classmate Date 24.02.24 Page

S.1

Probability and Statistics_1

Probability

Ex 1. Solution (Revision)

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S-1

Probability - Revision

Example 1. Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

- Event X is 'Exactly two of the selected balls have the same number.
- Event Y is 'The ball selected from bag A has number 4'

(a) Find $P(X)$ --- [5]

(b) Find $P(X \cap Y)$ and hence determine whether or not events X and Y are independent. --- [3]

(c) Find the prob. that two balls are numbered 2, given that exactly two of the selected balls have the same number. -- [2]

SP-20/05/Q7

Solution: $A = \{2,4,5,8\}$, $B = \{1,3,6,8,8\}$, $C = \{2,7,8,8,8,8,9\}$

(a) $P(X) = P$(Exactly 2 balls have the same number)

$P(2, N2, 2) = \frac{1}{4} \times 1 \times \frac{1}{7} = \frac{1}{28}$ —— ①

$P(8, 8, N8) = \frac{1}{4} \times \frac{2}{5} \times \frac{3}{7} = \frac{3}{70}$

$P(8, N8, 8) = \frac{1}{4} \times \frac{3}{5} \times \frac{4}{7} = \frac{3}{35}$

$P(N8, 8, 8) = \frac{3}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{6}{35}$

$P(X) = \frac{1}{28} + \frac{3}{70} + \frac{3}{35} + \frac{6}{35} = \frac{47}{140}$ (or 0.336) ✓

(b) $P(X \cap Y) = \{4, 8, 8\} = \frac{1}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{2}{35}$

$P(Y) = \frac{1}{4}$

$P(X) \times P(Y) = \frac{47}{140} \times \frac{1}{4} \neq \frac{2}{35} \leftarrow P(X \cap Y)$

Not independent.

(c) Let Event D is two balls are number 2.

$P(D) = P(2, N2, 2) = \frac{1}{4} \times 1 \times \frac{1}{7} = \frac{1}{28}$ from ①

To find

$$P(D/X) = \frac{P(D \cap X)}{P(X)} = \frac{1/28}{\frac{47}{140}} = \frac{5}{47} \text{ (or } 0.106) ✓$$

Conditional Prob.
(Baye's Theorem)

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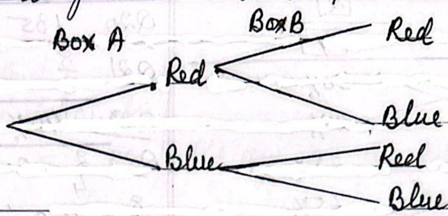
Example 2: Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. --[3]



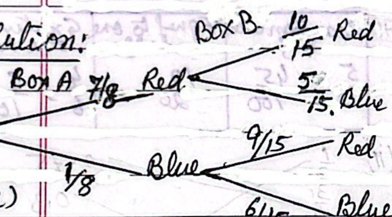
(b) Find the prob. that the two balls chosen are not the same colour. [2]

(c) Find the prob. that the ball chosen from box A is blue given that the ball chosen from box B is blue. $\boxed{M\text{-}20/52/Q6}$ --[4]

Solution:

Box A → 7 Red & 1 Blue
Box B → 9 Red & 5 Blue

(a)



(b) $P(RB \text{ or } BR) = \frac{7}{8}\times\frac{5}{15} + \frac{1}{8}\times\frac{9}{15}$

$= \frac{44}{120} = \frac{11}{30}$ (or 0.367)

(c) Conditional Prob (Baye's Theorem)

$P(A\to Blue / B\to Blue) = \frac{P(A\,blue \cap B\,blue)}{P(B\,blue)}$

$= \frac{(blue, blue)}{(blue, blue) + (Red, Blue)}$

$= \frac{\frac{1}{8}\times\frac{6}{15}}{\frac{1}{8}\times\frac{6}{15} + \frac{7}{8}\times\frac{5}{15}} = \frac{\frac{1}{20}}{\frac{41}{120}} = \frac{6}{41}$ (or 0.146)

3. Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The prob. for three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears a blue scarf, she wears a hat with prob. 0.4. When she wears a yellow scarf, she wears a hat with prob. 0.3.

(a) Find the probability that on a randomly chosen day Georgie wears a hat. --[2]

(b) Find the prob. that on a randomly chosen day Georgie wears a yellow scarf given that she does not wear a hat. --[3]

M-21/52/Q2

<u>Solution:</u> P(red) = 0.2, P(blue) = 0.45 and P(yellow) = 0.35

P(hat/red) = 1; P(hat/blue) = 0.4 and P(hat/yellow) = 0.3

(a) $P(\text{hat}) = P(\text{red}) \cdot P(\text{hat}/\text{red}) + P(\text{blue}) \cdot P(\text{hat}/\text{blue}) + P(\text{yellow}) \cdot P(\text{hat}/\text{yellow})$

$= 0.2 \times 1 + 0.45 \times 0.4 + 0.35 \times 0.3 = \underline{0.485}$ ✓

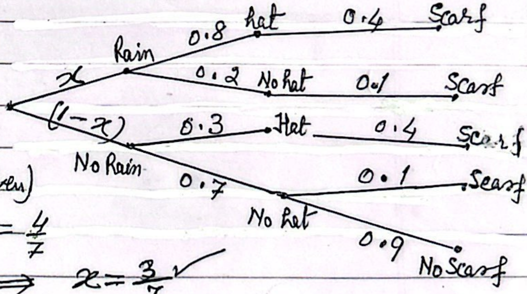
(b) $P(\text{yellow}/\text{No hat}) = \dfrac{P(\text{yellow and no hat})}{P(\text{no hat})} = \dfrac{0.35 \times 0.7}{1 - 0.485} = \underline{0.476}$ ✓

4. The probability that it will rain on any given day is x. If it is raining, the prob. that Aran wears a hat is 0.8 and if it is not raining, the prob. that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the prob. that he wears a scarf is 0.4. If he does not wear a hat, the prob. that he wears a scarf is 0.1. The prob. that on a randomly chosen day it is not raining and Aran is a not wearing a hat or a scarf is 0.36. Find the value of x. --[3]

M-23/52/Q4

Solution:

P(Not raining and not wearing hat or scarf)



$= (1-x) \times 0.7 \times .9 = 0.36$ (Given)

$\Rightarrow (1-x) = \frac{0.36}{0.63} = \frac{36}{63} = \frac{4}{7}$

$\Rightarrow x = 1 - \frac{4}{7} = \frac{3}{7} \Rightarrow \underline{x = \frac{3}{7}}$

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5. Marco has four boxes labelled K, L, M and N. He placed them in a straight line in the order K, L, M, N with K on the left. Marco also has four marbles, one red, one green, one white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows:

A: The white marble is either in box L or box M.

B: The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent. ---[3]

[M-23/52/Q5]

Solution: $P(A) = \frac{2}{4} = \frac{1}{2}$ ---① ← K L M N (W or W)

$P(B) = \frac{8}{24}$ ---②

Case I: K L M N: R ✓ ✓ ✓ (G or Y) $\left(\frac{{}^3P_2}{{}^3P_3} = \frac{6}{6}\right)$

$\frac{1}{4} \times \frac{6}{6} = \frac{6}{24}$

Case II: K L M N: × R ✓ ✓ → $\frac{1}{4} \times \frac{{}^2P_2}{{}^3P_2} = \frac{1}{4} \times \frac{2}{6} = \frac{2}{24}$

$= \frac{6}{24} + \frac{2}{24} = 8/24$

$P(A \cap B) =$ K L M N
R W G Y
R W Y G
R G W Y
R Y W G
$= \frac{4}{24} = \frac{1}{6}$ ---③

$\therefore P(A) \times P(B) = \frac{1}{2} \times \frac{8}{24} = \frac{1}{6}$ ---④

$\Rightarrow P(A) \cdot P(B) = P(A \cap B)$ from ③ & ④

$\therefore$ "A and B are independent".

Example 6 On Mondays, Rani cooks her evening meal. She has a pizza, a burger or a curry with prob. 0.35, 0.44, 0.21, respectively. When she cooks a pizza, Rani has some fruits with prob. 0.3. When she cooks a burger, she has some fruit with prob. 0.8. When she cooks curry, she never has any fruit.

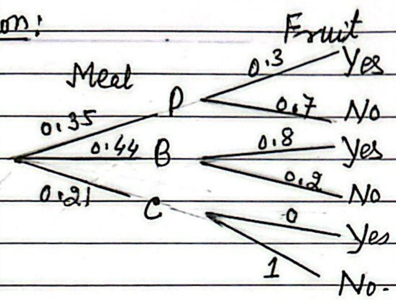
(a) Draw a fully labelled tree diagram to represent this information. --[2]

(b) Find the prob. that Rani has some fruit. --[2]

(c) Find the prob. that Rani does not have a burger given that she does not have any fruit. $\boxed{\text{S-20/51/Q5}}$ --[4]

Solution:

(a)



(b) $P \times F + B \times F + C \times F$

$0.35 \times 0.3 + 0.44 \times 0.8 + 0.21 \times 0$

$= \underline{0.457}$ ✓

(c) $P(\text{not } B / \text{not Fruit}) = \dfrac{P(B' \cap F')}{P(F')}$

Conditional Prob.
(Baye's Theorem)

$= \dfrac{0.35 \times 0.7 + 0.21 \times 1}{1 - (0.457)}$

$= \dfrac{0.455}{0.543}$

$= \dfrac{455}{543}$ (or 0.838) ✓

Example 7. A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of student in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	550

(a) Find the prob. that a randomly chosen student is at Canton college and prefers hockey. --- [1]

(b) Find the prob that a randomly chosen student is at Devar college given that he prefers soccers. --- [2]

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justify your answer. $\boxed{S\text{-}20/52/Q2}$ [2]

Solution:

(a) $\frac{56}{500}$ or $\frac{14}{125}$ (or 0.112) ✓

(b) $P(D/S) = \frac{P(D \cap S)}{P(S)} = \frac{120}{280}$ or $\frac{3}{7}$ ✓

(c) $P(\text{hockey}) = \frac{220}{500} = 0.44$

$P(\text{Amos or Benn}) = \frac{242}{500} = 0.484$

$P(\text{hocky} \cap A \text{ or } B) = \frac{104}{500} = 0.288$

for Independent.

$P(H) \times P(A \cup B) = P(H \cap (A \cup B))$

But $\frac{220}{500} \times \frac{242}{500} \neq \frac{104}{500}$ $\therefore$ Not Independent.

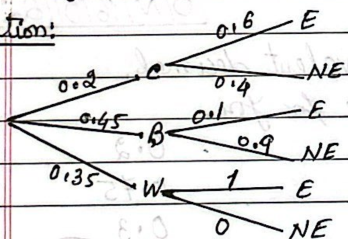
Example 8. Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the prob. that he goes by bus is 0.45 and the prob. that he walks is 0.35. When Juan goes by car, the prob the arrives early is 0.6. When he goes by bus, the prob. that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. ---[2]

(b) Find the prob. that Juan goes to college by car given that he arrives early. [S-20/53/Q1] ---[4]

Solution:

(a)



(b) $P(C/E) = \dfrac{P(C \cap E)}{P(E)}$

$= \dfrac{0.2 \times 0.6}{0.2 \times 0.6 + 0.45 \times 0.1 + 0.35 \times 1}$

$= \dfrac{0.12}{0.515} = 0.233 \left(\text{or } \dfrac{12}{515}\right)$ ✓

9. To gain a place at a science college, students first have to pass a written test and then a practical test.

Each student is allowed a maximum of two attempts at the written test. A student is only allowed a second attempt if they fail the first attempt. No student is allowed more than one attempt at the practical test. If the student fails both attempts at the written test, then they cannot attempt the practical test.

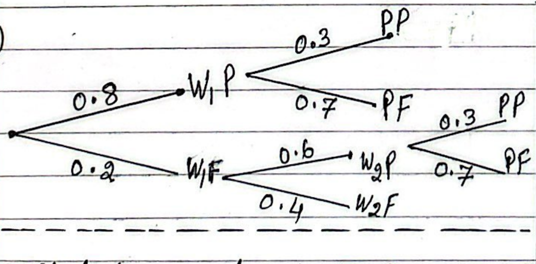
The probability that a student will pass the written test at the first attempt is 0.8. If a student fails the first attempt at the written test, the probability that they will pass at the second attempt is 0.6. The probability that a student will pass the practical test is always 0.3.

(a) Draw a tree diagram to represent this information, showing the probabilities on the branches. ---[3]

(b) Find the probility that a randomly chosen student will succeed in gaining a place at the college. -- [2]

(c) Find the probability that a randomly chosen student passes the written test at the first attempt given that the student succeeds in gaining a place at the college. [S-21/51/Q4]--[2]

Solution (a)



(b) P(Student gain a place at College)

$= P(W_1P) \times P(PP) + P(W_1F) \cdot P(W_2P) \times P(PP)$

$= 0.8 \times 0.3 + 0.2 \times 0.6 \times 0.3$

$= 0.24 + 0.036$

$= 0.276$ ✓

(c) $P(W_1P/P) = \dfrac{P(W_1 \cap \text{Practical})}{P(\text{getting place})}$

$= \dfrac{0.8 \times 0.3}{0.276}$

$= \dfrac{0.24}{0.276}$

$= 0.87$ ✓

10 On each day that Alexa goesto work, the probabilities that she travels by bus, by train or by car are 0.4, 0.35 and 0.25 respectively. When she travels by bus, the probability that arrives late is 0.55. When she travels by train, the probability that she arrives late is 0.7. When she travels by car, the prob. that the arrives late is x.

On a randomly chosen day when Alexa goes to work, the probability that she does not arrive late is 0.48.

(a) Find the value of x. ---[3]

(b) Find the probability that Alexa travels to work by train given that she arrives late. ---[3]

S-21/52/Q3

Solution: P(Alexa does not arrive late) = 0.48

(a) $\Rightarrow$ P(Alexa arrives late) = $1-0.48=0.52$ ---(i)

$\therefore$ P(Alexa arrives late) $=0.4\times0.55+0.35\times0.7+0.25x=0.52$ from (i)

$$\Rightarrow 0.22+0.245+0.25x=0.52$$

$$\Rightarrow 0.25x=0.52-0.465=0.055$$

$$\Rightarrow x=\frac{0.055}{0.25}=0.22$$

$$\Rightarrow \underline{x=0.22}$$

(b) $$P(\text{Train}/\text{Late})=\frac{P(\text{Train}\cap\text{Late})}{P(\text{Late})}=\frac{0.35\times0.7}{0.52}$$

$$=\frac{0.245}{0.52}$$

$$=\underline{0.471}$$

(Q11 is below)

12 In the region of Arka, the total number of households in three villages of Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a)(i) Find the probability that a randomly chosen household is in Shan and has poor broadband service. ---[1]

(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in shan. ---[2]

[S-21/53/Q7(a)]

Solution (a) (i): $P(\text{household in Shan has poor broadband}) = \frac{40}{800} = \frac{1}{20}$ (or 0.05) ✓

(a) (ii) $P(\text{Good broadband}/\text{from Shan}) = \frac{177}{233+177+40} = \frac{177}{440} = 0.402$ ✓

Alternate method for question (a)(ii) (conditional probability)

$$P(G/S) = \frac{P(G \cap S)}{P(S)} = \frac{\frac{177}{800}}{\frac{233+177+40}{800}} = \frac{177}{440} = 0.402$$ ✓

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11 Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6 are are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.
Find the probability that the score is 4 on a single throw of three dice. [3]

[S-21/53/Q4(a)]

Solution: Single throw of dice → 1, 2, 3, 4, 5, 6
for total of 4 in three throws → 1,1,2; 1,2,1, 2,1,1

$$\text{Probability}(\text{Total } 4) = 3 \times \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{72}$$ ✓

13. Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the prob. that Janice completes it at the first attempt is 0.6. If she fails her first attempt, the prob. that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately move on to level 2.
- For level 2, the prob. that Janice completes it at the first attempt is 0.4. If she fails her first attempt, the prob. that she completes it at the second attempt is 0.2.

(a) Show that the prob. that Janice move on to level 2 is 0.72. --[1]

(b) Find the prob. that Janice finishes the game. --[3]

(c) Find the prob. that Janice fails exactly one attempt, given that she finishes the game. --[4]

[S-22/S1/Q6]

Solution: (a) P(Janice move to level 2) = P(Level 1 completes in first attempt) + P(Level 1 fails in first & complete in 2nd)

$$= 0.6 + (1-0.6)\times 0.3 = 0.6 + 0.4\times 0.3 = \underline{0.72} \quad (1)$$

(b) P(Janice finish the game) = P(Janice complete level 1) × (Completes level 2)

$$= 0.72 \times (0.4 + (1-0.4)\times 0.2)$$

$$= 0.72 \times (0.4 + 0.6\times 0.2) = 0.72 \times 0.52$$

$$= \underline{0.3744} \quad (2)$$

(C) P(fails first or second attempt / finishes the game)

$$= \frac{P(\text{fails first or second level} \cap \text{finishes game})}{\text{Part (b)}} \quad (3)$$

Numerator of (3) $= P(S_1 \cdot F_2 \cdot S_2) + P(F_1 S_1 S_2) = 0.6\times(1-0.4)\times 0.2 + (1-0.6)\times 0.3\times 0.4$

$$= 0.6\times 0.6\times 0.2 + 0.4\times 0.3\times 0.4$$

$$= 0.072 + 0.048 = 0.12 \quad (4)$$

from (2) and (4) in (3)

Required Prob $= \dfrac{0.12}{0.3744} = \underline{0.321}$ ✓

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14. Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the prob. that all 3 eggs chosen contain the same colour sweet. ---[4]

(b) Find the prob. that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. ---[2]

(c) Find the prob. that at least one of Hanna's three children chooses an egg that contains an orange sweet. ---[3]

S-22|52|Q7

Solution: 3 red + 4 orange + 5 yellow = 12 sweets

(a) $P(\text{Same colour on all 3}) = P(RRR) + P(OOO) + P(YYY)$

$$= \frac{3}{12}\times\frac{2}{11}\times\frac{1}{10} + \frac{4}{12}\times\frac{3}{11}\times\frac{2}{10} + \frac{5}{12}\times\frac{4}{11}\times\frac{3}{10} =$$

$$= \frac{6}{1320} + \frac{24}{1320} + \frac{60}{1320} = 90/1320 = \underline{\frac{3}{44}} \checkmark \text{---} 1$$

(b) $P(YYY / \text{all Same colour})$

$$= \frac{P(YYY) \cap P(\text{all same colour})}{P(\text{all Same Colour})}$$

$$= \frac{\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10}}{3/44} = \frac{60}{1320}\times\frac{44}{3} = \underline{2/3} \ (\text{or } 0.667)$$

(c) $P(\text{atleast one orange}) = 1 - P(\text{None of three has orange})$

$$1 - \frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$$

$\left\{ \begin{array}{l} \text{No orange} \\ = Y \text{ or red} \\ = 5+3=8 \end{array} \right.$

$$= 1 - 14/55$$

$$= \underline{41/55} \checkmark$$

15. Sajid is practising for a long jump complition. He counts any jump that is longer than 6m as a success. On any day, the prob. that he has a success with his first jump is 0.2. For any subsequent jump, the prob. of a success is 0.3 if the previous jump was a sccess and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. ---[2]

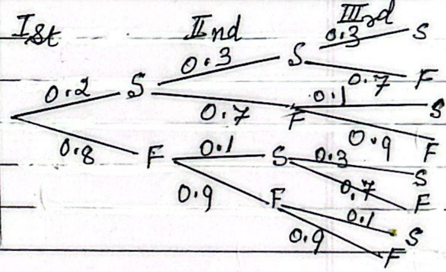
(b) Find the prob. that Sajid has exactly one success given that he has at least one success. --[5]

On another day, Sajid makes six jumps.

(c) Find the prob. that only his first three jumps are successes or only his last three jumps are successes. --[3]

S-22 | 53 | Q6

Solution (a) Tree diagram.



(b) P(Exactly one Success)

$= SFF + FSF + FFS$

$= 0.2\times .7\times .9 + 0.8\times 0.1\times 0.7 + 0.8\times 0.9\times 0.1$

$= 0.254$ ✓

P(atleast one Success) $= 1 - P(\text{No Success}) = 1 - 0.8\times 0.9\times 0.9 = 0.352$ ✓

P(exactly one success / atleast one success) $= \frac{0.254}{0.352} = 0.722$

$= 0.722$ ✓

(c) P(first three jumps success) $= SSSFFF$

$= 0.2\times 0.3\times 0.3\times 0.7\times 0.9\times .9$

$= 0.010206$ — ①

and P(Last three Success) $= FFFSSS$

$= 0.8\times 0.9\times 0.9\times 0.1\times 0.3\times 0.3 = 0.005832$ — ②

∴ P(First three success or the last three success)

$= 0.010206 + 0.005832$

$= 0.0160$ ✓ (38)

Probability

16. Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events A and B are defined as follows:

A: The sum of the two numbers is less then 6.

B: The difference between the two numbers is atmost 2.

(a) Determine whether or not the events A and B are independent. ---[4]

(b) Find $P(B/A')$ ---[3]

S-23/53/Q5

Solution: Two dice; $n(S) = 6\times6 = 36$

(a) A = Sum of two numbers is less than 6 = Sum $\{2,3,4,5\}$

$n(A) = 10$ ✓ $= \{\{(1,1),(1,2),(2,1),(1,3),(2,2),(3,1),(1,4),(2,3),(3,2),(4,1)\}$

B = Difference is $\leq 2 = \{0,1,2\}$

$= \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$
$(1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(4,5),(5,4),(6,5),(5,6)$
$(1,3),(3,1),(2,4),(4,2),(3,5),(5,3),(4,6),(6,4)\}$

$n(B) = 24$

$(A\cap B) = \{(1,1),(2,2),(1,2),(2,1),(2,3),(3,2),(1,3),(3,1)\}$

$n(A\cap B) = 8$

$\therefore P(A) = \frac{10}{36} = \frac{5}{36}$ ✓

$P(B) = \frac{24}{36} = 2/3$

$P(A\cap B) = \frac{8}{36}$

Now $P(A)\times P(B) = \frac{5}{36}\times\frac{2}{3} \neq \frac{8}{36}$ $\therefore$ A and B are **not** independent events.

(b) $P(B/A') = \frac{P(B\cap A')}{P(A')}$ $\quad P(B\cap A') = \frac{16}{36}$

$= \frac{16}{36} \Big/ \left(1-\frac{10}{36}\right)$

$= \frac{8}{13}$ ✓

17 Two ordinary fair dice, one red and other blue, are thrown,
Event A is "the score on the red die is divisible by 3"
Event B is "the sum of the two scores is at least 9"

(a) Find $P(A \cap B)$ --- [2]

(b) Hence determine whether or not the event A and are independent. --- [2]

[W-20 / 51 / Q 1]

Solution:

Blue \ Red	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(a) $A = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

$B = \{(3,6), (6,3), (4,5), (5,4), (6,4), (4,6), (5,5), (5,6), (6,5), (6,6)\}$

$A \cap B =$ "red die is div by 3 and sum is atleast 9.

$\{(3,6), (6,3), (6,4), (6,5), (6,6)\}$

$P(A \cap B) = \frac{5}{36}$ ✓

(b) $P(A) = \frac{12}{36}$, $P(B) = \frac{10}{36}$

$P(A) \times P(B) = \frac{12}{36} \times \frac{10}{36} = \frac{5}{54} \neq \frac{5}{36}$ $(P(A \cap B))$

∴ A and B are **not** independent.

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18. The probability that a student at a large music college plays in the band is 0.6. For a student who plays in a band, the prob that she also sings in choir is 0.3. For a student who does not play in the band, the prob. that she sings in the choir is x. The prob. that a randomly chosen student from the college does not sing in the choir is 0.58.

(a) Find the value of x. ---[3]

Two students from the college are chosen at random.

(b) Find the prob. that both students play in the band and both student sing in the choir. [W-20/51/Q2.] ---[2]

Solution:

(a) $P(B) = 0.6$, $P(B^c) = 1 - 0.06 = 0.4$; $P(B^c \cap C) = x$

$P(B \cap C) = 0.3$ $P(B \cap C)^c = 1 - .3 = 0.7$

$P(B) \times (B \cap C)^c + P(B)^c \times (1 - x) = .58$

$0.6 \times 0.7 + 0.4(1 - x) = 0.58$

$\Rightarrow 0.42 + 0.4 - 0.4x = 0.58$

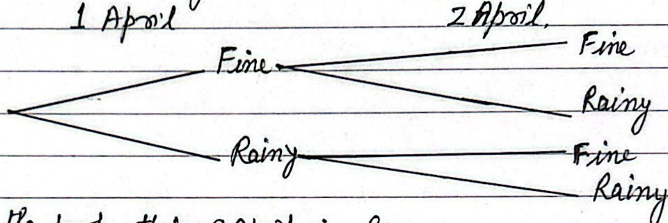
$\Rightarrow 0.4x = 0.24$

$\Rightarrow x = \underline{0.6}$ ✓

(b) $(0.6 \times 0.3)^2 = \underline{0.0324}$ ✓

19. In a certain country, the whether each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the prob that a rainy day is followed by a fine day is 0.4. The prob that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. ---[1]



(b) Find the prob that 2 April is fine. --[2]

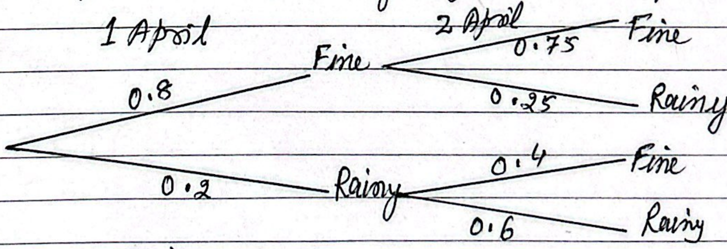
Let X be the event that 1 April is fine and Y be event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$ $\boxed{W\text{-}20/52/Q4}$ ---[3]

(d) Find the prob. that 1 April is fine given that 3 April is rainy.--[3].

Solution:

(a)



(b) $P(\text{2 April is fine}) = 0.8 \times 0.75 + 0.2 \times 0.4 = 0.6 + 0.08 = \underline{0.68}$ ✓

(c) $P(X \cap Y) = 0.8 \times 0.75 \times 0.25 + 0.8 \times 0.25 \times 0.6 = 0.15 + 0.12$
$= \underline{0.27}$ ✓

(d) $P(Y) = P(\text{3 April is rainy}) = \text{their (c)} \rightarrow 0.27 + 0.2 \times 0.4 \times 0.25$
$+ 0.2 \times 0.6 \times 0.6 = 0.362$

$$P(X/Y) = \frac{P(X \cap Y)}{P(Y)} = \frac{0.27}{0.362} \text{ (Part (c))}$$

$$= \underline{0.746} \checkmark$$

20. Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $4/5$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $4/9$. ---[3]

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. ---[3]

(c) Given that $E(X) = \frac{32}{15}$, find $var(X)$. ---[2]

W-20/53/Q6

Solution (a) Different case of getting exactly two heads.

In order ABC

$$\begin{cases} P(H_A) = 2/3 \\ P(H_B) = 2/3 \\ P(H_C) = 4/5 \end{cases}$$

(i) $P(HHT) = \frac{2}{3} \times \frac{2}{3} \times \frac{1}{5} = \frac{4}{45}$

(ii) $P(HTH) = \frac{2}{3} \times \frac{1}{3} \times \frac{4}{5} = 8/45$

(iii) $P(THH) = \frac{1}{3} \times \frac{2}{3} \times \frac{4}{5} = 8/45$

$P(\text{Exact } 2H) = 4/45 + 8/45 + 8/45 = \frac{20}{45} = 4/9$ ✓

(b)

x	0	1	2	3
$P(x)$	$1/45$	$8/45$	$20/45$	$16/45$

$P(0) = TTT = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{5}$

$P(1H) = TTH + THT + HTT$

$= \frac{1}{3} \times \frac{1}{3} \times \frac{4}{5} + \frac{1}{3} \times \frac{2}{3} \times \frac{1}{5} + \frac{2}{3} \times \frac{1}{3} \times \frac{1}{5} = 8/45$

$P(2H) = 4/9$

$P(3H) = 2/3 \times \frac{2}{3} \times \frac{4}{5} = \frac{16}{45}$

(c) $Var(X) = \Sigma p_i x_i^2 - [E(X)]^2$ --- (1)

$\Sigma p_i x_i^2 = \frac{1}{45} \times 0^2 + \frac{8}{45} \times 1^2 + \frac{20}{45} \times 2^2 + \frac{16}{45} \times 3^2 = 232/45$

for (1)

$Var(X) = \frac{132}{45} - (32/15)^2$ [Given $E(X) = \frac{32}{15}$]

$= \frac{232}{45} - \frac{1024}{225} = \frac{1160 - 1024}{225} = \frac{136}{225}$ ✓ (or 0.604)

21. For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.20 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the prob. that she has a biscuit is 0.6. When she has milk, she never has a biscuit. Find the prob. that Suki has tea given that she does not have a biscuit. --[5]

[W-21/51/Q3]

Solution: $P(T/B') = \frac{P(T\cap B')}{P(B')}$ --- ①

$P(B') = P(C)\times P(B'/C) + P(T)\times P(B'/T) + P(M)\cdot P(B'/M)$

$= 0.45\times0.7 + 0.35\times0.4 + 0.2\times1 = 0.655$ --- ②

$P(T\cap B') = 0.35\times0.4 = 0.14$ --- ③

from ① $P(T/B') = \frac{0.14}{0.655} = 0.214$ ✓ from ② & ③

22. Each of the 180 students at a college play exactly one of the piano, the guitar and drums. The numbers of male and female students who play piano, the guitar and drums are given as:

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the prob. that the student plays guitar --[1]

(b) Find the prob. that the student is male given that the student plays the drums. ---[2]

(c) Determine whether the events "the student plays the guitar" and "the student is female" are independent, justify your answer. --[2]

[W-21/52/Q1]

Solution (a) $P(\text{student plays Guitar}) = \frac{44+38}{180}$

$= \frac{82}{180}$ (or $\frac{41}{90}$) ✓

(b) $P(M/D) = \frac{P(M\cap D)}{P(D)}$ ——— ①

$= \frac{11/180}{\frac{20}{180}+\frac{11}{180}} = \frac{11}{180}\times\frac{180}{31}$

$= \frac{11}{31}$ ✓ (or 0.355)

(c) $P(F) = \frac{42+38+20}{180} = 5/9$ --- ①

$P(G) = \frac{44+38}{180} = \frac{82}{180} = \frac{41}{90}$ --- ②

$P(F\cap G) = 38/180 = \frac{19}{90}$ ——— ③

Now $P(F)\times P(G) = \frac{5}{9}\times\frac{41}{90} = \frac{41}{162} \neq \frac{19}{90}$

or $\neq P(F\cap G)$

$\therefore$ Not independent.

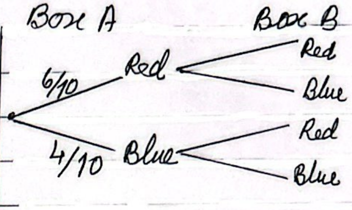
23. Box A contain 6 red balls and 4 blue balls. Box B contains x red balls and 9 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Complete the tree diagram, giving the remaining four prob. in terms of x. --- [3]

(b) Show that the probability that both balls chosen are blue is $4/(x+10)$ --- [2]

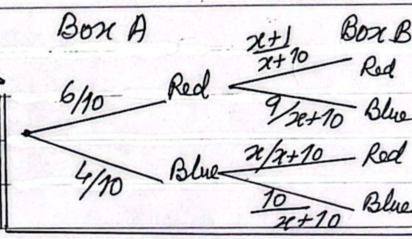
It is given that the prob. that both balls chosen are blue is $1/6$.

(c) Find the prob. correct to 3 s.f, that the ball chosen from box A is red given that the ball chosen form box B is red. --- [5]



W-21/53/Q7

Solution (a) Tree diagram



(b) $P(\text{Both blue}) = \frac{4}{10} \times \frac{10}{x+10} = \frac{4}{x+10}$

(c) $P(\text{both blue}) = \frac{4}{x+10} = \frac{1}{6}$ (given)

$\Rightarrow x+10 = 24 \Rightarrow x = 14$

Now $P(A\text{-red}/B\text{-red}) = \frac{P(A\,red \cap B\,red)}{P(B\,red)} = \frac{\frac{6}{10} \times \frac{x+1}{x+10}}{\frac{6}{10} \times \frac{x+1}{x+10} + \frac{4}{10} \times \frac{x}{x+10}}$

$= \frac{\frac{6}{10} \times \frac{15}{24}}{\frac{6}{10} \times \frac{15}{24} + \frac{4}{10} \times \frac{14}{24}} = \frac{3/8}{73/120} = \frac{45}{73}$ (or 0.616)

24. A game is played with an ordinary fair 6-sided die. A player throws the die once. If the result is 2,3,4 or 5, that result is the player's score and the player does not throw the die again. If the result is 1 or 6, the player throws the die a second time and the player's score is the sum of the two numbers from the two scores.

(a) Draw a fully labelled tree diagram to represent this information. --[2]

Events A and B are defined as; A: the player's score is 5, 6, 7, 8, or 9.

B: the player has two throws

(b) Show that $P(A) = \frac{1}{3}$ ---[3]

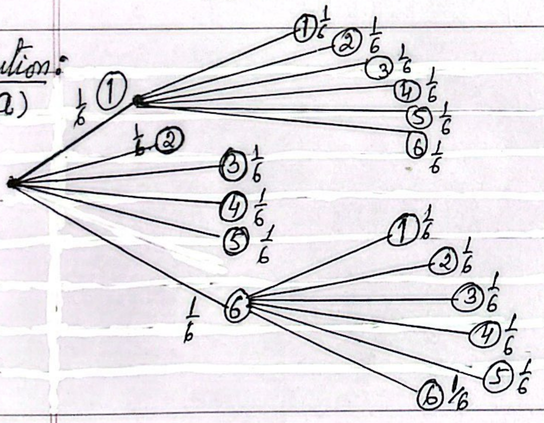
(c) Determine whether or not events A and B are independent. --[2]

(d) Find $P(B/A')$ --[3]

W-22/51/Q5

Solution:

(a)



(b) 5 Comes (1+4) or 5 $P = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6} = \frac{7}{36}$

6 Comes (1+5), $P = \frac{1}{6}\times\frac{1}{6} = 1/36$

7 Comes (1+6) or (6+1) $P = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6} = 2/36$

8 Comes (6+2) $P = \frac{1}{6}\times\frac{1}{6} = 1/36$

9 Comes (6+3), $P = \frac{1}{6}\times\frac{1}{6} = 1/36$

$P(A) = \frac{7}{36}+\frac{1}{36}+2/36+1/36+\frac{1}{36} = \frac{12}{36} = 1/3$ ✓

(c) $P(B) = \frac{2}{6} = \frac{1}{3}$ ✓

$P(A) = 1/3$

$P(A\cap B) = 6/36$ ✓

$P(A)\times P(B) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$ ✓

here $P(A\cap B) \neq P(A)\cdot P(B)$

$\{as\ \frac{6}{36} \neq \frac{1}{9}\}$

$\therefore$ Events A and B are **not independent**.

(d) $P(B/A') = \frac{P(B\cap A')}{P(A')}$ ---①

$B\cap A' = (1,1), (1,2), (1,3), (6,4), (6,5), (6,6)$

$P(B\cap A') = 6/36$

$P(A') = 1-\frac{1}{3} = 2/3$

from ① $P(B/A') = \frac{6/36}{2/3}$

$= \frac{6}{36}\times\frac{3}{2} = \frac{1}{4}$

$= 0.25$ ✓

25. On any day, kino travels to school by bus, by car or on foot with probabilities 0·2, 0·1 and 0·7 respectively. The probabilities that he is late when he travels by bus is x. The prob. that he is late when he travels by car is $2x$ and the prob. he is late when he travels on foot is 0·25.

The prob. that, on a randomly chosen day, kino is late is 0·235.

(a) Find the value of x. ---[3]

(b) Find the prob. that, on a chosen randomly chosen day, kino travels to school by car given that he is not late. --[2]

W-22/52/Q1

Solution: P(kino is late) = P(Bus)·P(late/Bus) + P(Car)P(late/car) + P(Foot)·P(late/Foot)

(a)

$$= 0\cdot2\times x + 0\cdot1\times 2x + 0\cdot7\times0\cdot25 = 0\cdot235 \text{ Given}$$

$$\Rightarrow 0\cdot2x + 0\cdot2x + 0\cdot175 = 0\cdot235$$

$$\Rightarrow 0\cdot4x = 0\cdot060$$

$$x = \frac{0\cdot06}{0\cdot4} = \underline{0\cdot15}$$

(b)

$$P(\text{Car}/\text{not late}) = \frac{P(\text{Car and not late})}{P(\text{not late})} \qquad \left\{P(B/A) = \frac{P(B\cap A)}{P(A)}\right\}$$

$$= \frac{0\cdot1\times(1-0\cdot3)}{1-0\cdot235} \qquad \left\{\begin{aligned}&\text{Given } P(\text{Car and late})\\ &= 2x = 2\times0\cdot15\\ &= 0\cdot3\end{aligned}\right.$$

$$= \frac{0\cdot1\times0\cdot7}{0\cdot765}$$

$$= \frac{0\cdot07}{0\cdot765} = \underline{0\cdot0915}$$

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26. Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is 1/4, independently of all other throws. Eric throws all three coins at the same time. Events A and B are defined as follows:

A: all three coins show the same result.

B: at least one of the biased coins shows a head.

(a) Show that $P(B) = 7/16$. ---[2]

(b) Find $P(A/B)$ ---[2]

The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw the prob. distribution table for X. ---[3]

W-22/52/Q5

Solution: (a) Two biased coins: $P(H) = 1/4$, $P(T) = 3/4$
① & ②

Case I. $P(BH_1, BT_2) = \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

Case II $P(BT_1, BH_2) = \frac{3}{4} \times \frac{1}{4} = 3/16$

Case III $P(BH_1, BH_2) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

$\therefore P(B) = 3/16 + 3/16 + \frac{1}{16} = \frac{7}{16}$ ✓

Alternate method:

(a) P(atleast one biased coin show head)

1 − P(None of the biased coin show head)

$= 1 - P(BT_1, BT_2) = 1 - \left(\frac{3}{4}\right)^2 = \frac{7}{16}$ ✓

(b) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(\text{All three heads})}{P(B)}$

$= \frac{\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4}}{\frac{7}{16}} = \frac{\frac{1}{32}}{\frac{7}{16}}$

$= \frac{1}{14} = 0.0714$ ✓

(c) $P(0) = P(TTT) = \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = 9/32$

$P(1H) = \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{3}{4} = \frac{15}{32}$ (TTH, THT, HTT)

$P(2H) = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4} + \frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} = 7/32$ (THH, HTH,

$P(3H) = \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$ (HHH)

X	0	1	2	3
P(X)	9/32	15/32	7/32	1/32
	0.28125	0.46875	0.21875	0.03125

27. Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game. Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on the first turn. ---[2]

(b) Find the prob. that Tom wins the game on his second turn. ---[4]

(c) Find the prob. that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. ---[2]

W-22/53/Q7

Solution: White - 5 and Red - 3; Total = 8 discs

Given: Sam removes the first disc. (drawn without replacement)

(a) P(Tom removes Red on first turn)

$= P(S_R \cdot T_R) \text{ or } P(S_W \cdot T_R)$

$= \frac{3}{8} \times \frac{2}{7} + \frac{5}{8} \times \frac{3}{7} = \frac{6+15}{56} = \frac{21}{56}$

$= 0.375$ ✓

(b) P(Tom wins the game on his Second turn)

$= P(S_R \cdot T_R \cdot S_W \cdot T_R) + P(S_W \cdot T_R \cdot S_R \cdot T_R) + P(S_W \cdot T_R \cdot S_W \cdot T_R)$

$= \frac{3}{8} \times \frac{2}{7} \times \frac{5}{6} \times \frac{1}{5} + \frac{5}{8} \times \frac{3}{7} \times \frac{2}{6} \times \frac{1}{5} + \frac{5}{8} \times \frac{3}{7} \times \frac{4}{6} \times \frac{2}{5}$

$= \frac{30}{1680} + \frac{30}{1680} + \frac{120}{1680}$

$= \frac{180}{1680} = \frac{3}{28} = 0.107$ ✓

(C) $P(S_{R\,1st} / T \text{ wins on Second Turn})$

$= \frac{P(S_R\, T_R\, S_W\, T_R)}{P \text{ in Part (b)}}$

$= \frac{30}{1680} \Big/ \frac{180}{1680}$

$= \frac{30}{180}$

$= \frac{1}{6}$

$= 0.167$ ✓

(Conditional Probability:)

$P(A/B) = \frac{P(A \cap B)}{P(B)}$