classmate

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Probability and Statistics-1

Permutations and Combinations

Ex-1 Solution (Revision)

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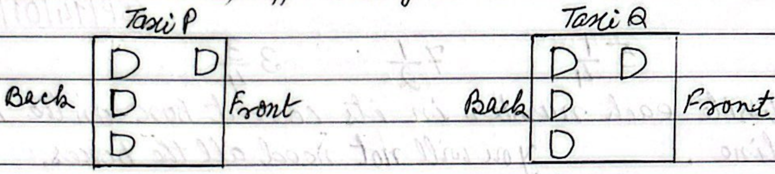
Suresh Goel
(Former Director)
Alliance World School,
Noida, Delhi-NCR
INDIA

(+91 9810444804)

Permutations and Combinations Revision.

Example 1: A group of 8 friends travels to the airport in two taxies, P and Q. Each taxi can take 4 passengers.

(a) The 8 friends divide themselves into two groups of 4, one group for taxi P and one group for taxi Q, with Jon and Sarah travelling in the same taxi.
Find the number of different ways in which this can be done. --- [3]



Each taxi can take 1 passenger in front and 3 passengers in the back. Mark sits the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(b) Find the number of different seating arrangements that are now possible for the 8 friends. SP-20/05/Q6 --- [4]

Solution: Jon and Sarah both may sit in taxi P or Q = 2 way —— ①

(a) Now out of remaining 6, two may sit in Jon & Sarah Taxi = ${}^6C_2$ ways

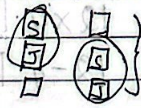
Rest of the 4 may sit in another taxi any way = ${}^4C_4$.

∴ Total number numer of ways = $2 \times {}^6C_2 \times {}^4C_4$

$= 2 \times \frac{6 \times 5}{2 \times 1} \times 1 = \underline{30 \text{ ways}}$ ✓

(b) In Taxi P → Mark sits on front. → 1 way

Sarah & Jon sit at back next to each other



→ May sit in 2 way ✓
but they can rearrage themselves = 2 ways ✓
and the remain 1 seat will be taken by one of remaining 5 = ${}^5P_1$ ways

For Taxi P = $1 \times (2 \times 2) \times {}^5P_1 = \underline{20 \text{ ways}}$ ✓

The remaing 4 may sit in any way in Taxi Q → ${}^4P_4 = 4! = 24$ ways

∴ Total number of seating arrangement $= 20 \times 24$

$= \underline{480}$ ✓

Example 2: The 40 members of a club include Ranuf and Saed. All 40 members will travel to a concert. 35 members will travel in a coach and the other 5 will travel in a car. Ranuf will be in coach and Saed will be in the car.
In how many ways can the members who will in the coach be chosen. [M-20/52/Q1] --[3]

Solution: Ranuf in the coach → Remain seats in coach $= 35-1=34$
Ranuf in coach & Saed have taken their seats → Remaining No $= 40-2=38$.
$\therefore$ No of way the member may be in coach $= {}^{38}C_{34} = \dfrac{38!}{34!\,4!}$

$$= \frac{38\times37\times36\times35}{4\times3\times2\times1} = \underline{73815}$$

Example 3: Richard has 3 blue candles, 2 red candles and 6 green candles. The candles are identical apart from their colours. He arranges 11 candles in a line.

(a) Find the number of different arrangements of the 11 candles if there is a red candle at each end. --[2]

(b) Find the number of different arrangements of the 11 candles if all the blue candles are together and red candles are not to together. --[4]

[M-20/52/Q4]

Solution (a) R, × × × × × × × × × R
Two red candles are at the end. Remaing 3 blue & 6 green may be arranged any way, (but 3 blue are of one kind and 6 green are of another one kind)
$\therefore$ Total number of arrangements $= \dfrac{9!}{3!\,6!} = \underline{84}$

(b) Case I when 3 blue are together → 1 bundle & Rest 6 green & 2 Red any way

$$\text{No} = \frac{9!}{2!\,6!} \quad \text{—①}$$

Case II when 3 blue together & 2 Red together $= \dfrac{8!}{6!}$ —②

$\therefore$ Req number $= \dfrac{9!}{2!\,6!} - \dfrac{8!}{6!} = 252 - 56 = \underline{196}$

4 (a) Find the total number of different arrangement of the 11 letters in the word CATERPILLAR. --[2]

(b) Find the total number of different arrangements of the 11 letters in the words CATERPILLAR in which there is an R at the beginning and an R at the end, and two A's are not together. --[4]

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contains both Rs and atleast one A and atleast one L. --[4]

[M-21/52/Q6]

Solution (a)

CATERPILLAR → C–1, A–2, T–1, E–1, R–2, P–1, I–1, L=2 — Total. 11

No of 11 letter Words $= \frac{11!}{2!\,2!\,2!} = \underline{4989600}$ ✓

(b) R --(AA)$^{\text{together}}$ ----- R $= \frac{8!}{2!} = 20160$

R -- No restriction -- R $= \frac{9!}{2!\,2!} = 90720$

∴ The required number, when A's are not together $= 90720 - 20160$
$= \underline{70560}$ ✓

(c) RRAL → ${}^5C_2 = 10$ [Two out of C, T, E, P, I (5)]

RRAAL → ${}^5C_1 = 5$ [one of C, T, E, P, I (5)]

RRAALL ______ $= 1$ (all 6)

RRALL → ${}^5C_1 = 5$ [one of C, T, E, P, I)

Total ways $= \underline{21}$ ✓

5. A group of 12 people consists of 3 boys, 4 girls and 5 adults.

(a) In how many ways a team of 5 people be chosen from the group if exactly one adult is included? ---[2]

(b) In how many ways can a team of 5 people be chosen from the group if the team includes at least 2 boys and at least 1 girl? --[4]

The same group of 12 people stand in a line.

(c) How many different arrangement are there in which the 3 boys stand together and an adult is at each end of the line? --[4]

| M-22 | 52 | Q5 |

Solution: 3 Boys + 4 Girls + 5 Adults = 12.

(a) exactly one adult $= {}^5C_1 \times {}^7C_4$

$= \underline{175}$ ✓

(b) Atleast 2 boys & atleast 1 Girl :

$2B\,1G\,2A = {}^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$ ✓

$2B\,2G\,1A = {}^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$

$2B\,3G = {}^3C_2 \times {}^4C_3 = 12$

$3B\,1G\,1A = {}^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$

$3B\,2G = {}^3C_3 \times {}^4C_2 = 6$

Total = $\underline{248}$

(c) $\underline{A_1}$ $b_1 b_2 b_3$ · G_1, G_2, G_3, G_4 · A_3, A_4, A_5 · $\underline{A_2}$

At the end 2 adults $= {}^5P_2$ ✓

3 boys together $= {}^3P_3$

all ⟶ $8!$

∴ Total number of arrangements

$= 8! \times 3! \times {}^5P_2$

$= \underline{4838400}$ ✓

6. (a) Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three E's are together and the two Ds are not next to each other. ---[4]

(b) Find the prob. that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two D's. ---[5]

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. ---[3]

[M-23/52/Q7]

Solution (a) DELIVERED $\longrightarrow \begin{cases} 3E \\ 2D \\ L, I, V, R \rightarrow 4 \text{ differerent} \end{cases}$

EEE, –, –, –, –, D, D

Only 3E are together $= \frac{7!}{2!}$ --- ①

EEE DD – – – – '3E' are together and '2D' are together $= {}^6P_6 = 6!$ --- ②

∴ Arrangement when '3E' are together but '2D' are not together

$= \frac{7!}{2!} - 6! = 1800$ ✓ (frm ① & ②)

(b) D, –, –, –, –, D, –, –, –

$= 7! \times 4 \times 2$ (Two D's are in one position with four letters in between - repeats allowed)

$= 40320$ ✓

(c) Case I DE – – – $= {}^4C_3 = 4$ (DE & 3 different out of 4)

II DEE – – $= {}^4C_2 = 6$

III DEEE – $= {}^4C_1 = 4$

IV DDE – – $= {}^4C_2 = 6$

V DDEE – $= {}^4C_1 = 4$

VI DDEEE $= {}^4C_0 = 1$

Total $= 25$ ✓

Example-7: Find the number of different arrangements that can be
(a) made from the 9 letters of the word JEWELLERY in which the three Es are together and 2Ls are together, --[2]
(b) Find the different arrangement that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [S-20/51/Q2] --[4]

Solution:
(a) Three Es together & 2Ls together.
$\therefore$ Effective No $\rightarrow$ 6
$\therefore$ No of arrangements $= 6! = 720$ ✓

(b) Total number any way $= \frac{9!}{3!\,2!}$
when two Ls together $= \frac{8!}{3!}$
$\therefore$ Req No when two Ls not together $= \frac{9!}{3!\,2!} - \frac{8!}{3!}$
$= 30240 - 6720 = 23520$ ✓

Example 8. In a music competition, there are 8 pianists, 4 guitarists and 6 violinists. 7 of these musicians will be selected to go through to the final.
How many different selections of 7 finalists can be made if there must be at least 2 pianists, at least 1 guitarist and more violinists than guitarists. [S-20/51/Q4] --[4]

Solution: 8P, 4G and 6V $\Rightarrow$ Total = 18, To be selected = 7, 'V > G'
Total 7 $\rightarrow$ Case I 2P, 1G, 4V $= {}^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$
" $\rightarrow$ Case II 2P, 2G, 3V $= {}^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$
" $\rightarrow$ Case III 3P, 1G, 3V $= {}^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$
" $\rightarrow$ Case IV 4P 1G, 2V $= {}^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$
$\therefore$ Total = 13,720 ✓

Example 9 (a) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. --- [2]

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that Es are not together. S-20/52/Q6 --- [4]

Solution: SUMMERTIME { S-1, U-1, M-3, E-2, R-1, T-1, I-1 }

E × × × × × × × × E

3M

∴ Number of Ways = $\frac{8!}{3!}$

$= \underline{6720}$ ✓

(b) Total 10, 2E and 3M

Total number of ways any way = $\frac{10!}{2!\,3!}$ —— ①

and with 2Es together = $\frac{9!}{3!}$ —— ②

hence the number of ways when 2Es are not together = $\frac{10!}{2!\,3!} - \frac{9!}{3!}$ from ① & ②

$= 302400 - 60480 = \underline{241920}$ ✓

Example 10 (a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. --- [1]

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. --- [2]

(c) Find the prob. that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. --- [4]

• 5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and atmost on L. S-20/53/Q7 --- [3]

Solution (a) $\frac{9!}{2!\,2!} = 90720$ ✓ (∵ 2E & 2L in Total 9.)

(b) C - - - T - - - E

$\frac{6!}{2!} = 360$ ✓ (∵ 2L in 6 letters)

(c) 2Es together = $\frac{8!}{2!} = 20160$

⇒ 2Es not together = $90720 - 20160 = 70560$

∴ Prob = $\frac{70560}{90720} = \frac{7}{9}$ (or 0.778) ✓

(d) EL - - - $= {}^5C_3 = 10$

or EEL - - $= {}^5C_2 = 10$

E - - - - $= {}^5C_4 = 5$

EE - - - $= {}^5C_3 = 10$

∴ Total number of ways

$= 10 + 10 + 5 + 10$

$= \underline{35}$ ✓

11. A bag contains 12 marbles, each of a different size. 8 of the marbles are red and 4 of the marbles are blue.
How many different selections of 5 marbles contain at least 4 marbles of the same colour? ---[4]

S-21/51/Q1

Solution: 8 red & 4 blue → Total 12.
Selections:
(i) 4 red & 1 blue $= {}^8C_4 \times {}^4C_1 = 280$
(ii) 4 blue & 1 red $= {}^8C_1 \times {}^4C_4 = 8$
(iii) 5 Red $= {}^8C_5 = 56$

Total = 344 ✓

12 (a) How many different arrangements are there of the 8 letters in the word RELEASED. ---[1]

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order. ---[3]

(c) An arrangement of the 8 letters in the word RELEASED is is chosen at random.
Find the probability that the letters A and D are not together.
[S-21/51/Q3] ---[4]

<u>Solution</u>: (a) RELEASED $\longrightarrow$ R − 1, E − 3, L − 1, A − 1, S − 1, D − 1

Number of arrangements

$$= \frac{8!}{3!} = \underline{6720} \quad \text{---(i)}$$

(b) --'LED'--- together (LED) + 5 more $\rightarrow$ with 2E

$$\text{Number of arrangements} = \frac{6!}{2!} = \underline{360}$$

(c) (A&D) together $\rightarrow \dfrac{7! \times 2}{3!} = 1680$ ---(ii)

$\therefore$ Number of arrangement when A&D are not together
$= 6720 - 1680 = 5040$ (from (i), (ii))

$$\therefore P(\text{A\&D are not together}) = \frac{5040}{6720} = \underline{\frac{3}{4}} \ (\text{or } 0.75)$$

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13(a) Find the total number of different arrangement of the 8 letters in the word TOMORROW. ---[2]

(b) Find the total number of different arrangements of 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three O.s are not together. ---[3]

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. ---[5]

S-21/52/Q6

Solution (a) TOMORROW → T-1, O-3, M-1, R-2, W-1

Total number of arrangements $= \frac{8!}{2!\,3!} = \underline{3360}$ ✓

(b) R — — — — — — R

(i) Number of arrangement Rs at the ends $= \frac{6!}{3!}$ ---(i)

(ii) R - (OOO) - - R

Number of arrangements Rs at the end and Os together $= 4!$ ---(ii)

∴ Number of arrangement Rs together but three Os are not together $= \frac{6!}{3!} - 4! = \underline{96}$ ✓ [from (i) & (ii)].

(c)
$$
\left.\begin{aligned}
OORR &= {}^3C_2 \times {}^2C_2 \times [{}^3C_0] = 3\times1\times1 = 3\\
ORR- &= {}^3C_1 \times {}^2C_2 \times {}^3C_1 = 3\times1\times3 = 9\\
OOR- &= {}^3C_2 \times {}^2C_1 \times {}^3C_1 = 3\times2\times3 = 18\\
OR-- &= {}^3C_1 \times {}^2C_1 \times {}^3C_2 = 3\times2\times3 = 18\\
OOOR &= {}^3C_3 \times {}^2C_1 \times [{}^3C_0] = 1\times2 = 2
\end{aligned}\right\}\ \text{Total} = 50
$$

$$\text{Probability} = \frac{50}{{}^8C_4} = \frac{50}{70} = \underline{0.714} \checkmark$$

14(a) How many different arrangement are there of the 11 letters in the word REQUIREMENT. ---[2]

(b) How many different arrangement are these of the 11 letters in the word REQUIREMENT, in which the two R.s are together and the three Es are together. ---[1]

(c) How many different arrangement are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs. ---[3]

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and atleast one R? [S-21/53/Q 6] ---[4]

Solution (a) REQUIREMENT

R – 2
E – 3
Q – 1
U – 1
I – 1
M – 1
N – 1
T – 1

Number of arrangements

$= \frac{11!}{2!\,3!}$

$= \underline{3326400}$

(b) RR EEE ------- = 8

Number arrangements $= 8!$

$= \underline{40320}$ ✓

(c) R - - - R _ _ _ _ _ _ = 7

I II III IV V VI VII th

R - - - R Can take 7 different positions.

In each case number of arrangements

$= \frac{9!}{3!}$

$\therefore$ Total No $= \frac{9!}{3!} \times 7 = \underline{423360}$ ✓

(d) (EEE)(RR)

EER - - - = ${}^6C_2 = 15$

EERR - - = ${}^6C_1 = 6$

EEER - = ${}^6C_1 = 6$

EEERR = ${}^6C_0 = 1$

Total no of selections

$= 15 + 6 + 6 + 1 = \underline{28}$ ✓

15 (a) Find the number of arrangement of the 8 letters in the word DECEIVED in which all three Es are together and the two two D's are together. ---[2]

(b) Find the number of different arrangement of 8 letters in the word DECEIVED in which three 3Es are not all together. ---[4]

[S-22 | 51 | Q1]

Solution: DECEIVED $\begin{cases} E-3 \\ D-2 \\ C-1 \\ I-1 \\ V-1 \end{cases}$

(a) EEE DD - 3 more

$= 5!$

$= 120$ ✓

(b) Total no. of ways $= \frac{8!}{2!\,3!} = 3360$ — ①

with 3E's together $= \frac{6!}{2!} = 360$ — ②

∴ 3Es are not together

$= 3360 - 360$ (from ① & ②)

$= 3000$ ✓

16. There are 6 men and 8 women in a Book Club. The committee of the club consists of five of its members. Mr Lan and Mrs Lan are members of the club.

(a) In how many different ways can the committee be selected if exactly one of Mr Lan and Mrs Lan must be on the committee ---[2]

(b) In how many different ways can the committee be selected if Mrs Lan be on the committee and there must be more women than men on the committee. ---[4]

[S-22 | 51 | Q2]

Solution: 6 men and 8 women = 14 (Total)

(a) out of Mr Lan and Mrs Lan exactly one selected $= {}^{2}C_{1}$ — ①

out of remaining 12 any 4

$= {}^{12}C_{4}$ — ②

from ① & ② Total no of selections

$= {}^{12}C_{4} \times {}^{2}C_{1} = 495 \times 2$

$= 990$ ✓

(b) Mrs Lan is on the committee, +

(i) 2W & 2M $= {}^{7}C_{2} \times {}^{6}C_{2} = 315$

(ii) 3W & 1M $= {}^{7}C_{3} \times {}^{6}C_{1} = 210$

(iii) 4W $= {}^{7}C_{4} = 35$

Total $= 315 + 210 + 35 = 560$ ✓

17.(a) Find the number of different arrangements of the 9 letters in the word CROCODILE. ---[1]

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. ---[3]

(c) Four letters are selected from the 9 letters in the word CROCODILE. Find the number of selections in which the number of Cs is not the same as the number of Os. ---[3]

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. ---[3]

[S-22/52/Q6]

Solution: "CROCODILE" → Two Cs, two Os and rest 5 different.

(a) $\frac{9!}{2!\,2!} = \underline{90720}$ ✓

(b) Required number of arrangement = 2Cs at ends − 2Cs at ends and 2Os are together ---①

No. of arrangement when 2C are at the end. C $\underbrace{-----}_{\text{with 2Os}}$ C

$= \frac{7!}{2!}$ ---②

No. of arrangents when two Cs are at the ends and 2Os togther

C (OO) ----- C $= 6!$ ---③

hence the required no. of arrangements $= \frac{7!}{2!} - 6!$ {from ② and ③ in ①}

$= 2520 - 720$

$= \underline{1800}$ ✓

(c) Cs are not same no as Os.

CCO − $^5C_1 = 5$

CC −− $^5C_2 = 10$

OOC − $= {}^5C_1 = 5$

OO −− $= {}^5C_2 = 10$

C −−− $= {}^5C_3 = 10$

O −−− $= {}^5C_3 = 10$

Total $= \underline{50}$ ✓

(d) OOC, C∧∧, ∧∧∧ $= {}^5C_2 = 10$

OO∧, C∧∧, C∧∧ $= {}^5C_1 \times {}^3C_2 = 30$

OC∧, O∧∧, ∧∧∧ $= {}^5C_1 \times {}^4C_2 = 30$

OC∧, OC∧, ∧∧∧ $= \frac{{}^5C_1 \times {}^4C_1}{2!} = 10$

Total $= 10 + 30 + 30 + 10$

$= \underline{80}$ ✓

18(a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. ---[2]

A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

(b) Find the number of ways in which this committee can be chosen if there are no restrictions on the number of men and women, but it must include no more than two of the brothers. ---[3]

S-23/51/Q2

Solution(a) Men = 6, Women = 8

Committee of 6 → with 3 men & 3 women;

$= {}^6C_3 \times {}^8C_3 = \underline{1120}$ ✓

(b) Men = 6 (3 brothers); Women = 8, Total = 14

Case I → 0 brother: ${}^3C_0 \times {}^{11}C_6 = 462$

1 brother: ${}^3C_1 \times {}^{11}C_5 = 1386$

2 brothers: ${}^3C_2 \times {}^{11}C_4 = 990$

Total ways = $\underline{2838}$ ✓

19 (a) Find the number of different arrangement of the 8 letters in the word COCOONED. ---[1]

(b) Find the number of different arrangements of 8 letters in the word COCOONED in which the first letter is O and the last letter is N. ---[2]

(c) Find the prob. that a randomly chosen arrangement of 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. ---[3]

S-23/51/Q3

Solution(a)

COCOONED
C → 2
O – 3
N, E, D, 1 each.

Number of arragement of 8 letter words $= \dfrac{8!}{2! \times 3!} = \underline{3360}$ ✓

(b) O(C$\cancel{O}$COO$\cancel{N}$ED)N

$\underleftrightarrow{\quad 6 \quad}$

$= \dfrac{6!}{2!\,2!}$

$= \underline{180}$

(c) $P(CC) = \dfrac{7!}{3!}$ —— ①

{(CC)OOONED}

$P(OOO \cap CC) = {}^5P_5 = 5!$ ... ②

{(OOO)(CC)NED}

Hence $P(OOO/CC) = \dfrac{P(OOO \cap CC)}{P(CC)}$

$= \dfrac{5!}{\frac{7!}{3!}}$ from ② & ①

$= \dfrac{120}{840} = \dfrac{1}{7} = \underline{0.143}$ ✓

20. In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person completes in only one of these sports. A team of 7 is selected from these 25 people to take part a competition.

(a) Find the number of different ways in which the team of 7 can be selected if it consist of exactly 1 swimmer, at least 2 cyclists and at most 2 runners. ---[4]

For another competion, in a team of 9 people consist of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

(b) How many different arrangents are there of 9 people if the swimmers stand together, the cyclists stand together and the runners stand together. ---[2]

(c) How many different arrangements are there of the 9 people if none of the cyclist stand next to each other. ---[4]

S-23/52/Q6

Solution Swimmers = 6, Cyclists = 8, Runners = 11 → Total = 25

(a) Team of 7 (1S + atleast 2C + at most 2 runners)

Case (I) $1S + 4C + 2R = {}^6C_1 \times {}^8C_4 \times {}^{11}C_2 = 23100$

(ii) $1S + 5C + 1R = {}^6C_1 \times {}^8C_5 \times {}^{11}C_1 = 3696$

III. $1S + 6C + 0R = {}^6C_1 \times {}^8C_6 \times {}^{11}C_0 = 168$

Total = $\underline{26964}$ ✓

(b) Team of 9 people. (2S + 3C + 4R)

Arrangement (2S) . (3C) . (4R) $= (2! \times 3! \times 4!) \times 3!$

$= \underline{1728}$ ✓

(c) Team of 9 (2S + 3C + 4R)

3C → C _ C _ C _ _ _ _ $= {}^7P_3$

and $2S + 4R = 6!$

∴ Total numumber of arrangement $= 6! \times {}^7P_3$

$= \underline{151,200}$ ✓

21(a) Find the number of differenent arrangements of 10 letters in the word CASABLANCA in which the two Cs are not together. ---[3]

(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2Cs. ---[3]

Five letters are selected from the 10 letters in the word "CASABLANCA"

(c) Find the number of different selections in which the five letters include at least 2As and atmost one C. ---[3]

S-23	53	Q7

Solution(a) CASABLANCA $\begin{cases} C-2 \\ A-4 \\ B, L, N, S \end{cases}$

Number arrangement with no restrictions $= \dfrac{10!}{2!\,4!}$ ---①

with 2Cs together. CC ① 8 more.

$= \dfrac{9!}{4!}$ ——— ②

Arrangement with 2Cs not together $\longrightarrow \dfrac{10!}{2!\,4!} - \dfrac{9!}{4!} = \boxed{60480}$ ✓

(b) A C - - - C - - - A

four Cases. $\begin{cases} C\ \overline{C}\ - - -\ \overline{C}\ C \\ \quad C - - - C \end{cases}$ $= \dfrac{6!}{2!} \times 4 = \boxed{1440}$

(c) 5 letters are selected out of 10.

(One C atmost)

(5 diff C, L, B, N S)

Case I – AA – – – $= {}^5C_3 = 10$

II – AAA – – $= {}^5C_2 = 10$

III – AAAA – $= {}^5C_1 = 5$

Total $= \underline{25}$ ✓

22.(a) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. --- [2]

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that Ps are not next to each other. --- [4]

(c) Find the probability that a randomly choosen arragement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. --- [2]

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [W-20/51/Q7] --- [3]

Solution (a) SHOPP(EEE)KR

$$\frac{8!}{2!} = 20160 \checkmark$$

SHOPKEEPER → S – 1, H – 1, O – 1, P – 2, E – 3, K – 1, R – 1

(b) Total number of way (any way) $= \frac{10!}{2!\,3!} = 302400$ —— ①

with 2Ps together $= \frac{9!}{3!} = 60480$ —— ②

from ① & ② with Ps' not together $= 302400 - 60480 = \underline{241920} \checkmark$

(c) Prob = $\dfrac{\text{Number of ways Es at the beginning and at the end}}{\text{Total number of ways}}$

$$= \frac{8!}{2!} \Big/ \frac{10!}{2!\,3!} = \frac{20160}{302400} = \frac{1}{15} = \underline{0.0667} \checkmark$$

(d) PEEE $= {}^5C_0 = 1$

PEE– $= {}^5C_1 = 5$

PE – – $= {}^5C_2 = 10$

P – – – $= {}^5C_3 = 10$

$\therefore$ Total $= \underline{26} \checkmark$

23. Mr and Mrs Ahmed with their two children, and Mr and Mrs Baker with their three children, are visiting an activity centre together. They will divide into groups for some of the activities.

(a) In how many way can 9 people be divided into a group of 6 and a group of 3. --[2]

5 of the 9 people are selected at random for a particular activity.

(b) Find the probability that this group of 5 people contains all 3 of Baker children. ---[3]

All 9 people stand in a line.

(c) Find the number of different arrangements in which Mr Ahmed is not standing next to Mr Baker. ---[3]

(d) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. --[3]

W-20/52/Q6

Solution (a) $^{9}C_3 \times {}^{3}C_3 = 84 \times 1 = \underline{84}$ ✓ (Total number = 9)

(b) Total number of selection of 5 out of 9 = $^{9}C_5 = 126$

number of ways of 3 baker children and remaing 2 out of 6 $= {}^{3}C_3 \times {}^{6}C_2 = 15$

$\therefore$ Required Probability $= \frac{15}{126} = \underline{0.119}$ ✓

(c) Total no. of arrangement of 9 persons = $9!$

No. of arrangement when Mr Ahmed and Mr Baker are together = (Baker, Ahmed), –, –, –, –, –, –, – $= 8! \times 2!$

$\therefore$ The two are not together $= 9! - 8! \times 2! = \underline{282240}$ ✓

(d) (Ahmed, $\underset{}{\overset{\text{one out of 7}}{—}}$, Baker), –, –, –, –, –, –

$= 7! \times 7 \times 2! = \underline{70560}$ ✓

Scanned with CamScanner

24. A committee of 6 people is to be chosen from 9 women and 5 men,
(a) Find the number of ways in which 6 people can be chosen if there must be more women then men on the committee. ---[3]

The 9 women and 5 men include a sister and brother.
(b) Find the number of ways in which the committee can be chosen if the sister and brother cannot both be on the committee. ---[3]

W-20/53/Q3

Solution (a)

	9 Women	&	5 Men	
More women in 6	6	&	0	$\rightarrow {}^9C_6 \times {}^5C_0 = 84$
	5	&	1	$\rightarrow {}^9C_5 \times {}^5C_1 = 126 \times 5 = 630$
	4	&	2	$\rightarrow {}^9C_4 \times {}^5C_2 = 126 \times 10 = 1260$

Total $= 84 + 630 + 1260 = \underline{1974}$ ✓

(b) Total number of selection of 14, 6 at a time $= {}^{14}C_6 = 3003$.
Number of ways when brother & sister both $= {}^{12}C_4 \times {}^2C_2 = 495$

$\therefore$ Required number of way $= 3003 - 495$
(Not including both of them) $= \underline{2508}$ ✓

25. The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. ---[3]

(b) Find the prob. that the arrangement has both Rs together given that given that all three Es are together ---[4]

W-20	53	Q5

Solution: Word RESERVED $\rightarrow \begin{cases} R \rightarrow 2 \\ E \rightarrow 3 \\ V \rightarrow 1 \\ S \rightarrow 1 \\ D \rightarrow 1 \end{cases}$

To arranges randomly; Total no of letters = 8

Total number of Ways $= \dfrac{8!}{3!\,2!} = 3360$ ✓

(a) V(RESERD)E

Number of ways (with V as the first and E as last) $= \dfrac{6!}{2! \times 2!}$

$= 180$

$\therefore$ Probability $= \dfrac{180}{3360} = \dfrac{3}{56}$ or $\underline{0.0536}$ ✓

(b) $X =$ Es together $=$ (EEE)RSRVD $= \dfrac{6!}{2!} = 360$

$Y =$ Rs together and Es together $=$ (RR)(EEE)SVD $= 5! = 120$

Prob $= \dfrac{120}{360} = \underline{\dfrac{1}{3}}$ ✓

26 Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, the 9 members will stand in a line.

(a) How many different arrangement of the 9 members are possible in which Raman will be at the centre of the line? --[1]

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other? --[3]

For the second photograph, the members in stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4? --[2]

(d) for a random division into a group of 5 and a group of 4, find the prob. that Raman and Sanjay are in the same group as each other. --[4]

W-21/51/Q5

Solution: (a) - - - - Raman - - - -

∴ Raman in the middle $= 8! = \underline{40320}$ ✓

(b) Total arrangements any way $= 9!$ --(1)

when Raman and Sanjay are together - - - (R·S) - - - or SR

$= 8! \times 2$ (as R & S can interchange) --(2)

∴ Arrange when R & S are not together $= 9! - 8! \times 2$ (from (1) & (2))

$= \underline{282240}$ ✓

(c) — — — — —
— — — —

Combination of 9 members when 5 are at the back and 4 in front

$= {}^9C_5 \times {}^4C_4$

$= \underline{126}$ --- (3)

(d) when R & S are in the back row $= {}^7C_3 \times {}^4C_4 = 35$

when R & S are in the front row

$= {}^7C_2 \times {}^5C_5 = 21$

∴ Total $= 35 + 21 = 56$ —— (4)

from (3) & (4)

∴ Probability $= \dfrac{56}{126} = \dfrac{4}{9}$

$= \underline{0.444}$ ✓

27. A group of 6 people is to be chosen from 4 men and 11 women.
(a) In how many different ways can a group of 6 be chosen if it must contain exactly 1 man? --[2]

Two of the 11 women are sisters Jane and Kate.

(b) In how many different ways can a group of 6 be chosen if Jane and Kate can not both be in the group? --[3]

|W-21|52|Q2|

Solution: 4 men + 11 women.

(a) Selections of 6 with exactly 1 man $= {}^4C_1 \times {}^{11}C_5 = \underline{1848}$ ✓

(b) Combination of 6 anyway $= {}^{15}C_6$ --- 1

When both Jane and Kate are in the same group $= {}^{13}C_4$ —— 2

∴ Selections when J & K are not in the same group $= {}^{15}C_6 - {}^{13}C_4 = 5005 - 715$
$= \underline{4290}$ ✓

28 (a) In how many different ways can the 9 letters of the word TELESCOPE be arranged. ---[2]

(b) In how may different ways can the 9 letters of the word TELESCOPE be arrange so that there exactly two letters between the T and the C? --[4]

|W-21|52|Q4|

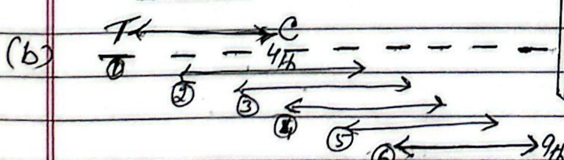
Solution: (a) TELESCOPE

$\left\{ \begin{array}{l} T=1 \\ E=3 \\ L=1 \\ S=1 \\ C=1 \\ P=1 \\ O=1 \end{array} \right.$

∴ Number of arranged $= \dfrac{9!}{3!}$

$= \underline{60480}$ ✓

(b)



T & C with two letters in the middle can take 6 different positions and T & C can interchange. $= 6 \times 2$ — 1

And rest of the seven letters (with 3E) can be arranged
$= \dfrac{7!}{3!}$ --- 2

∴ Total number arrangements
$= \dfrac{7!}{3!} \times 6 \times 2$ from 1 & 2

$= \underline{10080}$ ✓

29: The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is room for only 20 people to attend.
In how many ways can the 20 people be chosen from 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [W-21|53|Q1] ---[2]

Solution: $26-3=23$; $20-3=17$

$\therefore$ Number of combinations $= {}^{23}C_{17} = \underline{100947}$ ✓

30. A security code consists of 2 letters followed by a 4-digit number. The letters are chosen from {A, B, C, D, E} and the digits are chosen from {1, 2, 3, 4, 5, 6, 7}. No letter or digit may appear more than once. An example of a code is BE3216.

(a) How many different codes can be formed. ---[2]

(b) Find the number of different codes that include the letter A or the digit 5 or both. ---[3]

A security code is formed at random.

(c) Find the probability that code is DE followed by a number between 4500 and 5000. ---[3]

[W-21|53|Q5]

Solution Letters = {A, B, C, D, E}

Digits = {1, 2, 3, 4, 5, 6, 7}

(a) 2 letters and 4 digits:

No. of Codes $= {}^5P_2 \times {}^7P_4 =$

$= 5\times4\times7\times6\times5\times4 = \underline{16800}$

(b) With A, ${}^4P_1 \times {}^7P_4 \times 2 = 4\times2\times7\times5\times4\times3$

$= 8\times840 = 6720$ ✓

With 5, ${}^5P_2 \times {}^6P_3 \times 4 = 20\times120\times4 = 9600$ ✓

With A & 5 $= {}^4P_1 \times 2 \times {}^6P_3 \times 4 = 4\times8\times120$

$= 3840$ ✓

Required number

$= 6720 + 9600 - 3840 = \underline{12480}$ ✓

(c) DE → 1 way ✓

$4500 < \text{No} < 5000$

$\text{No} = {}^5P_2 \times 3$

$\text{No} = 1 \times {}^5P_2 \times 3 = 60$

$\therefore$ Req. Prob $= \dfrac{60}{16800}$ (from (a))

$= \dfrac{1}{280}$

$= \underline{0.00357}$ ✓

31 A social club has 15 members, of whom 8 are men, 7 are women. The committee of the club consists of 5 of its members.

(a) Find the number of different ways in which the committee can be formed from the 15 members if it must include more men than women. --[4]

The 15 members are having their photograph taken. They stand in three rows, with 3 people in the front row, 5 people in the middle row and 7 people in the back row.

(b) In how many different ways can the 15 members of the club be divided into a group of 3, a group of 5 and a group of 7? --[3]

In one photograph Abel, Betty, Celly, Dong, Eve, Freya and Gino are the 7 members in the back row.

(c) In how many different ways can these 7 members be arranged so that Abel and Betty are next to each other and Freya and Gino are not next to each other? --[3]

W-22/51/Q 6

Solution: Men = 8, Women = 7, Total 15.

(a) No of committee members = 5 (more men)

Case I. 5 M & 0 W = ${}^8C_5 \times {}^7C_0 = 56$

Case II. 4 M & 1 W = ${}^8C_4 \times {}^7C_1 = 490$

Case III 3 M & 2 W = ${}^8C_3 \times {}^7C_2 = 1176$

Total = 56 + 490 + 1176 = $\underline{1722}$ ✓

(b) Groups of 3, 5 & 7

--------7
------5
---3

${}^{15}C_3 \times {}^{12}C_5 \times {}^7C_7 = 455 \times 792 \times 1$
$= \underline{360360}$ ✓

Interchange

(c) $\boxed{\text{Abel \& Betty}}$ and 5 more = $6! \times 2$ = 1440 --- ①

$\boxed{\text{Abel \& Betty}}$ and $\boxed{\text{Freya \& Gino}}$ & 3 more

= $5! \times 2 \times 2$ = 480 ----- ②

∴ The number of arrangements of 7, when Abel & Betty are next to each other but Freya & Gino are not together.

= ① − ② = 1440 − 480 = $\underline{960}$ ✓

32 (a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. ---[2]

(b) The 9 letters in the word ALLIGATOR are arranged in a random order. Find the probability that the two Ls are together and there are 6 letters between the two As. ---[5]

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contains at least one A and at most one L.

[W-22 | 52 | Q 7] ---[3]

Solution (a) AA, LL and 5 different } Total 7

ALLIGATOR
A-2
L-2
G, T, O, R, I → 5 different

$\therefore$ No. of arrangement $= 7! = 5040$ ✓

(b) one of five diff

⊗ A (LL) (_ _ _ _ four diff) A or A (LL) - - - - A ⊗ ← one of five diff

$= 5 \times 5! \times 2$ ← (or maybe)

$= 10 \times 120 = \underline{1200}$ ✓

and the number of arrangement of 9 letters (2A & 2L)

$= \dfrac{9!}{2!\,2!} = 90720$ ✓

$\therefore$ Req. Prob $= \dfrac{1200}{90720} = \dfrac{5}{37}$ or $(\underline{0.0132})$ ✓

(c) To select 5 letters,
at least one A & at most one L

9 Letters
A-2
L-2
G, T, O, R, I - 5 diff.

Case I : A - $\underbrace{- - - -}_{4\text{ diff}}$ $= {}^5C_4 = 5$ ✓
(NO → L)

Case II. (No-L) AA $\underbrace{- - -}_{3\text{ diff}}$ $= {}^5C_3 = 10$

Case III. AL, $\underbrace{- - -}_{3\text{ diff}}$ $= {}^5C_3 = 10$

Case IV AAL, $\underbrace{- -}_{2\text{ diff}}$ $= {}^5C_2 = 10$

Total $= 5 + 10 + 10 + 10 = \underline{35}$ ✓

33(a) Find the number of different arrangement of the 9 letters in the word ACTIVATED. ---[2]

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. ---[3]

Five letters are selected at random from the 9 letter in the word ACTIVATED.

(c) Find the probability that the selection does not contain more Ts than As. ---[5]

W-22/53/Q6

Solution (a) No of arrangements of 9 letters $= \frac{9!}{2!\cdot 2!} = \underline{90720}$

ACTIVATED
A = 2
T = 2
C, I, V, E, D → 5 diff.

(b) A $\underline{5\text{ letters}}$ A

A - - - - - A ⊗ ⊗
⊗ A - - - - - A ⊗
⊗ ⊗ A - - - - - A

5 letters between 2 As $= \frac{7!}{2!} \times 3$

6 letters between 2As $= \frac{7!}{2!} \times 2$

7 letters between 2As $= \frac{7!}{2!} \times 1$

∴ Total number of arrangements with atleast 5 letters between 2As $= \frac{7!}{2!}(3+2+1) = \frac{7!}{2!} \times 6 = \underline{15120}$

(c) Selection of 5 letters from 9. ⟶ with not more Ts than As

(i) AT - - - $= 2 \times 2 \times {}^5C_3 = 40$

(ii) A - - - - $= 2 \times {}^5C_4 = 10$

(iii) AATT - $= {}^5C_1 = 5$

(iv) AAT - - $= 2 \times {}^5C_2 = 20$

(v) AA - - - $= {}^5C_3 = 10$

(vi) - - - - - $= {}^5C_5 = 1$

Total number of ways not containing more Ts than As $= 40+10+5+20+10+1 = 86$

Hence the required probability $= \frac{86}{{}^9C_5} = \frac{86}{126} = \underline{0.683}$