PROBABILITY AND STATISTICS -1
9709
(March, June and November series 2020 – 2023 With marking scheme)

Probability

EXERCISE -1

MANJULA BALAJI

1) SP-2020_9709_5Q7

Bag A contains 4 balls numbered 2, 4, 5, 8. Bag B contains 5 balls numbered 1, 3, 6, 8, 8. Bag C contains 7 balls numbered 2, 7, 8, 8, 8, 8, 9. One ball is selected at random from each bag.

- Event X is 'exactly two of the selected balls have the same number'.
- Event Y is 'the ball selected from bag A has number 4'.

(a) Find $P(X)$. [5]

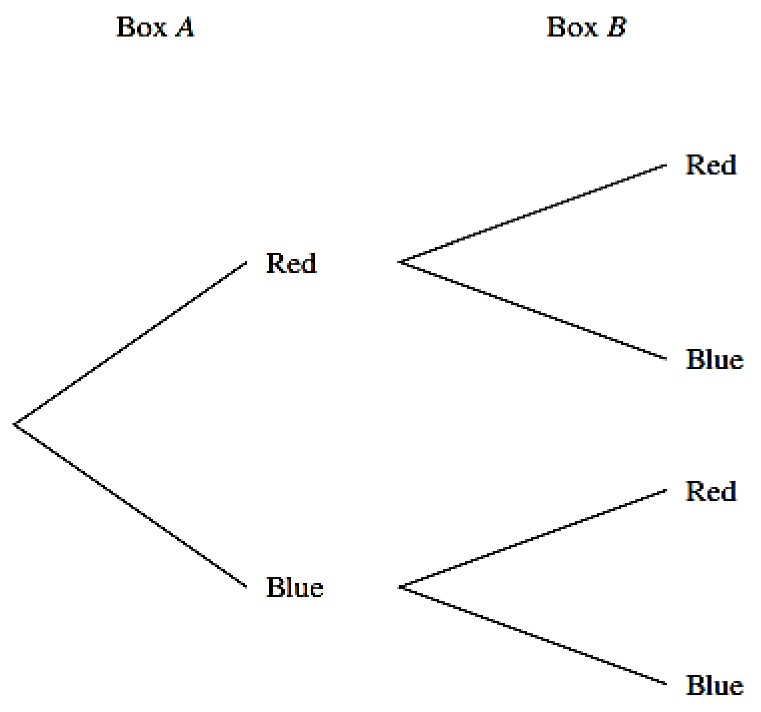
(b) Find $P(X \cap Y)$ and hence determine whether or not events X and Y are independent. [3]

(c) Find the probability that two balls are numbered 2, given that exactly two of the selected balls have the same number. [2]

2) MARCH 2020_9709_52 Q6

Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. [3]



(b) Find the probability that the two balls chosen are not the same colour. [2]

(c) Find the probability that the ball chosen from box A is blue given that the ball chosen from box B is blue. [4]

3) MARCH 2021_9709_52 Q2

Georgie has a red scarf, a blue scarf and a yellow scarf. Each day she wears exactly one of these scarves. The probabilities for the three colours are 0.2, 0.45 and 0.35 respectively. When she wears a red scarf, she always wears a hat. When she wears a blue scarf, she wears a hat with probability 0.4. When she wears a yellow scarf, she wears a hat with probability 0.3.

(a) Find the probability that on a randomly chosen day Georgie wears a hat. [2]

(b) Find the probability that on a randomly chosen day Georgie wears a yellow scarf given that she does not wear a hat. [3]

4) MARCH 2023_9709_52 Q4

The probability that it will rain on any given day is x. If it is raining, the probability that Aran wears a hat is 0.8 and if it is not raining, the probability that he wears a hat is 0.3. Whether it is raining or not, if Aran wears a hat, the probability that he wears a scarf is 0.4. If he does not wear a hat, the probability that he wears a scarf is 0.1. The probability that on a randomly chosen day it is not raining and Aran is not wearing a hat or a scarf is 0.36.

Find the value of x. [3]

5) MARCH 2023_9709_52 Q5

Marco has four boxes labelled K, L, M and N. He places them in a straight line in the order K, L, M, N with K on the left. Marco also has four coloured marbles: one is red, one is green, one is white and one is yellow. He places a single marble in each box, at random. Events A and B are defined as follows.

A: The white marble is in either box L or box M.

B: The red marble is to the left of both the green marble and the yellow marble.

Determine whether or not events A and B are independent. [3]

6) JUNE 2020_9709_51 Q5

On Mondays, Rani cooks her evening meal. She has a pizza, a burger or a curry with probabilities 0.35, 0.44, 0.21 respectively. When she cooks a pizza, Rani has some fruit with probability 0.3. When she cooks a burger, she has some fruit with probability 0.8. When she cooks a curry, she never has any fruit.

(a) Draw a fully labelled tree diagram to represent this information. [2]

(b) Find the probability that Rani has some fruit. [2]

(c) Find the probability that Rani does not have a burger given that she does not have any fruit. [4]

7) JUNE 2020_9709_52 Q2

A total of 500 students were asked which one of four colleges they attended and whether they preferred soccer or hockey. The numbers of students in each category are shown in the following table.

	Soccer	Hockey	Total
Amos	54	32	86
Benn	84	72	156
Canton	22	56	78
Devar	120	60	180
Total	280	220	500

(a) Find the probability that a randomly chosen student is at Canton college and prefers hockey. [1]

(b) Find the probability that a randomly chosen student is at Devar college given that he prefers soccer. [2]

(c) One of the students is chosen at random. Determine whether the events 'the student prefers hockey' and 'the student is at Amos college or Benn college' are independent, justifying your answer. [2]

8) JUNE 2020_9709_53 Q1

Juan goes to college each day by any one of car or bus or walking. The probability that he goes by car is 0.2, the probability that he goes by bus is 0.45 and the probability that he walks is 0.35. When Juan goes by car, the probability that he arrives early is 0.6. When he goes by bus, the probability that he arrives early is 0.1. When he walks he always arrives early.

(a) Draw a fully labelled tree diagram to represent this information. [2]

(b) Find the probability that Juan goes to college by car given that he arrives early. [4]

9) JUNE 2021_9709_51 Q4

To gain a place at a science college, students first have to pass a written test and then a practical test.

Each student is allowed a maximum of two attempts at the written test. A student is only allowed a second attempt if they fail the first attempt. No student is allowed more than one attempt at the practical test. If a student fails both attempts at the written test, then they cannot attempt the practical test.

The probability that a student will pass the written test at the first attempt is 0.8. If a student fails the first attempt at the written test, the probability that they will pass at the second attempt is 0.6. The probability that a student will pass the practical test is always 0.3.

(a) Draw a tree diagram to represent this information, showing the probabilities on the branches. [3]

(b) Find the probability that a randomly chosen student will succeed in gaining a place at the college. [2]

(c) Find the probability that a randomly chosen student passes the written test at the first attempt given that the student succeeds in gaining a place at the college. [2]

10) JUNE 2021_9709_52 Q3

On each day that Alexa goes to work, the probabilities that she travels by bus, by train or by car are 0.4, 0.35 and 0.25 respectively. When she travels by bus, the probability that she arrives late is 0.55. When she travels by train, the probability that she arrives late is 0.7. When she travels by car, the probability that she arrives late is x.

On a randomly chosen day when Alexa goes to work, the probability that she does not arrive late is 0.48.

(a) Find the value of x. [3]

(b) Find the probability that Alexa travels to work by train given that she arrives late. [3]

11) JUNE 2021_9709_53 Q4

Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

(a) Find the probability that the score is 4 on a single throw of the three dice. [3]

12) JUNE 2021_9709_53 Q7

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

13) JUNE 2022_9709_51 Q6

Janice is playing a computer game. She has to complete level 1 and level 2 to finish the game. She is allowed at most two attempts at any level.

- For level 1, the probability that Janice completes it at the first attempt is 0.6. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.3.
- If Janice completes level 1, she immediately moves on to level 2.
- For level 2, the probability that Janice completes it at the first attempt is 0.4. If she fails at her first attempt, the probability that she completes it at the second attempt is 0.2.

(a) Show that the probability that Janice moves on to level 2 is 0.72. [1]

(b) Find the probability that Janice finishes the game. [3]

(c) Find the probability that Janice fails exactly one attempt, given that she finishes the game. [4]

14) JUNE 2022_9709_52 Q7

Hanna buys 12 hollow chocolate eggs that each contain a sweet. The eggs look identical but Hanna knows that 3 contain a red sweet, 4 contain an orange sweet and 5 contain a yellow sweet. Each of Hanna's three children in turn randomly chooses and eats one of the eggs, keeping the sweet it contained.

(a) Find the probability that all 3 eggs chosen contain the same colour sweet. [4]

(b) Find the probability that all 3 eggs chosen contain a yellow sweet, given that all three children have the same colour sweet. [2]

(c) Find the probability that at least one of Hanna's three children chooses an egg that contains an orange sweet. [3]

15) JUNE 2022_9709_53 Q6

Sajid is practising for a long jump competition. He counts any jump that is longer than 6 m as a success. On any day, the probability that he has a success with his first jump is 0.2. For any subsequent jump, the probability of a success is 0.3 if the previous jump was a success and 0.1 otherwise. Sajid makes three jumps.

(a) Draw a tree diagram to illustrate this information, showing all the probabilities. [2]

(b) Find the probability that Sajid has exactly one success given that he has at least one success. [5]

On another day, Sajid makes six jumps.

(c) Find the probability that only his first three jumps are successes or only his last three jumps are successes. [3]

16) JUNE 2023_9709_53 Q5

Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events A and B are defined as follows.

A: The sum of the two numbers is less than 6.

B: The difference between the two numbers is at most 2.

(a) Determine whether or not the events A and B are independent. [4]

(b) Find $P(B \mid A')$. [3]

17) OCT 2020_9709_51 Q1

Two ordinary fair dice, one red and the other blue, are thrown.

Event A is 'the score on the red die is divisible by 3'.

Event B is 'the sum of the two scores is at least 9'.

(a) Find $P(A \cap B)$. [2]

(b) Hence determine whether or not the events A and B are independent. [2]

18) OCT 2020_9709_51 Q2

The probability that a student at a large music college plays in the band is 0.6. For a student who plays in the band, the probability that she also sings in the choir is 0.3. For a student who does not play in the band, the probability that she sings in the choir is x. The probability that a randomly chosen student from the college does not sing in the choir is 0.58.

(a) Find the value of x. [3]

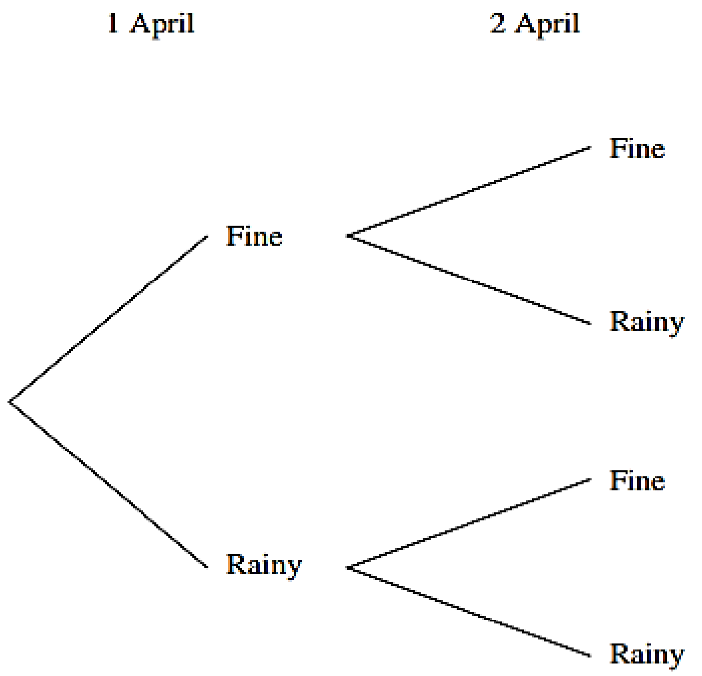
Two students from the college are chosen at random.

(b) Find the probability that both students play in the band and both sing in the choir. [2]

19) OCT 2020_9709_52 Q4

In a certain country, the weather each day is classified as fine or rainy. The probability that a fine day is followed by a fine day is 0.75 and the probability that a rainy day is followed by a fine day is 0.4. The probability that it is fine on 1 April is 0.8. The tree diagram below shows the possibilities for the weather on 1 April and 2 April.

(a) Complete the tree diagram to show the probabilities. [1]



(b) Find the probability that 2 April is fine. [2]

Let X be the event that 1 April is fine and Y be the event that 3 April is rainy.

(c) Find the value of $P(X \cap Y)$. [3]

(d) Find the probability that 1 April is fine given that 3 April is rainy. [3]

20) OCT 2020_9709_53 Q6

Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

(c) Given that $E(X) = \frac{32}{15}$, find $Var(X)$. [2]

21) OCT 2021_9709_51 Q3

For her bedtime drink, Suki has either chocolate, tea or milk with probabilities 0.45, 0.35 and 0.2 respectively. When she has chocolate, the probability that she has a biscuit is 0.3. When she has tea, the probability that she has a biscuit is 0.6. When she has milk, she never has a biscuit.

Find the probability that Suki has tea given that she does not have a biscuit. [5]

22) OCT 2021_9709_52 Q1

Each of the 180 students at a college plays exactly one of the piano, the guitar and the drums. The numbers of male and female students who play the piano, the guitar and the drums are given in the following table.

	Piano	Guitar	Drums
Male	25	44	11
Female	42	38	20

A student at the college is chosen at random.

(a) Find the probability that the student plays the guitar. [1]

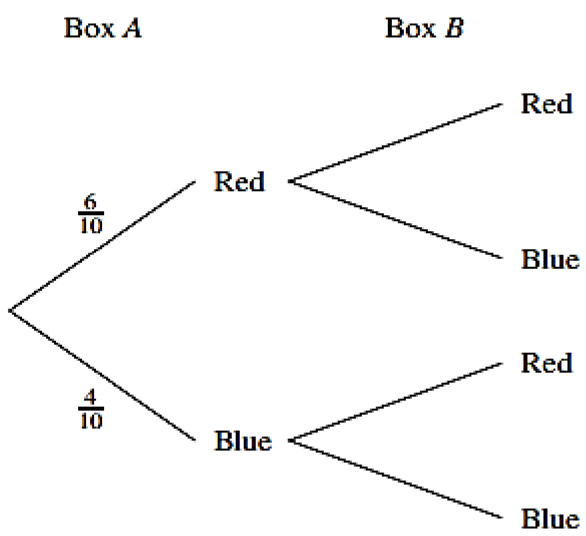
(b) Find the probability that the student is male given that the student plays the drums. [2]

(c) Determine whether the events 'the student plays the guitar' and 'the student is female' are independent, justifying your answer. [2]

23) OCT 2021_9709_53 Q7

Box A contains 6 red balls and 4 blue balls. Box B contains x red balls and 9 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B.

(a) Complete the tree diagram below, giving the remaining four probabilities in terms of x. [3]



(b) Show that the probability that both balls chosen are blue is $\frac{4}{x+10}$. [2]

It is given that the probability that both balls chosen are blue is $\frac{1}{6}$.

(c) Find the probability, correct to 3 significant figures, that the ball chosen from box A is red given that the ball chosen from box B is red. [5]

24) OCT 2022_9709_51 Q5

A game is played with an ordinary fair 6-sided die. A player throws the die once. If the result is 2, 3, 4 or 5, that result is the player's score and the player does not throw the die again. If the result is 1 or 6, the player throws the die a second time and the player's score is the sum of the two numbers from the two throws.

(a) Draw a fully labelled tree diagram to represent this information. [2]

Events A and B are defined as follows.

A: the player's score is 5, 6, 7, 8 or 9

B: the player has two throws

(b) Show that $P(A) = \frac{1}{3}$. [3]

(c) Determine whether or not events A and B are independent. [2]

(d) Find $P(B \mid A')$. [3]

25) OCT 2022_9709_52 Q1

On any day, Kino travels to school by bus, by car or on foot with probabilities 0.2, 0.1 and 0.7 respectively. The probability that he is late when he travels by bus is x. The probability that he is late when he travels by car is $2x$ and the probability that he is late when he travels on foot is 0.25.

The probability that, on a randomly chosen day, Kino is late is 0.235.

(a) Find the value of x. [3]

(b) Find the probability that, on a randomly chosen day, Kino travels to school by car given that he is not late. [2]

26) OCT 2022_9709_52 Q5

Eric has three coins. One of the coins is fair. The other two coins are each biased so that the probability of obtaining a head on any throw is $\frac{1}{4}$, independently of all other throws. Eric throws all three coins at the same time.

Events A and B are defined as follows.

A: all three coins show the same result

B: at least one of the biased coins shows a head

(a) Show that $P(B) = \frac{7}{16}$. [2]

(b) Find $P(A \mid B)$. [2]

The random variable X is the number of heads obtained when Eric throws the three coins.

(c) Draw up the probability distribution table for X. [3]

27) OCT 2022_9709_53 Q7

Sam and Tom are playing a game which involves a bag containing 5 white discs and 3 red discs. They take turns to remove one disc from the bag at random. Discs that are removed are not replaced into the bag. The game ends as soon as one player has removed two red discs from the bag. That player wins the game.

Sam removes the first disc.

(a) Find the probability that Tom removes a red disc on his first turn. [2]

(b) Find the probability that Tom wins the game on his second turn. [4]

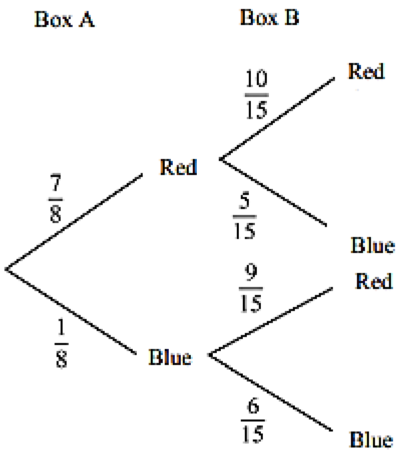
(c) Find the probability that Sam removes a red disc on his first turn given that Tom wins the game on his second turn. [2]

MARKING SCHEME

1) SP-2020_9709_5Q7

(a)	P(X) = P(exactly 2 balls have same number) $P(2, N2, 2) = \frac{1}{4} \times 1 \times \frac{1}{7} = \frac{1}{28}$	1	M1	Considering at least two options of 2s and 8s
	$P(8, 8, N8) = \frac{1}{4} \times \frac{2}{5} \times \frac{3}{7} = \frac{3}{70}$	1	M1	Considering three options for the 8s
	$P(8, N8, 8) = \frac{1}{4} \times \frac{3}{5} \times \frac{4}{7} = \frac{3}{35}$	1	M1	Summing their options if more than 3 in total
	$P(N8, 8, 8) = \frac{3}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{6}{35}$	1	B1	One option correct
	$P(X) = \text{sum} = \frac{47}{140}$ (0.336)	1	A1	
		5		
(b)	$P(X \cap Y) = P(4, 8, 8) = \frac{1}{4} \times \frac{2}{5} \times \frac{4}{7} = \frac{2}{35}$	1	B1	
	$P(Y) = \frac{1}{4}$ $\frac{2}{35} \neq \frac{47}{140} \times \frac{1}{4}$	1	M1	Attempt to compare $P(X \cap Y)$ with $P(X) \times P(Y)$ or using conditional probabilities
	Not independent	1	A1	Correct answer, correct working only
		3		
(c)	$P(2, 2 \text{ given same}) = \frac{1}{28} \div \frac{47}{140}$	1	M1	$\frac{1}{28}$ in numerator of a fraction
	$= \frac{5}{47}$ (0.106)	1	A1	
		2		

2) MARCH 2020_9709_52 Q6

a)		B1 B1 B1	Both correct probs, box A 2 probs correct for box B All correct probs for box B
		3	
b)	$\frac{7}{8} \times \frac{5}{15} + \frac{1}{8} \times \frac{9}{15}$	M1	Two 2 factor terms added, correct or FT *their* **6(a)**.
	$= \frac{44}{120} \left[\frac{11}{30} \text{ or } 0.367\right]$	A1	OE
		2	

:)	$P(\text{A blue} \mid \text{B blue}) = \dfrac{P(\text{A blue} \cap \text{B blue})}{P(\text{B blue})}$ $= \dfrac{\frac{1}{8}\times\frac{6}{15}}{\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}} = \dfrac{\frac{1}{20}}{\frac{41}{120}}$	M1	*their* $\frac{1}{8}\times\frac{6}{15}$ seen as numerator or denom of fraction
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen as denominator
	$= \frac{6}{41}$ or 0.146	A1	
		4	

3) MARCH 2021_9709_52 Q2

(a)	$0.2[\times 1]+0.45\times 0.4+0.35\times 0.3$	M1	$0{\cdot}2\ [\times 1] + 0{\cdot}45 \times b + 0{\cdot}35 \times c$, $b = 0{\cdot}4, 0{\cdot}6$ $c = 0{\cdot}3, 0{\cdot}7$
	$0{\cdot}485$ or $\frac{97}{200}$	A1	
		2	
(b)	$P(Y \mid \overline{H}) = \dfrac{P(Y \cap \overline{H})}{P(\overline{H})} = \dfrac{0.35\times 0.7}{1-\textit{their}\,\textbf{(a)}} = \dfrac{0.245}{0.515}$	B1	$0{\cdot}35 \times 0{\cdot}7$ or $0{\cdot}245$ seen as numerator or denominator of fraction.
		M1	$0{\cdot}515$ or $1 -$ *their* **(a)** or $[0{\cdot}3 \times 0 +]\ 0{\cdot}45 \times d + 0{\cdot}35 \times e$, where d = *their* b', e = *their* c' seen as denominator of fraction.
	$0{\cdot}476$ or $\frac{49}{103}$	A1	$0{\cdot}4757 \leqslant p \leqslant 0{\cdot}476$
		3	

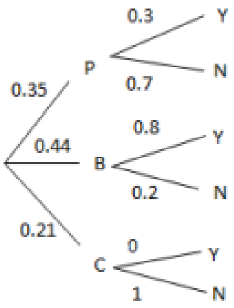
4) MARCH 2023_9709_52 Q4

$(1-x)\times 0.7\times 0.9 = 0.36$	M1	$(1-x)\times a\times b = 0.36$, $a = 0.7$ or 0.3, $b = 0.9$ or 0.1
	B1	$(1\text{-}x)\times 0.7\times 0.9=0.36$, $(1\text{-}x)\times 0.63=0.36$, $0.63-0.63x=0.36$ or $1-x=\frac{0.36}{0.63}$ seen. Condone recovery from omission of brackets.
$x = \frac{3}{7}$	A1	Accept 0.428571 to at least 3 sf. Condone 0.4285 rounding to 0.429 . If M0 awarded, SC B1 for $x = \frac{3}{7}$ or 0.428571 to at least 3 sf.
	3	

5) MARCH 2023_9709_52 Q5

$P(A) = \frac{1}{2}$, $P(B) = \frac{8}{24} = \frac{1}{3}$,	B1	Both stated, accept unsimplified.
$P(A \cap B) = \frac{1}{6}$	M1	Evidence that independence properties not used.
$P(A) \times P(B) = \frac{1}{2}\times\frac{1}{3} = \frac{1}{6}$ so events are independent	A1	Evaluated and conclusion stated. $P(A) \times P(B)$ and $P(A \cap B)$ seen.
	3	

6) JUNE 2020_9709_51 Q5

a)	 Fully correct labelled tree for method of transport with correct probabilities.	B1
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0	B1
		2
b)	$0.35 \times 0.3 + 0.44 \times 0.8\ (+0)$	M1
	0.457	A1
		2
c)	P(not B|not fruit) = $\frac{P(B' \cap F')}{P(F')}$	M1
	$\frac{0.35 \times 0.7 + 0.21 \times 1}{1 - \mathit{their}(\mathbf{b})}$	M1
	$\frac{0.455}{0.543}$ (**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator)	M1
	0.838 or $\frac{455}{543}$	A1

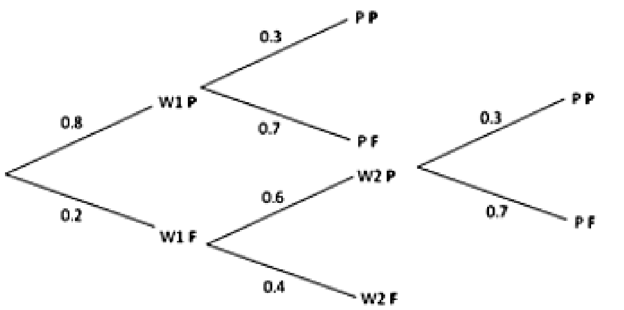
7) JUNE 2020_9709_52 Q2

(a)	$\frac{56}{500}$ or $\frac{14}{125}$ or 0.112	B1
		1
(b)	$P(D\|S) = \frac{P(D \cap S)}{P(S)} = \frac{120}{280}$	M1
	$\frac{120}{280}$ or $\frac{3}{7}$	A1
		2
(c)	P(hockey) = $\frac{220}{500} = 0.44$ P(Amos or Benn) = $\frac{242}{500} = 0.484$ P(hockey $\cap$ A or B) = $\frac{104}{500} = 0.208$ P(H) × P(A U B) = P(H ∩ (A U B)) if independent	M1
	$\frac{220}{500} \times \frac{242}{500} = \frac{1331}{6250}$ so not independent	A1
		2

8) JUNE 2020_9709_53 Q1

(a)	$\frac{1}{\frac{1}{4}} = 4$	B1
		1
(b)	$\frac{9}{64}$ $(=0.141)$	B1
		1
(c)	$P(X<6) = 1 - \left(\frac{3}{4}\right)^5$ **(FT *their* probability/mean from part (a))**	M1
	0.763	A1
		2
(d)	Mean $= 80 \times 0.25 = 20$ Var $= 80 \times 0.25 \times 0.75 = 15$	M1
	P(more than 25) $= P\left(z > \frac{25.5-20}{\sqrt{15}}\right)$	M1
	$P(z > 1.42)$	M1
	$1 - 0.9222$	M1
	0.0778	A1
		5

9) JUNE 2021_9709_51 Q4

(a)		B1	Fully correct labelled tree diagram for each pair of branches clearly identifying written and practical, pass and fail for each intersection (no additional branches)
		B1	'One written test' branch all probabilities (or %) correct
		B1	'Two written tests' branch all probabilities (or %) correct, condone additional branches after W2F with probabilities 1 for PF and 0 for PP
		3	
(b)	[P(W1P) × P(PP) + P(W1F) × P(W2P) × P(PP)] $0.8 \times 0.3 + 0.2 \times 0.6 \times 0.3$	M1	Consistent with *their* tree diagram or correct
	0.276 or $\frac{69}{250}$	A1	
		2	
(c)	$P(W1 \mid P) = \frac{P(W1 \cap \text{Practical})}{P(\text{getting place})} = \frac{0.8 \times 0.3}{their(b)} \left[= \frac{0.24}{0.276}\right]$	M1	Correct expression or FT *their* (b)
	$\frac{20}{23}$ or 0.87[0]	A1	
		2	

10) JUNE 2021_9709_52 Q3

a)	P(not late) = 0.4 × 0.45 + 0.35 × 0.3 + 0.25 × (1 – x) or P(late) = 0.4 × 0.55 + 0.35 × 0.7 + 0.25x	M1	$0.4\times p+0.35\times q+0.25\times r$, p = 0.45, 0.55, q = 0.3, 0.7 and r = (1 – x), x
	0.18 + 0.105 + 0.25 (1 – x) = 0.48 or 0.22 + 0.245 + 0.25x = 0.52	A1	Linear equation formed using sum of 3 probabilities and 0.48 or 0.52 as appropriate. Accept unsimplified.
	x = 0.22	A1	Final answer
		3	
b)	$\left[P(train\|late)=\frac{P(train\cap late)}{P(late)}\right]$ $=\frac{0.35\times0.7}{1-0.48}$ or $\frac{0.35\times0.7}{0.4\times0.55+0.35\times0.7+0.25\times their\,0.22}$	B1	0.35×0.7 or 0.245 seen as numerator of fraction
		M1	P(late) seen as a denominator with *their* probability as numerator (Accept $\frac{their\ p}{0.52}$ or $\frac{their\ p}{0.22+0.245+0.25\times their\,0.22}$)
	= 0.471 or $\frac{49}{104}$	A1	
		3	

11) JUNE 2021_9709_53 Q4a

(a)	[Possible cases: 1 1 2, 1 2 1, 2 1 1] Probability = $\left(\frac{1}{6}\right)^3\times3$	M1	$\left(\frac{1}{6}\right)^3\times k$, where k is an integer.
		M1	Multiply a probability by 3, not +, – or ÷
	$\frac{1}{72}$	A1	Accept $\frac{3}{216}$ or $0.01\dot{3}\dot{8}$ or 0.0139
		3	

12) JUNE 2021_9709_53 Q7a

(a)(i)	$\frac{40}{800}$ or $\frac{1}{20}$ or 0.05	B1	
		1	
(a)(ii)	$\frac{177}{223+177+40}$	M1	*Their* 223 + 177 + 40 seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	A1	CAO
	Alternative method for Question 7(a)(ii)		
	$P(G\mid S)=\frac{P(G\cap S)}{P(S)}=\frac{\frac{177}{800}}{\frac{223+177+40}{800}}=\frac{\frac{177}{800}}{\frac{440}{800}}=\frac{\frac{177}{800}}{\frac{11}{20}}$ or 0.55	M1	*Their* P(S) seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	A1	CAO
		2	

13) JUNE 2022_9709_51 Q6

(a)	$0.6 + 0.4 \times 0.3 = 0.72$ or $1 - 0.4 \times 0.7 = 0.72$	B1	Clear identified calculation AG
		1	
(b)	$0.72 \times (0.4 + 0.6 \times 0.2)$	M1	$0.72 \times u$, $0 < u < 1$
		M1	$v \times (0.4 + 0.6 \times 0.2)$, or $v \times (1 - 0.6 \times 0.8)$ $0 < v \leqslant 1$ no additional terms **SC B1** for $0.72 \times (0.4 + 0.12)$ or $0.72 \times (1 - 0.48)$
	0.3744	A1	WWW. Condone 0.374. **SC B1** for 0.3744 only
		3	
	Alternative method for question 6(b)		
	[p(P1P2) + p(F1P1P2) + p(P1F2P2) + p(F1P1F2P2)] = $0.6 \times 0.4 + 0.4 \times 0.3 \times 0.4 + 0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.6 \times 0.2$	M1	Any two terms unsimplified and correct
		M1	Summing 4 appropriate scenarios by listing or on a tree diagram **SC B1** for $0.24 + 0.048 + 0.072 + 0.0144$
	0.3744	A1	WWW. Condone 0.374. **SC B1** for 0.3744 only
		3	
(c)	$P(\text{fails first or second level} \mid \text{finishes game}) = \dfrac{P(\text{fails first or second level} \cap \text{finishes game})}{\textit{their}\,(\mathbf{b})}$	M1	Either $0.6 \times 0.6 \times 0.2$ or $0.4 \times 0.3 \times 0.4$ seen Condone 0.072 or 0.048 if seen in **(b)**
	Numerator = P(S SF) + P(FS S) = $0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.4 = 0.072 + 0.048 = 0.12$	A1	Both correct accept unsimplified expression. No additional terms
	Required probability = $\dfrac{0.12}{\textit{their}(\mathbf{b})}$	M1	$\dfrac{\textit{Their}\text{ sum of two 3-term probabilities as numerator}}{\textit{their}\ (\mathbf{b})\text{ or correct}}$
	0.321 or $\dfrac{25}{78}$	A1	$0.3205 < p \leqslant 0.321$
		4	

14) JUNE 2022_9709_52 Q7

a)	YYY: $\dfrac{5}{12} \times \dfrac{4}{11} \times \dfrac{3}{10} = \dfrac{60}{1320}, \dfrac{1}{22}$ OOO: $\dfrac{4}{12} \times \dfrac{3}{11} \times \dfrac{2}{10} = \dfrac{24}{1320}, \dfrac{1}{55}$ RRR: $\dfrac{3}{12} \times \dfrac{2}{11} \times \dfrac{1}{10} = \dfrac{6}{1320}, \dfrac{1}{220}$	M1	**Either** $12 \times 11 \times 10$ in denominator **or** $a \times (a-1) \times (a-2)$, $a = 5, 4, 3$ in numerator seen in at least one expression.
		A1	One expression $\dfrac{a}{12} \times \dfrac{a-1}{11} \times \dfrac{a-2}{10}$, $a = 5, 4, 3$ (consistent in expression). Correct order of values in the numerator is essential.
		M1	$\dfrac{5}{12} \times \dfrac{4}{d} \times \dfrac{3}{e} + \dfrac{4}{12} \times \dfrac{3}{d} \times \dfrac{2}{e} + \dfrac{3}{12} \times \dfrac{2}{d} \times \dfrac{1}{e}$, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$. Condone $\dfrac{1}{22} + \dfrac{1}{55} + \dfrac{1}{220}$ OE
	[Total =] $\dfrac{90}{1320}, \dfrac{3}{44}, 0.0682$	A1	0.06818. Dependent only upon the second M mark.

(a)	**Alternative method for question 7(a)**		
	YYY: $\frac{{}^5C_3}{{}^{12}C_3} = \frac{10}{220}, \frac{1}{22}$ OOO: $\frac{{}^4C_3}{{}^{12}C_3} = \frac{4}{220}, \frac{1}{55}$ RRR: $\frac{{}^3C_3}{{}^{12}C_3} = \frac{1}{220}$	M1	Either ${}^{12}C_3$ in denominator or ${}^aC_3$ in numerator seen in at least one expression.
		A1	One expression $\frac{{}^aC_3}{{}^{12}C_3}$ $a = 5, 4, 3$
		M1	$\frac{{}^5C_3}{{}^{12}C_3}+\frac{{}^4C_3}{{}^{12}C_3}+\frac{{}^3C_3}{{}^{12}C_3}$ Condone $\frac{1}{22}+\frac{1}{55}+\frac{1}{220}$ OE
	[Total =] $\frac{90}{1320}, \frac{3}{44}$, 0.0682	A1	0.06818. Dependent only upon the second M mark.
		4	
(b)	[P(YYY | all same colour) =] $\frac{60}{1320} \div \frac{90}{1320}$	M1	$\dfrac{\textit{their}\ P(YYY) \text{ or } \frac{60}{1320} \text{ or } \frac{1}{22}}{\textit{their}\ 7(a) \text{ or } \frac{90}{1320} \text{ or } \frac{3}{44}}$
	$\frac{2}{3}$, 0.667	A1	OE
		2	
(c)	In each method, the M mark requires the scenarios to be identifiable. This may be implied by a list of scenarios and then the calculations which will be assumed to be in the same order. A correct value/expression will be condoned as identifying the connected scenario.		
	Method 1		
	[1 – no orange =] $1-\frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ or $1-\frac{{}^8C_3}{{}^{12}C_3} = 1-\frac{14}{55}$	B1	$\frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ or $\frac{{}^8C_3}{{}^{12}C_3}$ seen, condone $\frac{336}{1320}$ or $\frac{56}{220}$ only, not OE.
		M1	$1-\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ Either $d = 11, e = 10$ or $d = 12, e = 12$ or 1 - $\frac{{}^8C_3}{{}^{12}C_3}$. Condone $1-\frac{14}{55}$ OE (not $\frac{41}{55}$).
	$\frac{41}{55}$	A1	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

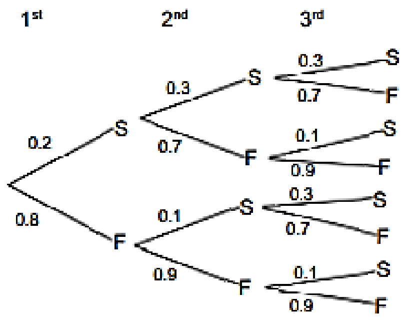
(c)	Method 2		
	P(1 O)= $\left(\frac{4}{12}\times\frac{3}{11}\times\frac{2}{10}+\frac{4}{12}\times\frac{5}{11}\times\frac{4}{10}+ 2\times\frac{4}{12}\times\frac{5}{11}\times\frac{3}{10}\right)\times 3=\frac{672}{1320}$ P(2O) = $\frac{4}{12}\times\frac{3}{11}\times\frac{8}{10}\times 3=\frac{288}{1320}$ P(3O) = $\frac{24}{1320}$	**B1** **M1**	P(1 O)or P(2 O) correct, accept unsimplified. 3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11, e = 10$ or $d = 12, e = 12$.
	[Total =] $\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

	Method 3		
	O Y R = ${}^4C_1\times{}^5C_1\times{}^3C_1$ = 60 O R R = ${}^4C_1\times{}^3C_2$ = 12 O Y Y = ${}^4C_1\times{}^5C_2$ = 40 O O Y = ${}^4C_2\times{}^5C_1$ = 30 O O R = ${}^4C_2\times{}^3C_1$ = 18 O O O = ${}^4C_3$ = 4 Total = 164 Prob = $\frac{164}{{}^{12}C_3}$	**B1** **M1**	Number of ways either 1 or 2 orange sweets obtained correctly (112 or 48). Accept unsimplified Note ${}^4C_1\times{}^8C_2 = 112$ or ${}^4C_2\times{}^8C_1 = 48$ are correct alternatives. 3 correct scenarios (1, 2 or 3 orange sweets) added on numerator, denominator ${}^{12}C_3$
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

(c)	Method 4		
	P(R R O) = $\frac{3}{12}\times\frac{2}{11}\times\frac{4}{10}=\frac{1}{55}$ P(R O) = $\frac{3}{12}\times\frac{4}{11}=\frac{1}{11}$ P(R Y O) = $\frac{3}{12}\times\frac{5}{11}\times\frac{4}{10}=\frac{1}{22}$ P(O) = $\frac{4}{12}=\frac{1}{3}$ P(Y R O) = $\frac{5}{12}\times\frac{3}{11}\times\frac{4}{10}=\frac{1}{22}$ P(Y O) = $\frac{5}{12}\times\frac{4}{11}=\frac{5}{33}$ P(Y Y O) = $\frac{5}{12}\times\frac{4}{11}\times\frac{4}{10}=\frac{2}{33}$	**B1** **M1**	P(R ^ ^) = $\frac{17}{110}$ or P(Y ^ ^) = $\frac{17}{66}$. Accept unsimplified. 3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11, e = 10$ or $d = 12, e = 12$.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

(c)	Method 5		
	P(O) = $\frac{4}{12}=\frac{1}{3}$ P(^ O) = $\frac{8}{12}\times\frac{4}{11}=\frac{8}{33}$ P(^ ^ O) = $\frac{8}{12}\times\frac{7}{11}\times\frac{4}{10}=\frac{28}{165}$	**B1** **M1**	P(^ O) = $\frac{8}{33}$ or P(^ ^ O) = $\frac{28}{165}$. Accept unsimplified. 3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11, e = 10$ or $d = 12, e = 12$ with correct numerator.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

15) JUNE 2022_9709_53 Q6

(a)		B1	First and second jumps correct with probabilities and outcomes identified.
		B1	Third jump correct with probabilities and outcomes identified.
		2	
(b)	SFF $0.2\times0.7\times0.9 = 0.126$ FSF $0.8\times0.1\times0.7 = 0.056$ FFS $0.8\times0.9\times0.1 = 0.072$	M1	Two or three correct 3 factor probabilities added, correct or FT from part 6(a). Accept unsimplified.
	[Total = probability of 1 success =] 0.254 $\left(\frac{127}{500}\right)$	A1	Accept unsimplified.
	[Probability of at least 1 success = $1-0.8\times0.9\times0.9$ =]0.352 $\left(\frac{44}{125}\right)$	B1 FT	Accept unsimplified.
	P(exactly 1 success | at least 1 success)= $\frac{\textit{their}\ 0.254}{\textit{their}\ 0.352}$	M1	Accept unsimplified.
	0.722, $\frac{127}{176}$	A1	$0.7215 < p \leqslant 0.722$
		5	

(c)	$0.8\times0.9\times0.9\times0.1\times0.3\times0.3 = 0.005832$ [FFFSSS] $0.2\times0.3\times0.3\times0.7\times0.9\times0.9 = 0.010206$ [SSSFFF]	M1	$a\times b\times c\times d\times e\times f$ FT from *their* tree diagram. Either a, b and c all = 0.8 or 0.9 (at least one of each) and d, e and f all = 0.1 or 0.3 (at least one of each). Or a, b, c = 0.2 or 0.3 (at least one of each) and d, e, f = 0.7 or 0.9 (at least one of each).
		A1	Either correct. Accept unsimplified.
	[Total =] 0.0160[38]	A1	
		3	

16) JUNE 2023_9709_53 Q5

(a)	$P(A) = \frac{10}{36}$ $P(B) = \frac{24}{36}$	B1	Accept $P(A) = \frac{10}{36}, \frac{5}{18}, 0.278$ and $P(B) = \frac{24}{36}, \frac{2}{3}, 0.667$.
	$P(A \cap B) = \frac{8}{36}$	B1	
	$\frac{10}{36}\times\frac{24}{36}$	M1	*Their* P(*A*) × *their* P(*B*) seen numerically, $0 \leqslant$ their P(*A*), P(*B*) $\leqslant 1$.
	$= \frac{5}{27}, 0.185 \left[\neq \frac{8}{36}\right]$ Events are not independent	A1 FT	Multiplication evaluated correctly and compared with intersection that is not a product of multiplication, conclusion stated, notation P(*A*), P(*B*) and P(*A* ∩ *B*) used.
		4	

b)	$\left[P(B\|A') = \frac{P(B \cap A')}{P(A')} = \right]$ $\frac{16}{36} \Big/ \left(1 - \frac{10}{36}\right)$	M1	$[P(B \cap A') =]\frac{16}{36}, 0.4444$ or *their* P(B) – *their* P(A ∩ B) seen as numerator or denominator of conditional probability fraction.
		M1	$[P(A') =]\left(1 - \frac{10}{36}\right), \frac{26}{36}, 0.7222$ or 1 – *their* P(A) seen as denominator of conditional probability fraction.
	$= \frac{8}{13}$	A1	Final answer $\frac{16}{26}, \frac{8}{13}, 0.6153846$ to at least 3SF.
	Alternative Method for Question 5(b): Direct from outcome tables		
	$\left[P(B\|A') = \frac{\text{Number of outcomes}(B \cap A')}{\text{Number of outcomes}(A')} = \right]$ $\frac{16}{26}$	M1	[Number of outcomes$(B \cap A')$ =]16 seen as numerator or denominator of conditional probability fraction.
		M1	[Number of outcomes(A') =]26 seen as denominator of conditional probability fraction.
		A1	Final answer $\frac{16}{26}, \frac{8}{13}, 0.6153846$ to at least 3SF.
		3	

17) OCT 2020_9709_51 Q1

(a)

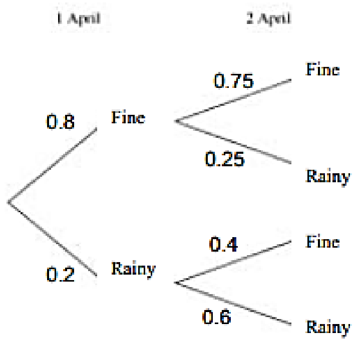
		Red					
		1	2	3	4	5	6
Blue	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

(a)	(outcome table above)	M1	Complete outcome space or or listing A and B outcomes or listing A∩B outcomes
	$P(A \cap B) = \frac{5}{36}$	A1	With evidence
		2	
(b)	$P(A) \times P(B) = \frac{1}{3} \times \frac{10}{36}$	M1	*Their* $\frac{1}{3} \times$ *their* $\frac{10}{36}$ seen
	$\frac{5}{54} \neq \frac{5}{36}$ so not independent	A1	$\frac{5}{54}, \frac{5}{36}$, P(A) × P(B) and P(A∩B) seen in workings and correct conclusion stated Condone $\frac{5}{36}$ being stated in (a)
	Alternative method for question 1(b)		
	P(B|A) = P(B) $P(B\|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{5}{36}}{\frac{1}{3}}$	M1	OE, $\frac{\textit{their}\ 1(a)}{\textit{their}\ P(A)}$ seen
	$\frac{5}{12} \neq \frac{5}{18}$ so not independent	A1	P(A|B), P(B), $\frac{5}{12}, \frac{5}{18}$ seen in workings and correct conclusion stated Condone $\frac{5}{18} = \frac{10}{36}$ being identified in (a)
		2	

18) OCT 2020_9709_51 Q2

(a)	$0{\cdot}6 \times 0{\cdot}7 + 0{\cdot}4(1-x) = 0{\cdot}58$ $\equiv 0{\cdot}42 + 0{\cdot}4(1-x) = 0{\cdot}58$	M1	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}58$; $a = 0{\cdot}3, 0{\cdot}7$, $b = x, (1-x)$
		B1	Single correct product seen, condone 0·42, in an equation of appropriate form
	$x = 0{\cdot}6$	A1	
	Alternative method for question 2(a)		
	$0{\cdot}6 \times 0{\cdot}3 + 0{\cdot}4x = 0{\cdot}42$ $\equiv 0{\cdot}18 + 0{\cdot}4x = 0{\cdot}42$	M1	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times b = 0{\cdot}42$; $a = 0{\cdot}3, 0{\cdot}7$, $b = x, (1-x)$
		B1	Single correct product seen, condone 0·18, in an equation of appropriate form
	$x = 0{\cdot}6$	A1	
		3	
(b)	$(0.6\times0.3)^2$	M1	$(a \times b)^2$, $a = 0{\cdot}6, 0{\cdot}4$ and $b = 0{\cdot}7, 0{\cdot}3, x, (1-x)$ or $0{\cdot}18^2$, alone.
	0.0324	A1	
		2	

19) OCT 2020_9709_52 Q4

(a)		B1	All probabilities correct, may be on branch or next to 'Fine/Rainy' Ignore additional branches.
		1	
(b)	$0{\cdot}8 \times 0{\cdot}75 + 0{\cdot}2 \times 0{\cdot}4$ $(= 0{\cdot}6 + 0{\cdot}08)$	M1	Correct or FT from *their* diagram unsimplified, all probabilities $0 < p < 1$. Partial evaluation only sufficient when correct. Accept working in 4(**b**) or by the tree diagram.
	$0.68, \frac{17}{25}$	A1	From supporting working
		2	

20) OCT 2020_9709_53 Q6

	Answer	Mark	Guidance
(a)	Scenarios: HHT: $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{5}=\frac{4}{45}$ HTH: $\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}=\frac{8}{45}$ THH: $\frac{1}{3}\times\frac{2}{3}\times\frac{4}{5}=\frac{8}{45}$	M1 M1	One 3 factor probability with 3, 3, 5 as denominators 3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios
	Total $=\frac{20}{45}=\frac{4}{9}$	A1	AG, Total of 3 products with clear context
		3	
(b)	x: 0, 1, 2, 3 Prob.: $\frac{1}{45}$, $\frac{8}{45}$, $\frac{20}{45}$, $\frac{16}{45}$	B1	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
		B1	2 of P(0), P(1) and P(3) correct
		B1 FT	3 or 4 probabilities sum to 1 with P(2) correct
		3	
(c)	$\text{Var}(X)=\frac{0^2\times1+1^2\times8+2^2\times20+3^2\times16}{45}-\left(\frac{32}{15}\right)^2$ $=\frac{8}{45}+\frac{80}{45}+\frac{144}{45}-\left(\frac{32}{15}\right)^2$	M1	Substitute *their* attempts at scores in correct variance formula, must have '– mean²' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values
	$\frac{136}{225}$ or 0.604	A1	
		2	

21) OCT 2021_9709_51 Q3

Answer	Mark	Guidance
$\left[P(T\|B')=\frac{P(T\cap B')}{P(B')}\right]$ $P(B')=0.45\times0.7+0.35\times0.4+0.2\times1$	M1	$0.45\times a+0.35\times b+0.2[\times1], a=0.7,0.3\ b=0.4,0.6$, seen anywhere.
$\left[=0.655,\frac{131}{200}\right]$	A1	Correct, accept unsimplified.
$P(T\cap B')=0.35\times0.4\ [=0.14,\ \frac{7}{50}]$	M1	Seen as numerator or denominator of a fraction.
$P(T\mid B')=\frac{\textit{their}\ 0.14}{\textit{their}\ 0.655}$	M1	Values substituted into conditional probability formula correctly. Accept unsimplified. Denominator sum of 3 two-factor probabilities (condone omission of 1 from final factor). If clearly identified, condone from incomplete denominator.
0.214, $\frac{28}{131}$	A1	If 0 marks awarded, **SC B1** 0.214 WWW.
	5	

22) OCT 2021_9709_52 Q1

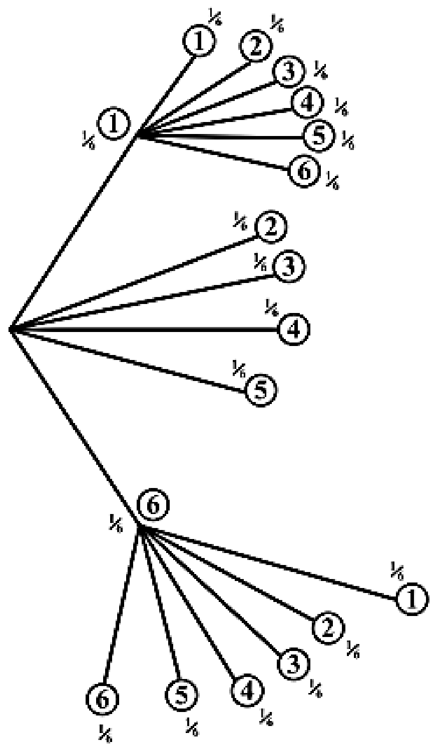
	Answer	Mark	Guidance
(a)	$\frac{82}{180},\frac{41}{90}$, 0.456	B1	
		1	
(b)	$\left[P(M\|D)=\frac{P(M\cap D)}{P(D)}\right]=\frac{\frac{11}{180}}{\frac{20}{180}+\frac{11}{180}}$ or $\frac{0.6011}{0.1722}$	M1	*Their* identified $\frac{P(M\cap D)}{P(D)}$ *or* from data table $\frac{11}{20+11}$, accept unsimplified, condone × 180.
	$\frac{11}{31}$, 0.355	A1	Final answer.
		2	

(c)	$P(F)=\frac{100}{180},\frac{5}{9},0.5556$ OE $P(G)=\frac{82}{180},\frac{41}{90}0.4556$ OE	M1	*Their* identified P(F) × *their* identified P(G) or correct seen, can be unsimplified.
	$P(F\cap G)=\frac{38}{180},\frac{19}{90},0.2111$ OE $P(F)\times P(G)=\frac{100}{180}\times\frac{82}{180}=\frac{41}{162},0.2531$ OE $\left[\neq\frac{38}{180}\right]$ Not independent	A1	$\frac{41}{162},\frac{38}{180},P(F\cap G)$ and $P(F)\times P(G)$ seen with correct conclusion, WWW. Values and labels must be seen.
	Alternative method for question 1(c)		
	$P(F\cap G)=\frac{38}{180},\frac{19}{90},0.2111$ OE $P(G)=\frac{82}{180},\frac{41}{90},0.4556$ OE	M1	P(F|G) (OE) unsimplified with *their* identified probs or correct
	$P(F\|G)=\frac{\frac{38}{180}}{\frac{82}{180}}=\frac{19}{41},0.4634$ OE $\neq P(F)=\frac{100}{180},\frac{5}{9},0.5556$ OE Not independent	A1	$\frac{19}{41},\frac{100}{180},P(F\cap G)$ and $P(F\|G)$ seen with correct conclusion WWW. Values and labels must be seen.
		2	

23) OCT 2021_9709_53 Q7

a)	Probabilities: $\frac{x+1}{x+10},\frac{9}{x+10},\frac{x}{x+10},\frac{10}{x+10}$	B1	One probability correct in correct position.
		B1	Another probability correct in correct position.
		B1	Other two probabilities correct in correct positions.
		3	
b)	$\frac{4}{10}\times their\frac{10}{x+10}$	M1	Method consistent with *their* tree diagram.
	$\frac{4}{x+10}$	A1	AG
		2	
(c)	$\frac{4}{x+10}=\frac{1}{6}$ $x+10=24,\quad x=14$	B1	Find value of x. Can be implied by correct probabilities in calculation.
	P(ARed|BRed) = P(ARed ∩ BRed) ÷ P(BRed) $\frac{\frac{6}{10}\times their\frac{x+1}{x+10}}{\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}}=\frac{\frac{6}{10}\times\frac{15}{24}}{\frac{6}{10}\times\frac{15}{24}+\frac{4}{10}\times\frac{14}{24}}=\frac{\frac{3}{8}}{\frac{73}{120}}$	B1 FT	$\frac{6}{10}\times their\frac{x+1}{x+10}$ as numerator or denominator of fraction.
		M1	$\frac{6}{10}\times their\frac{x+1}{x+10}+\frac{4}{10}\times their\frac{x}{x+10}$ seen anywhere.
		A1 FT	Seen as denominator of fraction.
	$\frac{45}{73}$, 0.616[4…]	A1	If B0 M0: **SC B1** for $\frac{\frac{3}{8}}{\frac{73}{120}}$ or $\frac{0.375}{0.6083}$ **SC B1** $\frac{45}{73}$ or 0.616.
		5	

24) OCT 2022_9709_51 Q5

Question	Answer	Marks	Guidance
(a)		B1	1st throw fully correct with probabilities and outcomes identified. (Probabilities $\left(\text{all } \frac{1}{6}\right)$ and outcomes (1,2,3,4,5,6) on branches).
		B1	2nd throw fully correct with probabilities and outcomes identified. (Probabilities $\left(\text{all } \frac{1}{6}\right)$ and outcomes (1,2,3,4,5,6) on branches).
		2	
(b)	5 comes from 1+4 or 5: $P(5) = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6} = \frac{7}{36}$ 6 comes from 1+5 $P(6) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$ 7 comes from 1+6 or 6+1 $P(7) = \frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6} = \frac{2}{36}$ 8 comes from 6+2 $P(8) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$ 9 comes from 6+3 $P(9) = \frac{1}{6}\times\frac{1}{6} = \frac{1}{36}$	B1	P(5) or P(7) identified and correct unsimplified, accept if supported by correct scenarios shown or from tree diagram .
	$P(A) = \frac{7}{36}+\frac{1}{36}+\frac{2}{36}+\frac{1}{36}+\frac{1}{36}$	M1	Adding only the values from 5 correct scenarios.
	$= \frac{12}{36} = \frac{1}{3}$	A1	Scenarios identified (may be on tree diagram in 5(a)), all probabilities seen, WWW AG.
		3	
(c)	$P(B) = \frac{1}{3}$, $P(A\cap B) = \frac{6}{36}$	M1	Both identified and evaluated, consistent with *their* tree diagram or correct.
	$P(A)P(B) = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$ $\frac{6}{36} \neq \frac{1}{9}$, so not independent	A1	P(A) × P(B) seen and evaluated, all notation present and correct. Correct conclusion WWW.
		2	
(d)	$P(B \mid A') = \frac{P(B\cap A')}{P(A')} = \frac{\textit{their } \frac{6}{36}}{\frac{2}{3}}$	B1	$\frac{6}{36}$ oe as numerator of a fraction.
		M1	$\frac{\textit{their } \frac{6}{36} \text{ or correct}}{\textit{their } 1-\frac{1}{3} \text{ or correct}}$ seen, consistent with *their* tree diagram.
	$\frac{1}{4}$, 0.25	A1	
		3	

25) OCT 2022_9709_52 Q1

(a)	$0.2\times x+0.1\times 2x+0.7\times 0.25=0.235$	M1	$0.2\times x+0.1\times 2x+0.7\times 0.25$ or $0.2x + 0.2x + 0.175$ seen.
		M1	Equating *their* 3 term expression (2 terms involving x) to 0.235
	$x=0.15$	A1	
		3	
(b)	$\left[P(car\mid not\ late)=\frac{P(car\ and\ not\ late)}{P(not\ late)}\right]$ $\frac{0.1\times(1-0.3)}{1-0.235}$	M1	$0.1\times(1-2\times their\ x)$ or 0.1×0.7 as numerator **and** $0.2 \times (1 - their\ x) + 0.1 \times (1 - 2 \times their\ x) + 0.7 \times 0.75$ with values substituted or $1 - 0.235$ or 0.765 as denominator of fraction. Condone $0.2 \times (1 - their\ x) + 0.1 \times (1 - \times their\ x) + 0.7 \times 0.75$ as denominator consistent with **1(a)**.
	$\left[\frac{0.07}{0.765}=\right]0.0915, \frac{70}{765}, \frac{14}{153}$	A1	0.091503267 to at least 3SF. If M0 scored **SC B1** for 0.091503267 to at least 3SF.
		2	

26) OCT 2022_9709_52 Q5

(a)	**Method 1: Scenarios identified ignoring unbiased coin**		
	$P(BH_1\ BT_2)=\frac{1}{4}\times\frac{3}{4}=\frac{3}{16}$ $P(BT_1\ BH_2)=\frac{3}{4}\times\frac{1}{4}=\frac{3}{16}$ $P(BH_1\ BH_2)=\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$	M1	All 3 different calculations seen unsimplified.
	$\frac{3}{16}+\frac{3}{16}+\frac{1}{16}=\frac{7}{16}$	A1	**Clear identification of all scenarios, linked probabilities and sum.** AG
(a)	**Method 2: Scenarios identified with all 3 coins**		
	$P(H\ BH_1\ BT_2)=\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{32}$ $P(T\ BH_1\ BT_2)=\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{32}$ $P(H\ BT_1\ BH_2)=\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{32}$ $P(T\ BT_1\ BH_2)=\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{32}$ $P(H\ BH_1\ BH_2)=\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{32}$ $P(T\ BH_1\ BH_2)=\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{32}$	M1	All 6 different calculations seen unsimplified.
	$P(B)=\frac{1+3+3+1+3+3}{32}=\frac{14}{32}=\frac{7}{16}$	A1	Clear identification of **all scenarios**, linked probabilities and sum. AG
	Method 3: 1- P(BT₁ BT₂) ignoring unbiased coin		
	$1 - P(BT_1\ BT_2) = 1-\left(\frac{3}{4}\right)^2$	M1	Calculation seen unsimplified **and** 1 – probability seen.
	$=\frac{7}{16}$	A1	Clear identification of scenario used, linked probability and calculation. AG

(a)	**Method 4: 1- P(BT₁ BT₂) with all 3 coins**		
	$1 - \text{P(H BT}_1\text{ BT}_2) - \text{P(T BT}_1\text{ BT}_2) = 1 - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right) - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right)$	M1	Both calculations seen unsimplified **and** 1 – 2 probabilities seen.
	$= 1 - \frac{9}{32} - \frac{9}{32} = \frac{7}{16}$	A1	Clear identification of all scenarios used, linked probabilities and calculation. AG
		2	
(b)	$\left[P(A\mid B) = \frac{P(A\cap B)}{P(B)} = \right] \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{7}{16}} = \frac{\frac{1}{32}}{\frac{7}{16}}$	M1	*Their* identified P(*HHH*) or correct as numerator **and** *their* identified P(B) or correct as denominator. Either numerical expression acceptable.
	$= \frac{1}{14}$, 0.0714	A1	Accept 0.071428… rounded to at least 3SF.
		2	
(c)	$\text{P(1H)} = \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4} + \frac{1}{2}\times\frac{3}{4}\times\frac{3}{4} = \frac{15}{32}$ $\text{P(2H)} = \frac{1}{2}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4} = \frac{7}{32}$ $\begin{array}{c\|c\|c\|c\|c} x & 0 & 1 & 2 & 3 \\ \hline p(X) & \frac{9}{32} & \frac{15}{32} & \frac{7}{32} & \frac{1}{32} \\ & 0.28125 & 0.46875 & 0.21875 & 0.03125 \end{array}$	B1	Table with correct *X* values and at least one probability. Condone any additional *X* values if probability stated as 0.
		B1	P(1) or P(2) correct, need not be in table, accept unsimplified.
		B1	4 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3 SF.
			SC B1 for 4 probabilities ($0 < p < 1$) sum to 1 ± 0.005 with P(1) **and** P(2) incorrect.
		3	

27) OCT 2022_9709_53 Q7

(a)	[P(SR TR) + P(SW TR) =] $\frac{3}{8}\times\frac{2}{7} + \frac{5}{8}\times\frac{3}{7}$	M1	$\frac{3}{8}\times\frac{2}{7} + k$ or $l + \frac{5}{8}\times\frac{3}{7}$ $\quad 0 < k, l < 1$
	$= \frac{21}{56}, \frac{3}{8}$, 0.375	A1	**SC B1** for $\frac{3}{8}$ with no explanation.
		2	
(b)	[RRWR, WRRR, WRWR] $\frac{3}{8}\times\frac{2}{7}\times\frac{5}{6}\times\frac{1}{5} + \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5} + \frac{5}{8}\times\frac{3}{7}\times\frac{4}{6}\times\frac{2}{5}$ $\left[= \frac{1}{56} + \frac{1}{56} + \frac{1}{14}\right]$	M1	$\frac{m}{8}\times\frac{n}{7}\times\frac{o}{6}\times\frac{q}{5}$ $\quad 1 \leqslant m,n,o,q \leqslant 5$, $m \neq n \neq o \neq q$
		A1	Probability for one scenario correct, accept unsimplified.
		M1	Adding probabilities for 3 correct scenarios and no incorrect.
	$= \frac{180}{1680}, \frac{3}{28}$, 0.107	A1	Or 0.1071428… to 4SF or better. **SC B1** for 3/28 with inadequate explanation.
		4	
(c)	[P(S first disc R |T2) =] $\frac{\frac{30}{1680}}{\frac{3}{28}} = \frac{\frac{1}{56}}{\frac{3}{28}}$	M1	$\frac{their\ P(RRWR)\ or\ \frac{3}{8}\times\frac{2}{7}\times\frac{5}{6}\times\frac{1}{5}}{their\ 7(b) - must\ be\ a\ prob\ or\ \frac{3}{28}}$
	$\frac{1}{6}$, 0.167	A1	
		2	