac{1}{15}$, 0·0667	A1	
	Alternative method for question 7(c)		
	Probability = $\frac{1}{10}\times\frac{1}{9}\times 3!$	M1	$\frac{1}{10}\times\frac{1}{9}\times m!, m=3,2$
	$\frac{1}{15}$, 0.0667	A1	
		2	

23) OCT2020_9709_52 Q6

a)	${}^9C_6\ (\times\ {}^3C_3)$	M1	${}^9C_k \times n$, $k = 6, 3$, $n = 1,2$ oe Condone ${}^9C_6 + {}^3C_3$, ${}^9P_6 \times {}^3P_3$
	84	A1	Accept unevaluated.
		2	
b)	Number with 3 Baker children = ${}^6C_2$ or 15	B1	Correct seen anywhere, not multiplied or added
	Total no of selections = ${}^9C_5$ or 126 Probability = $\frac{\text{number of selections with 3 Baker children}}{\text{total number of selections}}$	M1	Seen as denominator of fraction
	$\frac{15}{126}$, 0·119	A1	OE, e.g. $\frac{5}{42}$
	Alternative method for question 6(b)		
	$\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\left(\times\frac{6}{6}\right)\left(\times\frac{5}{5}\right)\times{}^5C_3$	B1	${}^5C_3$ (OE) or 10 seen anywhere, multiplied by fractions only, not added
		M1	$\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\left(\times\frac{6}{6}\right)\left(\times\frac{5}{5}\right)\times k$, $1 \leqslant k$, k integer
	$\frac{15}{126}$, 0·119	A1	OE, e.g. $\frac{5}{42}$
		3	
c)	[Total no of arrangements = 9!] [Arrangements with men together = 8! × 2] Not together: 9! –	M1	$9! - k$ or $362880 - k$, k an integer<362 880
	8! × 2	B1	8! × 2(!) or 80 640 seen anywhere
	282 240	A1	Exact value
	Alternative method for question 6(c)		
	7! × 8 × 7	B1	$7! \times k$, k positive integer > 1
		M1	$m \times 8 \times 7$, $m \times {}^8P_2$, $m \times {}^8C_2$ m positive integer > 1
	282 240	A1	Exact value
		3	
d)	7! × 2 × 7	M1	$7! \times k$, k positive integer > 1 If 7! not seen, condone $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times (1) \times k$ *or* $7 \times 6! \times k$ only
		M1	$m \times 2 \times 7$, m positive integer > 1
	70 560	A1	
		3	

24) OCT 2020_9709_53 Q3

3(a)	Scenarios: 6W 0M $^9C_6 = 84$ 5W 1M $^9C_5 \times {}^5C_1 = 126 \times 5 = 630$ 4W 2M $^9C_4 \times {}^5C_2 = 126 \times 10 = 1260$	M1	Correct number of ways for either 5 or 4 women, accept unsimplified
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios.
	Total = 1974	A1	
		3	
3(b)	Total number of ways = $^{14}C_6$ (3003) Number with sister and brother = $^{12}C_4$ (495) Number required = $^{14}C_6 -$	M1	$^{14}C_6$ – a value
	$^{12}C_4 = 3003 - 495$	M1	$^{12}C_x$ or $^nC_4$ seen on its own or subtracted from *their* total, $x \leqslant 6$, $n \leqslant 13$
	2508	A1	
	Alternative method for question 3(b)		
	Number of ways with neither = $^{12}C_6 = 924$	M1	$^{12}C_6$ + a value
	Number of ways with either brother or sister (not both) $= {}^{12}C_5 \times 2$ (= 792 × 2) = 1584	M1	$^{12}C_x \times 2$ or $^nC_5 \times 2$ seen on its own or added to *their* number of ways with neither, $x \leqslant 5$, $n \leqslant 12$
	Number required = 924 + 1584 = 2508	A1	
		3	

25) OCT 2020_9709_53 Q5

5(a)	Total number of ways = $\frac{8!}{3!2!}$ (= 3360)	B1	Correct unsimplified expression for total number of ways
	Number of ways with V and E in correct positions = $\frac{6!}{2!\times 2!}$ (= 180)	B1	$\frac{6!}{2!\times 2!}$ alone or as numerator in an attempt to find the number of ways with V and E in correct positions. No ×,±
	Probability = $\frac{180}{3360}$ $\left(=\frac{3}{56}\right)$ or 0.0536	B1 FT	Final answer from *their* $\frac{6!}{2!\times 2!}$ divided by *their* total number of ways
	Alternative method for question 5(a)		
	$\frac{1}{8}\times\frac{3}{7}$	M1	$\frac{a}{8}\times\frac{b}{7}$ seen, no other terms (correct denominators)
		M1	$\frac{1}{c}\times\frac{3}{d}$ seen, no other terms (correct numerators)
	$\frac{3}{56}$ or 0.0536	A1	
		3	

(b)	Rs together and Es together: 5! (120)	B1	Alone or as numerator of probability to represent the number of ways with Rs and Es together, no ×, +, –
	Es together: $\frac{6!}{2!}(=360)$	B1	Alone or as denominator of probability to represent the number of ways with Es together, no ×, + or –
	Probability = $\frac{5!}{\frac{6!}{2!}}$	M1	$\frac{their\ 5!}{their\ \frac{6!}{2!}}$ seen
	$\frac{1}{3}$	A1	OE
	Alternative method for question 5(b)		
	P(Rs together and Es together): $\frac{5!}{their \text{ total number of ways}}\left(=\frac{1}{28}\right)$	B1	
	P(Es together): $\frac{\frac{6!}{2!}}{their \text{ total number of ways}}\left(=\frac{3}{28}\right)$	B1	Alone or as numerator of probability to represent the P(Rs and Es together), no ×, +, –
	Probability = $\frac{\frac{1}{28}}{\frac{3}{28}}$	M1	Alone or as denominator of probability to represent the P(Es together), no ×, + or –
	$\frac{1}{3}$	A1	OE, $\frac{their\ \frac{1}{28}}{their\ \frac{3}{28}}$ seen
		4	

26) OCT 2021_9709_51 Q5

(a)	[8! =] 40 320	B1	Evaluated, exact value only.
		1	
(b)	**Method 1** [^ ^ ^ R ^ ^ S ^ ^]		
	$7! \times {}^8C_2 \times 2$	M1	$7! \times k$ seen, k an integer > 1.
		M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, $n = 7, 8$ or 9, m an integer > 1.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
	Method 2 [Total number of arrangements – Arrangements with R & S together]		
	$9! - 8! \times 2$	M1	$9! - k$, k an integer < 362 880.
		M1	$m - 8! \times n$, m an integer > 40 320, $n = 1, 2$.
	282 240	A1	Exact value only. **SC B1** for final answer 282 240 WWW.
		3	
(c)	${}^9C_5$ [$\times {}^4C_4$]	M1	${}^9C_x$ [$\times {}^{9-x}C_{9-x}$] $x = 4, 5$. Condone × 1 for ${}^{9-x}C_{9-x}$. Condone use of P.
	126	A1	WWW
		2	

(d)	[Number of ways with Raman and Sanjay together on back row =] ${}^7C_3$ [Number of ways with Raman and Sanjay together on front row =] ${}^7C_2$	M1	${}^7C_x$ seen, $x = 3$ or 2.
	[Total =] 35 + 21	M1	Summing two correct scenarios.
	56	A1	Evaluated – may be seen used in probability. If **M0** scored, **SC B1** for 56 WWW.
	Probability = $\frac{their\,56}{their(c)} = \frac{56}{126}, \frac{4}{9}$, 0.444	B1 FT	FT *their* 56 from adding 2 or more scenarios in numerator and *their* (c) or correct as denominator.
		4	

27) OCT 2021_9709_52 Q2

(a)	${}^{11}C_5 \times {}^4C_1$	M1	${}^{11}C_5 \times {}^4C_1$ condone ${}^{11}P_5 \times {}^4P_1$ no +, –, × or ÷.
	1848	A1	CAO as exact.
		2	
(b)	**Method 1** [Identifying scenarios]		
	[Neither selected =] ${}^{13}C_6$ [= 1716] [Only Jane selected =] ${}^{13}C_5$ [= 1287] [Only Kate selected =] ${}^{13}C_5$ [= 1287]	M1	Either ${}^{13}C_6$ seen alone or ${}^{13}C_5$ seen alone or × 2 (condone ${}^{13}P_n$, $n = 5,6$).
	[Total =] 1716 + 1287 + 1287	M1	Three correct scenarios only added, accept unsimplified (values may be incorrect).
	4290	A1	
	Method 2 [Total number of selections – selections with Jane and Kate both picked]		
	${}^{15}C_6 - {}^{13}C_4$ [= 5005 – 715]	M1	${}^{15}C_6 - k$, k a positive integer < 5005, condone ${}^{15}P_6$.
		M1	$m - {}^{13}C_4$, m integer > 715, condone $n - {}^{13}P_4$, $n > 17\,160$.
	4290	A1	
		3	
			SC Where the condition of **2(a)** is also applied in **2(b)**, the final answer is 1512 **SC M1 M1 A0** max. The method marks can be earned for the equivalent stages in each method. Method 1 ${}^4C_1 \times {}^9C_5 + {}^4C_1 \times {}^9C_4 \times 2$ Method 2 ${}^4C_1 \times {}^{11}C_5 - {}^4C_1 \times {}^9C_3$

28) OCT 2021_9709_52 Q4

(a)	$\frac{9!}{3!}$	M1	$\frac{9!}{e!}$, $e = 2, 3$
	60 480	A1	
		2	
(b)	$\frac{7!}{3!}\times 2\times 6$	M1	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times 2\times q$ $7 \leqslant m \leqslant 9, 1 \leqslant n \leqslant 3, 1 \leqslant q \leqslant 8$ all integers.
		M1	$\frac{m!}{n!}\times p\times 6$ $7 \leqslant m \leqslant 9, 1 \leqslant n \leqslant 3, 1 \leqslant p \leqslant 2$ all integers. (Accept 3P2 for 6) If **M0 M0 M0** awarded, **SC M1** for $t\times 12$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.
	Alternative method for question 4(b)		
	$\frac{{}^7P_2\times 6!\times 2}{3!}$	M1	$\frac{6!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times {}^7P_2\times q$ $m = 6,9, 1 \leqslant n \leqslant 3, 1 \leqslant q \leqslant 2$ all integers.
		M1	$\frac{m!}{n!}\times {}^7P_r\times 2$ $m = 6, 9, 1 \leqslant n \leqslant 3, 1 \leqslant r \leqslant 5$ all integers. If **M0 M0 M0** awarded, **SC M1** for $t\times 84$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.
4(b)	**Alternative method for question 4(b)**		
	$\frac{7!}{3!}\times 4P2$	M1	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$t\times {}^4P_2$ or 12, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
		M1	$\frac{m!}{n!}\times 4P2$ $7 \leqslant m \leqslant 9, 1 \leqslant n \leqslant 3$ all integers.
	10 008	A1	Exact value.
		4	

29) OCT 2021_9709_53 Q1

(a)	$\frac{82}{180}, \frac{41}{90}$, 0.456	B1	
		1	
(b)	$\left[P(M\|D) = \frac{P(M\cap D)}{P(D)}\right] = \frac{\frac{11}{180}}{\frac{20}{180}+\frac{11}{180}}$ or $\frac{0.6011}{0.1722}$	M1	*Their* identified $\frac{P(M\cap D)}{P(D)}$ *or* from data table $\frac{11}{20+11}$, accept unsimplified, condone × 180.
	$\frac{11}{31}$, 0.355	A1	Final answer.
		2	

(c)	$P(F) = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE $P(G) = \frac{82}{180}, \frac{41}{90} 0.4556$ OE	M1	*Their* identified P(F) × *their* identified P(G) or correct seen, can be unsimplified.
	$P(F \cap G) = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $P(F) \times P(G) = \frac{100}{180} \times \frac{82}{180} = \frac{41}{162}, 0.2531$ OE $\left[\neq \frac{38}{180}\right]$ Not independent	A1	$\frac{41}{162}, \frac{38}{180}, P(F \cap G)$ and $P(F) \times P(G)$ seen with correct conclusion, WWW. Values and labels must be seen.
	Alternative method for question 1(c)		
	$P(F \cap G) = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $P(G) = \frac{82}{180}, \frac{41}{90}, 0.4556$ OE	M1	P(F|G) (OE) unsimplified with *their* identified probs or correct
	$P(F \mid G) = \frac{\frac{38}{180}}{\frac{82}{180}} = \frac{19}{41}, 0.4634$ OE $\neq P(F) = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE Not independent	A1	$\frac{19}{41}, \frac{100}{180}, P(F \cap G)$ and $P(F \mid G)$ seen with correct conclusion WWW. Values and labels must be seen.
		2	

30) OCT 2021_9709_53 Q5

(a)	$[P(0, 1, 2) =]\ {}^{10}C_0 0.16^0 0.84^{10} + {}^{10}C_1 0.16^1 0.84^9 + {}^{10}C_2 0.16^2 0.84^8$ [= 0.17490 + 0.333145 + 0.28555]	M1	One term: ${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	A1	$0.7935 < p \leqslant 0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
(b)	$(0.84)^7 0.16$	M1	$(1-p)^7 p$, $0 < p < 1$
	0.0472	A1	0.0472144 to at least 3sf.
		2	
(c)	$4 \times 0.0472 \times (1 - 0.0472)^3$	M1	$4 \times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	A1	$0.163 \leqslant p \leqslant 0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

31) OCT 2022_9709_51 Q6

6(a)	5M0W $^8C_5$ [× $^7C_0$] = 56 4M1W $^8C_4$ × $^7C_1$ = 490 3M2W $^8C_3$ × $^7C_2$ = 1176	M1	$^8C_x \times {^7C_{5-x}}$ for x = 1, 2, 3, 4, or 5
		B1	Outcome for 4M1W or 3M2W correct and identified, accept unsimplified.
		M1	Add 3 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified. Addition may be implied by final answer.
	[Total =] 1722	A1	Value stated WWW.
	Alternative method for Question 6(a)		
	2M3W $^8C_2$ × $^7C_3$ = 980 1M4W $^8C_1$ × $^7C_4$ = 280 0M5W $^8C_0$ × $^7C_5$ = 21	M1	$^8C_x \times {^7C_{5-x}}$ for x = 1, 2, 3, 4, or 5
		B1	Outcome for 2M3W or 1M4W correct and identified, accept unsimplified.
	[Total = $^{15}C_5$ – (980 + 280 + 21)] 3003 – (980 + 280 + 21)	M1	Subtract 3 values of appropriate scenarios from *their* identified total or correct, no incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total =] 1722	A1	Value stated WWW.
		4	
6(b)	$^{15}C_3 \times {^{12}C_5}$ [× $^7C_7$] [= 455 × 792]	M1	$^{15}C_r \times q$, r = 3, 5, 7; q a positive integer >1
		M1	$^{15}C_s \times {^{15-s}C_t}$ [× $^{15-s-t}C_u$] s = 3,5,7; t = 3,5,7 ≠ s; u = 3,5,7 ≠ s,t
	360360	A1	Final answer. If A0 awarded SC B1 for final answer 360360.
		3	
6(c)	**Method 1: Total number of arrangements with AB together – Arrangements with AB and FG together**		
	6! × 2 – 5! ×2 ×2 [= 1440 – 480]	M1	$a!$ × 2! × b, a = 5, 6; b = 1,2 seen.
		M1	Either 6! × 2 – c , 1 < c < 1440 or d – 5! ×2 ×2, 1440 < d
	960	A1	
	Method 2: arrangements with AB together with F and G not together.		
	2 × 4! × 5 × 4	M1	2 × 4! × e, e positive integer >1
		M1	f × 5 × 4, f positive integer >1 condone f × 20, f × $^5C_2$, f positive integer >1
	960	A1	
		3	

32) OCT 2022_9709_52 Q7

(a)	7!	M1	$\frac{7!}{b! \times c!}$ b,c = 1,2 $7! \times \frac{2!}{2!} \times \frac{2!}{2!}$ oe, no further terms present.
	5040	A1	
		2	

b)	**Method 1 for first 3 marks: Arrangements of 6 letters including Ls between As**		
	$5! \times 5 \times 2$	M1	$5! \times d$, d integer > 1
		M1	$e! \times f \times g$, $e = 5, 6, 7$; $f = 1, 5$; $g = 1, 2$; $f \neq g$. 1 can be implicit.
	1200	A1	
	Method 2 for first 3 marks: Number of arrangements of LL^^^^^ – number of arrangements with the Ls split by an A		
	$6! \times 2 - 5! \times 2$	M1	$6! \times 2 - h$ h an integer $1 < h < 1440$
		M1	$k - 5! \times 2$ k an integer $k > 240$
	1200	A1	
	Method 3 for first 3 marks: Alternative approaches to Method 1		
	^A ^^^^^ A $\;{}^5P_1 \times {}^1P_1 \times {}^5P_5 \times {}^1P_1 = 600$	M1	LL treated as a single unit.
		M1	
	1200	A1	
(b)	**Final 2 marks of Question 7(b)**		
	[Total number of arrangements =] $\left[\frac{9!}{2!2!} = \right]$ 90720	B1	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\frac{1200}{90720}, \frac{5}{378}$, 0.0132	B1 FT	$\frac{\textit{their } 1200}{\textit{their } 90720}$ unsimplified B1 FT if *their* 1200 and *their* 90 720 supported by work in this part.
		5	
(c)	**Method 1: Scenarios identified Both As and Ls removed**		
	A _ _ _ _ ${}^5C_4 = 5$ AA _ _ _ ${}^5C_3 = 10$ AL _ _ _ ${}^5C_3 = 10$ AAL _ _ ${}^5C_2 = 10$	B1	1 correct, identified outcome/value for A, AL or AAL scenario, accept unsimplified ${}^5C_{5-x}$ cannot be used in place of ${}^5C_x$
		M1	Add 4 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	[Total =] 35	A1	Value stated WWW.
	Method 2: 1 A fixed, 1 L removed No other scenarios can be present anywhere in solution		
	A ^^^^ ${}^7C_4$	M1	${}^7C_h$, $3 \leqslant h \leqslant 5$
		B1	${}^7C_4$ oe, no other terms, scenario identified.
	[Total =] 35	A1	Value stated.
	Method 3: 1 A fixed, both Ls removed		
	A ^^^^ = ${}^6C_4 = 15$ A L ^^^ = ${}^6C_3 = 20$	B1	Correct outcome/value for 1 identified scenario, accept unsimplified. WWW
		M1	Add 2 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	[Total =] 35	A1	Value stated.
		3	

33) OCT 2022_9709_53 Q6

a)	$\frac{9!}{2!2!}$	M1	$\frac{h!}{2!\times j!}$, $h = 7, 8, 9; j = 1, 2$
	90720	A1	
		2	
b)	Arrangements with 5 letters between As + Arrangements with 6 letters between As + Arrangements with 7 letters between As		
	With gap of 5: $\frac{7!}{2!}\times 3$ [= 7560] With gap of 6: $\frac{7!}{2!}\times 2$ [= 5040] With gap of 7: $\frac{7!}{2!}\times 1$ [= 2520]	M1	$\frac{7!}{2!}\times k$, k positive integer $1 < k < 7$
		M1	Add their no of ways for 3 identified correct scenarios, no additional incorrect scenarios, accept unsimplified.
	[Total no = $\frac{7!}{2!}\times 6 =$] 15120	A1	
		3	
c)	**Method 1: Summing number of ways**		
	AT _ _ _ $2\times 2\times {}^5C_3$ 40 A _ _ _ _ $2\times {}^5C_4$ 10 AATT _ ${}^5C_1$ 5 AAT _ _ $2\times {}^5C_2$ 20 AA _ _ _ ${}^5C_3$ 10 _ _ _ _ _ ${}^5C_5$ 1	B1	Correct no of ways for 4 correctly identified scenarios, accept unsimplified.
		M1	Add no of ways for 5 or 6 identified correct scenarios, no additional incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total no of ways not containing more Ts than As =] = 40+10+5+20+10+1 [=86]	A1	All correct and added
	Probability = $\frac{86}{{}^9C_5}$	M1	$\frac{their\ 86}{9C5\ or\ their\ identified\ total}$ accept numerator unevaluated
	$\frac{86}{126}, \frac{43}{63}$, 0.683	A1	
	Method 2: Subtracting no of ways with more Ts from total		
	T _ _ _ _ $2\times {}^5C_4$ 10 TTA _ _ $2\times {}^5C_2$ 20 TT _ _ _ ${}^5C_3$ 10	B1	Correct no of ways for 2 correctly identified scenarios, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations
		M1	Add no of ways for 2 or 3 correct scenarios and subtract from their total no of ways All correct and subtracted
	Total no of ways with more Ts than As =40 ${}^9C_5 - 40 = 86$	A1	
	Probability = $\frac{86}{{}^9C_5}$	M1	$\frac{their\ 86}{9C5\ or\ their\ identified\ total}$ accept numerator unevaluated
c)	$\frac{43}{63}$, 0.683	A1	
		5	