S.1

Probability and Statistics_1

The Normal Distribution

Ex.1_Solution (Revision)

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S-1

Normal Distribution

Revision-1

Example 1: A Petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(a) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. ---[4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$

(b) Find the value of m. ---[3]

(c) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. ---[3]

SP-20/05/Q4

Solution: $\mu = 4520$, $\sigma = 560$, $x = 3900$

(a) $P(X > x) = P\left(z > \frac{x-\mu}{\sigma}\right) = P\left(z > \frac{3900 - 4520}{560}\right) = P(z > -1.107)$

$= P(z < 1.107)$

$= \phi(1.107) = \underline{0.8657}$ ✓

∴ Expected No. of days $= np = 365 \times 0.8657 = \underline{315 \text{ or } 316}$

(b) Now $\mu = m$, $\sigma = 560$, $P(X > 8000) = 0.122 \Rightarrow P(x < 8000) = 1 - 0.122$

$= 0.8780$

Now $P(X \leq 8000) = P\left(\frac{8000 - m}{560}\right) = 0.878$ given

$\Rightarrow \frac{8000 - m}{560} = 1.165$ $[\because \phi^{-1}(0.878) = 1.165$

$\Rightarrow m = 7350$ ✓

(c) Now $p = P(X > 8000) = 0.122$ and $q = 1 - 0.122 = 0.878$; $n = 6$

$P(r < 2) = P(0) + P(1)$, Using Binomial Prob. dis.

$\because P(r) = {}^nC_r \cdot p^r q^{n-r}$

$P(0) = q^n$

$= q^6 + {}^6C_1\, p q^5$

$= (0.878)^6 + 6 \times (0.122)(0.878)^5$

$= 0.4581 + 0.3819$

$= \underline{0.840}$ ✓

Example 2: A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6 is thrown 90 times.

(a) Use an approximation to find the probability that a 3 is obtained fewer than 18 times. ---[4]

(b) Justify your use of the approximation in part (a). --[1]

On another occasion, the same die is thrown repeatedly until a 3 is obtained.

(c) Find the prob. that obtaining a 3 requires fewer than 7 throws. -- [2]

SP-20/05/Q5

Solution: $p = \frac{1}{6}$, $q = \frac{5}{6}$; mean $\mu = np = 90 \times \frac{1}{6} = 15$

Binomial → approximated to Normal {Variance $\sigma^2 = npq = 90 \times \frac{1}{6} \times \frac{5}{6} = \frac{25}{2} = 12.5$

$X \sim B(n, p) \rightarrow N(\mu, \sigma^2)$ } $\therefore \sigma = \sqrt{12.5}$

(a) $P(X < 18) = P\left(Z < \frac{17.5^{*} - 15}{\sqrt{12.5}}\right)$

$= P(Z < 0.7071)$

$= \Phi(0.7071) = \underline{0.760}$ ✓

[$\because X < 18$ Should be considered $X < 17.5$ * Continuity Correction

(b) $np = 15 > 5$ and $nq = 75 > 5$,
so the normal distribution is justified.

(c) Now $P(\text{getting } 3) = \frac{1}{6}$, $q = \frac{5}{6}$

$P(r < 7) = P(r \leq 6) = 1 - q^6$ { Using Geometric distribution.

$= 1 - \left(\frac{5}{6}\right)^6$

$= 0.665$ ✓

3. The weights of apples of a certain variety are normally distributed with mean 82 grams. 22% of the apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples. ---[3]

(b) Find the prob. that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams. ...[4]

[M-20/52/Q3]

Solution (a) $P(X>87) = P\left(z > \frac{87-82}{\sigma}\right) = 0.22$ $\left[\text{Given } \mu = 82,\ z = \frac{X-\mu}{\sigma}\right)$

$$\Rightarrow P\left(z < \frac{5}{\sigma}\right) = 1 - 0.22 = 0.78$$

$$\Rightarrow \frac{5}{\sigma} = \phi^{-1}(0.78) = 0.772$$

$$\Rightarrow \underline{\sigma = 6.48} \quad (6.4767)$$

(b) $|x-\mu| < 4 \Rightarrow -4 < X-\mu < 4$

$$\Rightarrow \frac{-4}{\sigma} < \frac{X-\mu}{\sigma} < \frac{4}{\sigma} \Rightarrow -\frac{4}{\sigma} < z < \frac{4}{\sigma} \quad \left(\because z = \frac{X-\mu}{\sigma}\right)$$

$$P(|X-\mu| < 4) = P\left(-\frac{4}{\sigma} < Z < \frac{4}{\sigma}\right)$$

$$= P(-0.6176 < Z < 0.6176) \quad \left[\frac{4}{\sigma} = \frac{4}{6.4767} = 0.6176\right.$$

$$= \phi(0.6176) - (1 - \phi(0.6176))$$

$$= 2\phi(0.6176) - 1$$

$$= 2 \times 0.7317 - 1$$

$$= \underline{0.463}$$

Example 4. In Greenton, 70% of the adults own a car. A random sample of 120 adults from Greenton is chosen.
Use an approximation to find the probability that more than 75 of them own a car. [M-20/52/Q5(b)] -- [5]

Solution: Mean $\mu = np = 120 \times 0.7 = 84$ ✓, Var $= npq = 120 \times 0.7 \times 0.3 = 25.2$ ✓

$P(X > 75) \sim P(X \geq 76) \sim P(X \geq 75.5)$

$= P\left(Z > \frac{75.5 - 84}{\sqrt{25.2}}\right)$

$= P(Z > -1.693)$

$= P(Z < 1.693)$

$= \phi(1.693)$

$= 0.955$ ✓

[using continuity correction: for $B(n, p) \to N(\mu, \sigma^2)$ $P(X > 75) \sim P(X \geq 75.5)$

[$Z = \frac{X - \mu}{\sigma}$

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5. The time spent by shoppers in large shopping centre has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

(a) Find the probability that a shopper chosen at random spends between 85 and 100 minutes in the shopping centre. --[3]

88% of shoppers spend more than t minutes in the shopping centre.

(b) Find the value of t. --[3]

[M-21/52/Q3]

Solution

$\mu = 96$, $\sigma = 18$, normal distribution $N(\mu, \sigma^2) = N(96, 18^2)$

(a) $P(85 < X < 100) = P\left(\frac{85-96}{18} < Z < \frac{100-96}{18}\right)$

$= P(-0.611 < Z < 0.222)$

$= \phi(0.222) - \phi(-0.611)$

$= \phi(0.222) - [1 - \phi(0.611)]$

$= 0.5879 - 1 + 0.7294 = \underline{0.317}$ ✓

(b) Given $P(X > t) = 0.88$

$\Rightarrow z = -c \Rightarrow P(z > -c) = 0.88 \Rightarrow P(z < c) = 0.88$

$\Rightarrow c = 1.175$

$\therefore z = -1.175$

$\Rightarrow \frac{t-96}{18} = -1.175 \Rightarrow t = \underline{74.9 \text{ minutes}}$.

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6 There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling, or running and the results are given in the following table:

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

(a) (i) Find the probability that the students preffers swing. ---[1]

(ii) Determine whether the events 'the student is male' and the 'student preffers swing' are independent, justify your answer, --[2]

On average at all the schools in this country 30% of the students do not like any sports.

(Continued→)

6(b) (i) 10 of the students from this country are chosen at random. Find the prob. that at least 3 of these students do not like any sports. --[3]

(ii) 90 students from this country are now chosen at random. Use an approximation to find the prob. that fewer than 32 of them do not like any sports. --[5]

M-21/52 | Q7

Solution (a) (i) $P(\text{Swimming}) = \frac{135}{400} = \underline{0.338}$ ✓ (0.3375)

(ii) $P(\text{Swimming}) = \frac{135}{400} = \frac{27}{80}$

$P(M) = \frac{180}{400} = \frac{9}{20}$

$P(M \cap S) = \frac{31}{400}$

Now $P(S) \cdot P(M) = \frac{27}{80} \times \frac{9}{20} \neq \frac{31}{400}$ $P(S \cap M)$

$\therefore$ the two events are not independent.

(b) Now 30% of students don't like sports:

(i) P(don't like sports) = 0.3

$n = 10, \quad p = 0.3, \quad q = 0.7$

$P(X \geq 3) = 1 - P(X = 0, 1, 2)$

$= 1 - [(0.7)^{10} + {}^{10}C_1 \times 0.3 \times (0.7)^9 + {}^{10}C_2 \times 0.3^2 \times (0.7)^8]$

$= 1 - [0.2824 + 0.1211 + 0.2334]$

$= \underline{0.617}$ ✓

(ii) Now 90 students are chosen at random.

Use Normal distribution to find $P(X < 32)$

$\mu = np = 90 \times 0.3 = 27$ ($\geq$)

$\sigma^2 = npq = 9 \times 0.3 \times 0.7 = 18.9$ ($>$

$P(X < 32) = P\left(Z < \frac{31.5 - 27}{\sqrt{18.9}}\right)$

Continuity correction
$X < 32 \rightarrow X \leq 31$
$\rightarrow P(31.5)$

$= P(Z < 1.1035)$

$= \phi(1.1035)$

$= 0.8497$

$= \underline{0.850}$ ✓

7. The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

(a) Find the prob. that a randomly chosen male leopard from this region weights between 46 kg and 62 kg. ---[4]

The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

(b) Find the value of σ. ---[3]

The distribution of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

(c) Find the prob. that both the weights of these leopards are less than 46 kg. ---[4]

[M-22/52/Q4]

Solution: Male; Normal (μ, σ^2), $\mu = 55$ kg, $\sigma = 6$ kg

(a) $P(46 < M < 62) = P\left(\frac{46-55}{6} < z < \frac{62-55}{6}\right)$

$= P\left(-1.5 < Z < \frac{7}{6}\right)$

$= \phi(7/6) - \phi(-1.5)$

$= \phi(7/6) - [1 - \phi(1.5)]$

$= \phi(7/6) + \phi(1.5) - 1$

$= 0.8784 + 0.9332 - 1$

$= \underline{0.812}$ ✓

(b) For Female: $N(\mu, \sigma^2)$: $\mu = 42$ kg, $\sigma = \sigma$ kg?

$P(F < 36) = P\left(z < \frac{36-42}{\sigma}\right) = 0.25$ [25%]

$\Rightarrow P\left(z < -\frac{6}{\sigma}\right) = 0.25$

$\Rightarrow P\left(z < \frac{6}{\sigma}\right) = 1 - 0.25$

$\Rightarrow \phi\left(\frac{6}{\sigma}\right) = 0.75 \Rightarrow \frac{6}{\sigma} = \phi^{-1}(0.75)$

$\Rightarrow \frac{6}{\sigma} = 0.674$

$\Rightarrow \underline{\sigma = 8.90}$ ✓

(c) $P(\text{male} < 46) = P\left(Z < \frac{46-55}{6}\right)$

$= P(z < -1.5)$

$= \phi(-1.5) = 1 - \phi(1.5)$

$= 1 - 0.9332 = 0.0668$ — ①

$P(\text{female} < 46) = P\left(z < \frac{46-42}{8.9}\right)$

$= P(z < 0.449)$

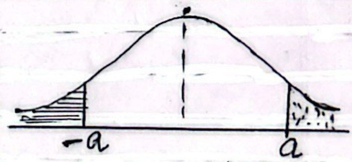
$= 0.6732$ — ②

$P(\text{both} < 46) = P(m < 46) \times P(f < 46)$

fm ① & ②

$= 0.0668 \times 0.6732$

$= \underline{0.0450}$ ✓



$\phi(-a) = 1 - \phi(a)$

$\Rightarrow \phi(a) = 1 - \phi(-a)$

8. In a cycling event the times taken to complete a course are modelled by a normal distribution with mean 62.3 minutes and standard deviation 8.4 minutes.

(a) Find the prob. that a randomly chosen cyclist has a time less than 74 minutes. --[2]

(b) Find the prob. that 4 randomly chosen cyclists all have times between 50 and 74 minutes. ---[4]

In a different cycling event, the times can also be modelled by a normal distribution. 23% of the cyclist have times less 36 minutes and 10% of the cyclist have times greater than 54 minutes.

(c) Find estimates for the mean and standard deviation of this distribution. --[5]

[M-23/52/Q6]

Solution (a) $N(\mu, \sigma^2)$; $\mu = 62.3$ and $\sigma = 8.4$

$$P(X<74) = P\left(Z < \frac{74-62.3}{84}\right) = P(Z<1.393) = \phi(1.393) = \underline{0.918}$$

(b) $$P(50<X<74) = P\left(\frac{50-62.3}{8.4} < Z < \frac{74-62.3}{8.4}\right) = P(-1.464 < Z < 1.393)$$

$$= \phi(1.393) - \phi(-1.464) = \phi(1.393) - [1-\phi(1.464)]$$

for each one cyclist: $= \phi(1.393) + \phi(1.464) - 1 = 0.9285 + 0.9182 - 1$
$= 0.847$

Hence required prob. for 4 randomly chosen cyclist $= (0.847)^4$
$= 0.514$

(c) Given $P(X<36) = 0.23 < 0.5$

$$\Rightarrow P\left(Z < \frac{36-\mu}{\sigma}\right) = 0.23 \Rightarrow \frac{36-\mu}{\sigma} = -b \text{ (let) } \quad \text{--- (1)}$$

$$P(Z<-b) = 0.23$$

$$\Rightarrow 1-\phi(b) = 0.23 \Rightarrow \phi(b) = 0.77 \Rightarrow b = \phi^{-1}(0.77) = 0.739$$

from (1) $\frac{36-\mu}{\sigma} = -.739 \Rightarrow 36-\mu = 0.739\sigma$ --- (2)

Again

$$P(X>54) = P\left(Z > \frac{54-\mu}{\sigma}\right) = 0.1 \Rightarrow P\left(Z < \frac{54-\mu}{\sigma}\right) = 0.9$$

$$\Rightarrow \phi\left(\frac{54-\mu}{\sigma}\right) = 0.9 \Rightarrow \frac{54-\mu}{\sigma} = \phi^{-1}(0.9) = 1.282$$

$\Rightarrow 54-\mu = 1.282\sigma$ --- (3)

Solving (2) and (3) $\quad \underline{\mu = 42.6}$ and $\underline{\sigma = 8.91}$

Example 9: The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm. (P-9)

(a) Find the prob. that a randomly chosen female snake of this species has length between 50 cm and 60 cm. ---[4]

The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find the estimate of the mean and standard deviation of the lengths of male snakes of this species. $\boxed{S\text{-}20/51/Q6}$ ---[5]

Solution: $\mu = 54,\ \sigma = 6.1$

(a) $P(50 < X < 60) = P\left(\frac{50-54}{6.1} < z < \frac{60-54}{6.1}\right)$

$= P(-0.6557 < z < 0.9836)$

$= \phi(0.9836) - \phi(-0.6557)$

$= \phi(0.9836) - [1 - \phi(0.6557)]$

$= 0.8375 + 0.7441 - 1$

$= 0.582$ ✓

(※) Note: here $0.16 < 0.5$

$\phi^{-1}(0.16) = -\phi^{-1}(1-0.16)$

(b) $P(X<45) = P\left(z < \frac{45-\mu}{\sigma}\right) = \frac{32}{200} = 0.16$ (※)

$\Rightarrow \frac{45-\mu}{\sigma} = -\phi^{-1}(1-0.16) = -\phi^{-1}(.84)$

$\frac{45-\mu}{\sigma} = -0.994$ —— ①

and $P(X>56) = P\left(z > \frac{56-\mu}{\sigma}\right) = \frac{17}{200} = 0.085$

$\Rightarrow P\left(z < \frac{56-\mu}{\sigma}\right) = 1 - 0.085 = 0.915$

$\therefore \frac{56-\mu}{\sigma} = \phi^{-1}(0.915) = 1.372$

$\Rightarrow \frac{56-\mu}{\sigma} = 1.372$ —— ②

Solving ① & ②

$\mu = 49.6,\ \sigma = 4.65$ ✓

Example 10. Trees in the Redian forest are classified as tall, medius or short. according to their height. The heights canbe modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short. ---[3]

Of the trees that are classified as tall or medium, one third are tall and two third are medium.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3d.p. --[2]

(c) Find height above which trees are classified as tall. --[3]

S-20/52/Q4

Solution: $\mu = 40$, $\sigma = 12$,

(a) $P(X<25) = P\left(Z < \frac{25-40}{12}\right) = P(Z<-1.25) = 1-\phi(1.25)$

$P(\text{Short tree}) = 1-0.8944 = 0.106$ (or 0.1056)

(b) $P(\text{tall or medium}) = 1-P(\text{Short}) = 1-0.1056 = 0.8944$

$P(\text{tall}) = \frac{1}{3} \times P(\text{tall or medium}) = \frac{1}{3} \times 0.8944 = \underline{0.298}$

(c) $P(\text{tall}) = 0.298$

Let height of trees above which considered tall $= h$

$P(X > h) = P\left(Z > \frac{h-40}{12}\right) = 0.298$

$\Rightarrow P\left(Z < \frac{h-40}{12}\right) = 1-0.298 = 0.702$

$\Rightarrow \frac{h-40}{12} = \phi^{-1}(0.702) = 0.53$

$\frac{h-40}{12} = 0.53$

$\Rightarrow \underline{h = 46.4\ m}$

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Example 11. On a given day, the prob. that Moena messages her friend Pasha is 0.72.
Use an approximation to find the prob. that in any period of 100 days Moena messages Pasha on fewer than 64 days. ---[5]
[S-20/52/Q7(c)]

Solution: $p = 0.72,\ q = 0.28,\ n = 100$

$\therefore$ Mean $\mu = np = 100 \times 0.72 = 72$; $Var = npq = 100 \times 0.72 \times 0.28$

$Var = 20.16$

$B(n,p) \approx N(\mu, \sigma^2)$

$P(X < 64) = P\left(z < \dfrac{63.5 - 72}{\sqrt{20.16}}\right)$

$= P(z < -1.893)$

$= 1 - P(z < 1.893)$

$= 1 - \phi(1.893)$

$= 1 - 0.9708 = 0.0292$

$B(n,p) \approx N(\mu, \sigma^2)$
for $X < 64 \approx X \leq 63$
Continuity correction
$X < 63.5$
$62.5 \leq 63 < 63.5$

Example 12: In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.
(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. --[2]
(b) Find the value of k such that $P(X < k) = 0.75$ [S-20/53/Q3] --[3]

Solution: mean $\mu = 15.8$, $\sigma = 4.2$

(a) $P(X < 21) = P\left(z < \dfrac{21 - 15.8}{4.2}\right)$

$= P(z < 1.238)$

$= \phi(1.238)$

$= 0.892$

(b) $P(X < k) = P\left(z < \dfrac{k - 15.8}{4.2}\right) = 0.75$

$\Rightarrow \dfrac{k - 15.8}{4.2} = \phi^{-1}(0.75) = 0.674$

$\Rightarrow k - 15.8 = 0.674 \times 4.2$

$\Rightarrow k = 18.6$ (18.6)

Example 13. A pair of fair coins is thrown 80 times.
Use an approximation to find the prob. that pair of tails is obtained more than 25 times. [S-20/53/Q5(d)] ---[5]

Solution: A pair of coin is thrown $S = \{HH, HT, TH, TT\}$; $P(TT) = \frac{1}{4} = 0.25$

$\therefore p = 0.25, q = 0.75, \Rightarrow$ Mean $\mu = np = 80 \times .25 = 20$ ✓

Var. $\sigma^2 = npq = 80 \times 0.25 \times 0.75 = 15$

$$P(X > 25) = P\left[z > \frac{25.5 - 20}{\sqrt{15}}\right]$$
$$= P[z > 1.42)$$
$$= 1 - P(z < 1.42)$$
$$= 1 - \phi(1.42)$$
$$= 1 - 0.922 = 0.0778$$ ✓

$B(n, p) \rightarrow N(\mu, \sigma^2)$
a continuity correct is req.
$X > 25 \sim X \geq 26$
$\sim X > 25.5$ ✓
for normal.

14. A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.
How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? ---[5]

S-21/51/Q2

Solution: $n = 500$; $\mu = 25.2$; $\sigma = 0.4$ cm

Now for $25.2 - 0.5 < x < 25.2 + 0.5$ $\left\{ P(X<a) = P\left(Z < \frac{a-\mu}{\sigma}\right)\right.$

$\Rightarrow 24.7 < x < 25.7$

$P(24.7 < X < 25.7) = P\left(\frac{24.7-25.2}{0.4} < Z < \frac{25.7-25.2}{0.4}\right)$

$= P\left(\frac{-0.5}{0.4} < Z < \frac{0.5}{0.4}\right)$

$= P(-1.25 < Z < 1.25)$

$= \phi(1.25) - (1 - \phi(1.25))$

$= 2\phi(1.25) - 1$

$= 2 \times 0.8944 - 1$

$p = 0.7888$ ✓

$\therefore$ Number rods $= np = 500 \times 0.7888 = 394$ or 395 ✓

15. In Questa, 60% of the adults travel to work by car.
A random sample of 150 adults from Questa is taken.
(i) Use an approximation to find the probability that the number who travel to work by car is less than 81. ---[5]
(ii) Justify the use your approximation in part (i) ---[1]

S-21/51/Q6(b),(c)

Solution: P(adult owns a car) $= 0.6 = p$ (60%)

(i) $n = 150$, $\mu = np = 150 \times 0.6 = 90$ ✓

Variance $= npq = 150 \times 0.6 \times 0.4 = 36$ ✓

$P(X < 81) = P\left(z < \frac{80.5 - 90}{6}\right)$

$= \phi(-1.5833)$

$= 1 - \phi(1.5833) = 1 - 0.9433 = 0.0567$ ✓

(ii) $np = 90$, $nq = 60$, both are greater than 5. Hence Normal distribution.

Continuity correction:
$X < 81 \Rightarrow X \le 80 \rightarrow 80.5 = x$

16. The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.
Find the value of σ. --[4]

[S-21/52/Q2]

Solution: $P(X > 1.10) = \frac{72}{2000} = 0.036$ --(i)

Also, $P(X > 1.1) = P\left(Z > \frac{1.1 - 1.04}{\sigma}\right)$

$= P\left(Z > \frac{0.06}{\sigma}\right)$

$= 1 - \phi\left(\frac{0.06}{\sigma}\right) = 0.036$ from (i)

$\Rightarrow \phi\left(\frac{0.06}{\sigma}\right) = 0.964$

$\Rightarrow \frac{0.06}{\sigma} = \phi^{-1}(0.964) = 1.798$

$\Rightarrow \sigma = \frac{0.06}{1.798} = \underline{0.0334}$

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17. The probability that the flight arrives early is 0.15. 60 days are chosen at random. Use an approximation to find the prob. that flight arrives early at least 12 times. --[5]

[S-21/52/Q5(c)]

Solution: $p = 0.15$, $q = 0.85$, $n = 60$; $\mu = np = 60 \times 0.15 = 9$

$\sigma^2 = \text{Variance} = npq = 60 \times 0.15 \times 0.85 = 7.65$

$P(X \geq 12) = P\left(Z > \frac{11.5 - 9}{\sqrt{7.65}}\right) \rightarrow \left\{\begin{array}{l}\text{Continuity correction:} \\ x \geq 12 \rightarrow x = 11.5\end{array}\right)$

$= P(Z > 0.9039)$

$= 1 - \phi(0.9039) = 1 - 0.8169 = \underline{0.183}$

18. The lengths of the leaves of a particular type of tree are modelled by a normal distribution. A scientist measures the leaves of a random sample of 500 leaves from this type of tree and finds that 42 are less than 4 cm long and 100 are more than 10 cm long.

(a) Find estimates for the mean and standard deviation of the lengths of leaves from this type of tree. ---[5]

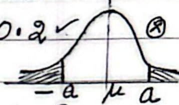
The lengths, in cm, of the leaves of a different type of tree have the distribution $N(\mu, \sigma^2)$. The scientist takes a random sample of 800 leaves from this type of tree.

(b) Find how many of these leaves the scientist would expect to have lengths, in cm, between $\mu - 2\sigma$ and $\mu + 2\sigma$. ---[4]

[S-21/53/Q5]

Solution $n = 500$; $x_1 < 4$ cm is 42 leaves; $x_2 > 10$ cm is 100 leaves

(a) $P(x_1 < 4) = \frac{42}{500} = 0.084$ ✓ and $P(x_2 > 10) = \frac{100}{500} = 0.2$ ✓



Now using Normal distribution:

$P(x_1 < 4) = P\left(z < \frac{4-\mu}{\sigma}\right) = 0.084$

$\Rightarrow \frac{4-\mu}{\sigma} = -a = -1.378$

$\Rightarrow 4 - \mu = -1.378\sigma$ ----(i)

Let $\frac{4-\mu}{\sigma} = -a$

$P(Z < -a) = 0.084$

$P(Z > a) = 0.084$

$\Rightarrow 1 - \phi(a) = 0.084$

$\Rightarrow \phi(a) = 0.916$ ✓

$\Rightarrow a = \phi^{-1}(0.916) = 1.378$

Again, $P(x_2 > 10) = P\left(z_2 > \frac{10-\mu}{\sigma}\right) = 0.2$

$\Rightarrow P\left(z_2 < \frac{10-\mu}{\sigma}\right) = 1 - .2 = 0.8$

$\Rightarrow \frac{10-\mu}{\sigma} = \phi^{-1}(0.8) = 0.842$

$\Rightarrow 10 - \mu = 0.842\sigma$ ----(ii)

Solving (i) and (ii) $\rightarrow$ $\sigma = 2.70$ ✓

and $\mu = 7.72$ ✓

(b) $P(\mu - 2\sigma < x < \mu + 2\sigma)$

$= P\left(\frac{\mu - 2\sigma - \mu}{\sigma} < Z < \frac{\mu + 2\sigma - \mu}{\sigma}\right)$

$= P(-2 < Z < 2)$

$= \phi(2) - \phi(-2)$

$= \phi(2) - [1 - \phi(2)]$

$= 2 \cdot \phi(2) - 1$

$p = 2 \times 0.9772 - 1 = 0.9544$

Given Total Number $n = 800$

$\therefore$ Expected Number $= np$

$= 800 \times 0.9544 = 763.52$

$\approx$ 763 (or 764)

19. In the whole of Akra there are a large number of households. A survey showed that 35% of households in Akra have no broadband service.

120 households in Akra are chosen at random.
Use an approximation to find the probability that more than 32 of these households have no broadband service. ...[5]

[S-21 | 53 | Q7b(ii)]

Solution: p = Probability of no broadband service $= 0.35$ (35%)

$q = 1 - 0.35 = 0.65$

Total Number of household $n = 120$

$\therefore$ Mean $\mu = np = 120 \times 0.35 = 42$

Variance $\sigma^2 = npq = 120 \times 0.35 \times 0.65 = 27.3$

Now $P(X > 32) = P\left(Z > \frac{X - \mu}{\sigma}\right)$

$$P(x > 32) = P\left(Z > \frac{32.5 - 42}{\sqrt{27.3}}\right)$$

⊗ Continuity correction
$X > 32 \Rightarrow X \geq 33$
$x \rightarrow 32.5$
(lower limit)

$= P(Z > -1.818)$

$= P(Z < 1.818)$

$= \phi(1.818)$

$= \underline{0.966}$

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20 The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2, 1.5^2)$

(a) Find the prob. that a randomly chosen leaf of this type has length less 6 cm. ---[2]

The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. ---[5]

(c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? ---[4]

[S-22/51/Q5]

Solution: $N(5.2, 1.5^2) \Rightarrow \mu = 5.2;\ \sigma = 1.5$

(a) $P(X<6) = P\left(Z < \frac{6-5.2}{1.5}\right)$

$= P(Z < 0.5333)$

$= \underline{0.703}$ ✓

(b) $P(X<3) = 46/500 = 0.092 < 0.5$

$\Rightarrow P\left(Z < \frac{3-\mu}{\sigma}\right) = 0.092$

[Let $\frac{3-\mu}{\sigma} = -a$] — ①

$P(Z < -a) = 0.092$

$\Rightarrow \phi(-a) = 0.092$

$\Rightarrow \phi(a) = 1 - 0.092 = 0.908$

$a = \phi^{-1}(0.908) = 1.329$

from ① $\frac{3-\mu}{\sigma} = -1.329$ --- ②

Also $P(X>8) = 95/500 = 0.19$

$\Rightarrow P\left(Z > \frac{8-\mu}{\sigma}\right) = 0.19$

$\Rightarrow P\left(Z < \frac{8-\mu}{\sigma}\right) = 1 - 0.19 = 0.81$

$\Rightarrow \frac{8-\mu}{\sigma} = \phi^{-1}(0.81) = 0.878$ — ③

(b) continued:

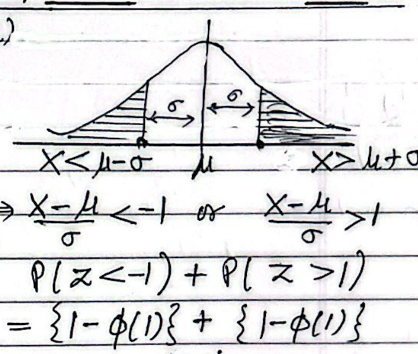
from ② $\sigma = \frac{\mu-3}{1.329}$ --- ④

fm ③ $\sigma = \frac{8-\mu}{0.878}$ — ⑤

from ④ & ⑤ $\frac{\mu-3}{1.329} = \frac{8-\mu}{0.878}$

$\Rightarrow \underline{\mu = 6.01}$ and $\underline{\sigma = 2.27}$ ✓

(c)



$\Rightarrow \frac{X-\mu}{\sigma} < -1$ or $\frac{X-\mu}{\sigma} > 1$

$P(Z<-1) + P(Z>1)$

$= \{1-\phi(1)\} + \{1-\phi(1)\}$

$= 2 - 2\phi(1)$

$= 2 - 2\times 0.8413$

$= \underline{0.3174}$ ✓

21, The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$

(a) Find the prob. that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. ---[3]

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ. ---[4]

[S'22/52/Q4]

Solution: $N(2.02, 0.03^2) \rightarrow \mu = 2.02; \sigma = 0.03$

(a) $P(1.98 < X < 2.03) = P\left(\frac{1.98-2.02}{0.03} < Z < \frac{2.03-2.02}{0.03}\right)$

$= P(-1.333 < Z < 0.333) = \phi(0.333) - \phi(-1.333)$

$= \phi(0.333) - [1 - \phi(1.333)]$

$= \phi(0.333) + \phi(1.333) - 1$

$= 0.6304 + 0.9087 - 1$

$= \underline{0.539}$ ✓

(b) for Binders: $N(\mu, \sigma^2)$: $\mu = 2.55$, $S.D = \sigma = ?$

$P(X > 2.6) = 134/5000 = 0.0268$

$\Rightarrow P(X < 2.6) = 1 - 0.0268 = 0.9732$

$\Rightarrow P\left(Z < \frac{2.6-2.55}{\sigma}\right) = 0.9732$

$\Rightarrow \frac{2.6-2.55}{\sigma} = \phi^{-1}(0.9732)$

$\Rightarrow \frac{0.05}{\sigma} = 1.93 \Rightarrow \sigma = \frac{0.05}{1.93} = 0.0259$

$\therefore \underline{\sigma = 0.0259}$

22. In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instruments. ---[3]

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the prob. that fewer than 40 of these students play exactly one musical instrument. ---[5]

[S-22/52/Q5]

Solution: P(do not play any) $= q = 28\% = 0.28$; P(one or more) $= 0.72 = p$,

(a) P(more than 9 play at least one) $= P(10, 11, 12)$, $n = 12$

$$= {}^{12}C_{10} \cdot 0.72^{10} \cdot 0.28^{2} + {}^{12}C_{11} \cdot 0.72^{11} \cdot 0.28 + {}^{12}C_{12}(0.72)^{12}$$

$\left\{ P(r) = {}^{n}C_{r} p^{r} \cdot q^{n-r} \right.$ Binomial prob. distribution

$$= 0.193725 + 0.090572 + 0.01940 = \underline{0.304}$$

(b) Now $n = 90$, $p =$ P(Exactly one) $= 0.52$, $q = 0.48$

Mean $\mu = np = 90 \times 0.52 = 46.8$

Variance $\sigma^2 = npq = 90 \times 0.52 \times 0.48 = 22.464$

$N(\mu, \sigma^2)$ Using Normal Prob. distribution

$$P(X < 40) = P\left(Z < \frac{39.5 - 46.8}{\sqrt{22.464}}\right)$$

$\left\{ \begin{array}{l} \text{Continuity correction} \\ X < 40 \rightarrow X \leq 39 \\ 38.5 \leq X < 39.5 \end{array} \right.$

$$= P(Z < -1.540)$$

$$= \phi(-1.540)$$

$$= 1 - \phi(1.54)$$

$$= 1 - 0.9382$$

$$= \underline{0.0618}$$

23. Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold at a super market.

(a) Find the prob. that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. ---[4]

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20000.

(b) Calculate an estimate for his total income from this year's apples. ---[3]

Farmer Tan also grows apples. The weight, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. ---[3]

S-22/53/Q5

Solution: $N(\mu,\sigma^2)$: $\mu = 170$, $\sigma = 25$

(a) $P(142 < X < 205)$

$= P\left(\frac{142-170}{25} < Z < \frac{205-170}{25}\right)$

$= P(-1.12 < Z < 1.4)$

$= \phi(1.4) - \phi(-1.12) = \phi(1.4) - (1-\phi(1.12))$

$= \phi(1.4) + \phi(1.12) - 1$

$= 0.9192 + 0.8686 - 1 = 0.788$ ✓

(b) $P(X > 205) = 1 - P(X < 205)$

$= 1 - \phi(1.4) = 1 - 0.9192$

$= 0.0808$ ✓

$P(142 < X < 205) = 0.788$ (part (a))

$\therefore$ number of apples $= 20{,}000$

Total income $= (0.0808 \times 0.30 + 0.788 \times 0.24) \times 20000$

$= \$4266.24$.

(c) Now $N(182, 20^2)$

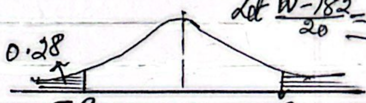
$\mu = 182$, $\sigma = 20$

$P(X > W) = 0.72$ (72%)

$\Rightarrow P\left(Z > \frac{W-182}{20}\right) = 0.72$

$\Rightarrow P\left(Z < \frac{W-182}{20}\right) = 1 - 0.72 = 0.28$

Let $\frac{W-182}{20} = -a$ —— (1)



$\phi(-a) = 0.28 \Rightarrow \phi(a) = 1 - 0.28 = 0.72$

$\Rightarrow a = \phi^{-1}(0.72) = 0.583$

$\Rightarrow$ from (1) $\frac{W-182}{20} = -0.583$

$\Rightarrow W - 182 = -11.66$

$\Rightarrow W = 170.34$

$\therefore W = 170$ ✓

24. A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and Standard deviation 5.2 minutes.

(a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle. ...[4]

All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean μ and standard deviation σ. It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

(b) Find the value of μ and the value of σ. ...[5]

S-23/51/Q4

Solution (a) $\mu = 14.6m$, $\sigma = 5.2$ minutes, $n = 250$

$P(X>20) = P\left(z > \frac{20-14.6}{5.2}\right)$

$= P(z > 1.03846)$

$= 1 - \phi(1.03846)$

$= 1 - 0.8504$

$= 0.150$

$\therefore$ Expection out of 250 students

$= np = 250 \times 0.1496$

$= 37 ($ or $38)$

(b) $P(X_1 < 14.5) = 0.2$

$\Rightarrow P\left(Z_1 < \frac{14.5-\mu}{\sigma}\right) = 0.2 < .5$

Let $\left(\frac{14.5-\mu}{\sigma}\right) = -a$ --(1)

$\Rightarrow P(z < -a) = 0.2$

$\Rightarrow 1 - \phi(a) = 0.2$

$\Rightarrow \phi(a) = 0.8$

$\Rightarrow a = \phi^{-1}(0.8) = 0.842$ --(2)

Hence from (1) & (2) $\frac{14.5-\mu}{\sigma} = -0.842$

$\Rightarrow 14.5 - \mu = -0.842\sigma$ --(3)

Also $P(X_2 > 18.5) = 0.67$

$\Rightarrow P\left(Z_2 > \frac{18.5-\mu}{\sigma}\right) = 0.67 > 0.5$

$\Rightarrow P(Z_2 > -b) = 0.67$ $\left\{ \text{Let } \frac{18.5-\mu}{\sigma} = -b \right.$ --(4)

$\Rightarrow P(Z_2 < b) = 0.67$

$\Rightarrow \phi(b) = 0.67$

$\Rightarrow b = \phi^{-1}(0.67) = 0.44$ --(5)

from (4) & (5)

$\frac{18.5-\mu}{\sigma} = -0.44$

$\Rightarrow 18.5 - \mu = -0.44\sigma$ --(6)

Solving (3) & (6)

$\mu = 22.9$; $\sigma = 9.95$

25 The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

(a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm. ---[4]

The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation σ cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

(b) Find the value of σ. ---[3]

A random sample of 120 Eastern bluebirds is chosen.

(c) Use an approximation to find the prob. that fewer than 80 of these 120 bluebirds have lengths greater than 17.1 cm. ---[5]

S-23/52/Q5

Solution (a) $\mu = 16.5$ cm, $\sigma = 0.6$ cm, $n = 150$

$P(15.4 < X < 16.8)$

$= P\left(\frac{15.4 - 16.5}{0.6} < Z < \frac{16.8 - 16.5}{0.6}\right)$

$= P(-1.833 < Z < 0.5)$

$= \phi(0.5) - \phi(-1.833)$

$= \phi(0.5) - (1 - \phi(1.833))$

$= 0.6915 + 0.9666 - 1$

$= 0.658$

$\therefore$ The Expected number $= np$

$= 150 \times 0.6581 = 98$ (or 99)

(b) Now $\mu = 18.4$, S.D $= \sigma$, and

$P(X > 17.1) = 0.72 > 0.5$

$\Rightarrow P\left(Z > \frac{17.1 - 18.4}{\sigma}\right) = 0.72$

$\Rightarrow P\left(Z > -\frac{1.3}{\sigma}\right) = 0.72$

$\Rightarrow P\left(Z < \frac{1.3}{\sigma}\right) \Rightarrow \phi\left(\frac{1.3}{\sigma}\right) = 0.72$

$\Rightarrow \phi\left(\frac{1.3}{\sigma}\right) = 0.72$

$\Rightarrow \frac{1.3}{\sigma} = \phi^{-1}(0.72) \Rightarrow \frac{1.3}{\sigma} = 0.583 \Rightarrow \sigma = \frac{1.3}{0.583} = 2.23$

(c) $n = 120$, $p = P(X > 17.1) = 0.72$

$q = 0.28$

Now

Mean $\mu = np = 120 \times 0.72 = 86.4$

Var $\sigma^2 = npq = 120 \times 0.72 \times 0.28 = 24.192$

$P(X < 80) = P\left(Z < \frac{79.5 - 86.4}{\sqrt{24.192}}\right)$ Continuity Correction: $X < 80 = X \le 79 \to 79.5$

$= P(Z < -1.4029)$

$= 1 - \phi(1.4029)$

$= 1 - 0.9196$

$= 0.0804$

26. Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.
Use an approximation to find the prob. that, of the 120 voters, between 36 and 54 inclusive voted for Anil. ---[5]

S-23|53|Q2

Solution: $p = 0.4,\ q = 0.6,\ n = 120$

Mean $\mu = np = 120 \times 0.4 = 48$ ✓

Var $\sigma^2 = npq = 120 \times 0.4 \times 0.6 = 28.8$

$B(n, p) \sim N(\mu, \sigma^2)$

$P(36 \leq X \leq 54)$

Continuity correction: $X \geq 36 \rightarrow 35.5$, $X \leq 54 \rightarrow 54.5$

$= P\left(\frac{35.5 - 48}{\sqrt{28.8}} < Z < \frac{54.5 - 48}{\sqrt{28.8}}\right)$

$= P(-2.3292 < Z < 1.211)$

$= P(Z < 1.211) - P(Z < -2.3292)$

$= \phi(1.211) - (1 - \phi(2.3292))$

$= 0.8871 + 0.9900 - 1 = \underline{0.877}$ ✓

27. The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of mass of grapes sold per day. ---[3]

The mass of grapes sold on any day is independent of the mass sold on any other day.

(b) 12 days are chosen at random. Find prob. that less than 16 kg of grapes are sold on more than 2 of these 12 days. ---[3]

(c) In a random sample of 365 days, on how many days would you expect the mass of grapes sold to be within 1.3 standard deviation of the mean. ---[4]

S-23/53/Q6

Solution (a) $\mu = 28$ kg, $\sigma = ?$

Given $P(X < 16\text{ kg}) = 0.1 < 0.5$

$\Rightarrow P\left(Z < \frac{16 - 28}{\sigma}\right) = 0.1$

$\Rightarrow \phi\left(-\frac{12}{\sigma}\right) = 0.1$

$\Rightarrow 1 - \phi\left(\frac{12}{\sigma}\right) = 0.1$

$\Rightarrow \phi\left(\frac{12}{\sigma}\right) = 0.9$

$\Rightarrow \frac{12}{\sigma} = \phi^{-1}(0.9) = 1.282$

$\Rightarrow \sigma = \frac{12}{1.282} = 9.36$

$\Rightarrow \underline{\sigma = 9.36}$

(b) Now $n = 12,\ p = 0.1,\ q = 0.9$

$P(r > 2) = 1 - P(0, 1, 2)$

$= 1 - \{ {}^{12}C_0 (0.9)^{12} + {}^{12}C_1 (0.1)(0.9)^{11} + {}^{12}C_2 (0.1)^2 (0.9)^{10} \}$

$= 0.111$ ✓

(c) $P\left(-\frac{1.3\sigma}{\sigma} < Z < \frac{1.3\sigma}{\sigma}\right) = P(-1.3 < Z < 1.3)$

$= \phi(1.3) - \phi(-1.3)$

$= \phi(1.3) - (1 - \phi(1.3))$

$= 2 \cdot \phi(1.3) - 1$

$= 2 \times 0.9032 - 1 = 0.806$

for $n = 365$ day. Expectation $= 365 \times 0.806$

$= \underline{294}$ (or 295) ✓

28. The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

(a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

(b) On 90% of days, Davin plays on his games machine for more than t hours. Find the value of t. [3]

(c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

W-20/51/Q5

Solution: $\mu = 3.5$, $\sigma = 0.9$

(a) $P(X > 4.2) = P\left(Z > \frac{4.2 - 3.5}{0.9}\right)$

$= P(Z > 0.7778)$

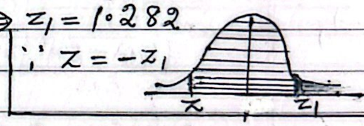
$= 1 - \phi(0.7778) = 1 - 0.7818$

$= \underline{0.218}$ ✓

(b) $P(Z > z) = 0.9 \Rightarrow P(Z < z_1),\ P(Z < z_1) = \phi(z_1) = 0.9 \ (z = -z_1)$

$\Rightarrow z_1 = 1.282$

$\because z = -z_1$



$P(X > t) = P\left(Z > \frac{t - 3.5}{0.9}\right) = -1.282$

$\Rightarrow t - 3.5 = -1.282 \times 0.9 = -1.1538$

$\Rightarrow t = 3.5 - 1.1538$

$= 2.346$

$\therefore t = \underline{2.35}$ ✓

(c) $P(2.8 < X < 4.2) = P\left(\frac{2.8 - 3.5}{0.9} < Z < \frac{4.2 - 3.5}{0.9}\right)$

$= \phi(0.7778) - (1 - \phi(0.7778))$

$= 0.7818 - 1 + 0.7818$

$= 0.5636.$

$\therefore$ Number of day $= 365 \times 0.5636 = 205.7$

or $= \underline{206 \text{ days}}$ ✓

29. Pia runs 2 Km every day and her times in minutes are normally distributed with mean 10.1 and Standard deviation 1.3.

(a) Find the probability that a randomly chosen day Pia takes longer than 11.3 minutes to run 2 Km. ---[3]

(b) On 75% of days, Pia takes longer than t minutes to run 2 Km, Find the value of t. --[3]

(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 Km --[3]

[W-20/52/Q3]

Solution (a) $P(X>11.3) = P\left(Z > \frac{11.3-10.1}{1.3}\right) = P(Z>0.9231)$ $\left[\because \mu = 10.1,\ \sigma = 1.3\right]$

$= 1-\phi(0.9231)$

$= 1-0.822 = \underline{0.178}$ ✓

(b) $P(X>t) = 0.75 \Rightarrow P\left(Z > \frac{t-10.1}{1.3}\right)$

$\left[P(Z>z) = 0.75 \Rightarrow z = -z_1\right.$

$\Rightarrow \frac{t-10.1}{1.3} = -0.674$

$\left\{\text{where } \phi(z_1) = .75 \Rightarrow z_1 = 0.674\right.$

$\Rightarrow t = \underline{9.22}$ ✓

(c) $P(8.9 < X < 11.3) = P\left(\frac{8.9-10.1}{1.3} < Z < \frac{11.3-10.1}{1.3}\right)$

$= \phi(0.9231) - (1-\phi(0.9231))$

$= 2\phi(0.9231) - 1$

$= 2\times 0.822 - 1$

$= 1.644 - 1 = 0.644$

$\therefore$ Number days $= 90 \times 0.644$

$= 57.96$

$= \underline{58 \text{ days}}$ ✓

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30. The times taken to swim 100 metres by members of a large swimming club have a normal distribution with mean 62 seconds and standard deviation 5 seconds.

(a) Find the probability that a randomly chosen member of the club takes between 56 and 66 seconds to swim 100 metres. ---[3]

(b) 13% of the members of the club take more than t minutes to swim 100 metres. Find the value of t. [W-20/53/Q1] ---[3]

Solution:
(a) $P(56 < x < 66) = P\left(\frac{56-62}{5} < z < \frac{66-62}{5}\right)$ $\left\{ \begin{array}{l} \mu = 62 \\ \sigma = 5 \\ z = \frac{x-\mu}{\sigma} \end{array} \right.$

$= P(-1.2 < z < 0.8)$

$= \phi(0.8) - (1-\phi(1.2))$

$= 0.7881 + 0.8849 - 1$

$= \underline{0.673}$ ✓

(b) $P(x > 60t) = P\left(z > \frac{60t-62}{5}\right) = 0.13$ $\left(\begin{array}{l} t = \text{minute} \\ \quad = 60t \text{ seconds} \end{array}\right)$

$= P\left(z < \frac{60t-62}{5}\right) = 1 - 0.13 = 0.87$

$\left\{ \begin{array}{l} \because \phi(z) = 0.87 \\ \Rightarrow z = 1.127 \end{array} \right.$ ✓

$\Rightarrow \frac{60t-62}{5} = 1.127$

$\Rightarrow 60t = 1.127 \times 5 + 62 = 67.635$

$\Rightarrow t = \underline{1.13}$ ✓

31. The 1300 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the prob. that the train arrives late on fewer than 3 days. ---[3]

A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [W-20/53/Q4] ---[5]

Solution: $P(\text{late}) = p = 0.35$, $q = 0.65$, $n = 7$.

(a) $P(r < 3) = P(0,1,2)$

$$= 0.65^7 + {}^7C_1 \times 0.35 \times 0.65^6 + {}^7C_2 \times 0.35^2 \times 0.65^5$$

$$= 0.049022 + 0.184776 + 0.29848$$

$$= \underline{0.532}$$

(b) Use Normal distribution.

Mean $\mu = np = 142 \times 0.35 = 49.7$

Variance $\sigma^2 = npq = 142 \times 0.35 \times 0.65 = 32.305$

$$P(X > 40) = P\left(Z > \frac{40.5 - 49.7}{\sqrt{32.305}}\right)$$

Continuity Correction: $P(X > 40) \Rightarrow P(X \geq 41) \Rightarrow P(Z > 40.5)$

$$= P(Z > -1.619)$$

$$= P(Z < 1.619)$$

$$= \phi(1.619)$$

$$= \underline{0.947}$$

32. The times in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) (i) On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? ---[5]

(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. ---[3]

(b) On 90% of days, Karli spends more than t minutes on social media. Find the value of t. --[3]

[W-21/51/Q7]

Solution: $N(\mu, \sigma^2)$, $\mu = 125$, $\sigma = 24$

(a)(i) $P(X>142) = P\left(Z > \frac{142-125}{24}\right)$

$= P(Z > 0.7083)$

$= 1 - \phi(0.7083)$

$= 1 - 0.7604$

$p = 0.2396$.

$\therefore$ No. of days $= np = 365 \times 0.2396 = 87.4$

$= \underline{87}$ or $\underline{88}$ days ✓

(a)(ii) $p = 0.2396$; $q = 0.7604$

$n = 10$, $\quad P(r) = {}^nC_r\, p^r\, q^{n-r}$

$P(r<2) = P(0,1)$

$= {}^{10}C_0\, p^0 q^{10} + {}^{10}C_1\, p^1 q^9$

$= (0.7604)^{10} + 10 \times 0.2396 \times (0.7604)^9$

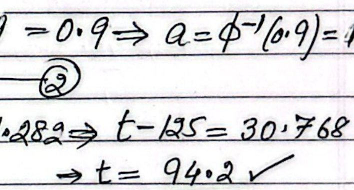
$= 0.064628 + 0.20364$

$\doteq \underline{0.268}$ ✓

(b) (90%)

$P(X>t) = P\left(Z > \frac{t-125}{24}\right) = 0.9$

$= P\left(Z < \frac{t-125}{24}\right) = 1 - 0.9$
$= 0.1$ — ①



$\Rightarrow z = -a \Rightarrow \quad \phi(-a) = 0.1$

$\rightarrow \phi(a) = 1 - 0.1 = 0.9 \Rightarrow a = \phi^{-1}(0.9) = 1.282$

$\Rightarrow z = -a = -1.282$ — ②

from ① and ② $\quad \frac{t-125}{24} = -1.282 \Rightarrow t - 125 = 30.768$

$\rightarrow t = \underline{94.2}$ ✓

33. The times taken, in minutes, to complete a particular task by employees at a large company are normally distributed with mean 32.2 and standard deviation 9.6.

(a) Find the probability that a randomly chosen employee takes more than 28.6 minutes to complete the task. ... [3]

(b) 20% of employees take longer than t minutes to complete the task. Find the value of t. ---[3]

(c) Find the prob. that the time taken to complete the task by a randomly chosen employee differs from mean by less than 15 minutes. --[4]

[W-21/52/Q6]

Solution: $N(\mu,\sigma^2)$; $\sigma = 32.2$, $\sigma = 9.6$

(a) $P(t > 28.6) = P\left(z > \frac{28.6-32.2}{9.6}\right)$

$= P(z > -0.375)$

$= P(Z < 0.375)$

$= \phi(0.375) = 0.6462$

$= \underline{0.646}$ ✓

(b) $P(X > t) = P\left(Z > \frac{t-32.2}{9.6}\right) = 0.2$ (20%)

$= P\left(Z < \frac{t-32.2}{9.6}\right) = 1-0.2$
$= 0.8$

$\phi\left(\frac{t-32.2}{9.6}\right) = 0.8$

$\Rightarrow \frac{t-32.2}{9.6} = \phi^{-1}(0.8)$
$= 0.842$

$t - 32.2 = 0.842 \times 9.6 = 8.0832$

$t = 8.0832 + 32.2$

$= 40.2832$

$\therefore t = \underline{40.3}$ ✓

(C) $P(-(t-\mu) < 15 < t-\mu)$

$= P\left(\frac{-15}{9.6} < Z < \frac{15}{9.6}\right)$

$= P(-1.5625 < Z < 1.5625)$

$= \phi(1.5625) - \phi(-1.5625)$

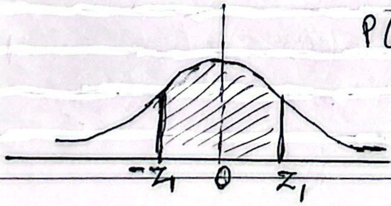
$= \phi(1.5625) - [1 - \phi(1.5625)]$

$= 2\phi(1.5625) - 1$

$= 2 \times 0.9409 - 1$

$= \underline{0.882}$ ✓

(C)



$P(-z_1 < Z < z_1) = \phi(z_1) - \phi(-z_1)$

$= \phi(z_1) - [1 - \phi(z_1)]$

$= 2\phi(z_1) - 1$ ✓

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34. Raj wants to improve his fitness, so every day he goes for a sun. The times, in minutes, of his suns have a normal distribution with mean 41.2 and standard deviation 3.6.

(a) Find the prob. that a randomly chosen day Raj suns for more than 43.2 minutes. ---[3]

(b) Find an estimate for the number of days in a year (365 days) on which Raj suns for less than 43.2 days. ---[2]

(c) On 95% of days, Raj suns for more than t minutes. Find the value of t. ---[3]

W-21/53/Q4

Solution: $N(\mu,\sigma^2)$: $\mu = 41.2$, $\sigma = 3.6$

(a) $P(X>43.2) = P\left(z > \frac{43.2-41.2}{3.6}\right)$
$= P(z>0.5556)$
$= 1-P(z<0.5556)$
$= 1-\phi(0.5556)$
$= 1-0.7108 = \underline{0.289}$ ✓

(b) $P(X<43.2) = 1-0.289$
$= 0.7108$

$\therefore$ No. of day $= np = 365 \times 0.7108$
$= 259.4$
$= \underline{259, 260 \text{ days}}$. ✓

(c) $P(X>t) = 0.95$ (95%)

$\Rightarrow P\left(z > \frac{t-41.2}{3.6}\right) = 0.95$

$\Rightarrow P\left(z < \frac{t-41.2}{3.6}\right) = 1-0.95 = 0.05$ --- ①

$\Rightarrow \phi\left(\frac{t-41.2}{3.6}\right) = \phi(-a) = 0.05$

$\Rightarrow \phi(a) = 1-0.05 = 0.95$

$\Rightarrow a = \phi^{-1}(0.95)$

$a = 1.645$ --- ②

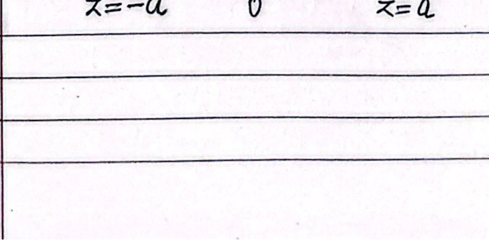
$\therefore$ from ① and ② $\left\{ \frac{t-41.2}{3.6} = -1.645 \right.$

$\Rightarrow t-41.2 = -5.922$

$t = 35.278$

$\therefore \underline{t = 35.3}$ ✓

(c)



35. The residents of Persham were surveyed about the reliability of their internet service. 12% rated the service as 'poor', 36% rated it as satisfactory and 52% rated it as 'good'.

A random sample of 8 residents of Persham is chosen.

(a) Find the probability that more than 2 and fewer than 8 of them rate their internet service as poor or satisfactory. --[3]

A random sample of 125 residents of Persham is now chosen.

(b) Use an approximation to find the prob. that more than 72 of these residents rate their internet service as good. ---[5]

[W-22/51/Q2]

Solution (a) Poor 12%, satisfactory 36% and good 52% ; $n=8$

$p = P(\text{poor or satisfactory}) = (12+36)\% = 0.48$, $q = 0.52$

Now $P(2<x<8) = P(3,4,5,6,7) = 1 - P(0,1,2,8)$

$= 1 - [{}^8C_0 \cdot 0.48^0 \times (0.52)^8 + {}^8C_1 \times 0.48^1 \times 0.52^7 + {}^8C_2 \times 0.48^2 \times 0.52^6 + {}^8C_8 \times 0.48^8 \times 0.52^0]$

$= 1 - [0.00534597 + 0.039478 + 0.127544 + 0.0028179]$

$= \underline{0.825}$ ✓

[using Binomial prob dis. $P(X=r) = {}^nC_r p^r q^{n-r}$]

(b) Now $n = 125$, $P(\text{Good}) = p = 0.52$, $q = 0.48$, $np = 125 \times 0.52 = 65 > 5$

we can use normal distribution $\rightarrow$ $nq = 125 \times 0.48 = 60 > 5$

$N(\mu, \sigma^2)$

Mean $\mu = np = 65$ ✓

Variance $\sigma^2 = npq = 125 \times 0.52 \times 0.48$

$\sigma^2 = 31.2$

Now $P(X>72) = P\left(Z > \frac{72.5 - 65}{\sqrt{31.2}}\right)$

[Continuity correction. $X > 72 \rightarrow x \geq 73 \rightarrow X = 72.5$ ✓

$= P(Z > 1.343)$

$= 1 - \phi(1.343)$

$= 1 - 0.9104$

$= \underline{0.0896}$ ✓

36. In a large population, the systolic blood pressure (SBP) of adults is normally distributed with mean 125.4 and standard deviation 18.6.

(a) Find the prob. that the SBP of a randomly chosen adults is less than 132. ---[2]

The SBP of 12 year-old children in the same population is normally distributed with mean 117. of these children 88% have SBP more than 108.

(b) Find the standard deviation of this distribution. ---[3]

Three adults are chosen at random from this population.

(c) Find the prob. that each of these three adults has SBP within 1.5 standard deviations of the mean. --- [4]

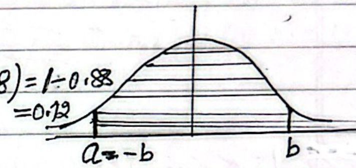
[W-22/51/Q4]

Solution $N(125.4, 18.6^2) \leftrightarrow N(\mu, \sigma^2)$

(a) $P(X < 132) = P\left(Z < \dfrac{132 - 125.4}{18.6}\right) = P(Z < 0.3548) = \phi(0.3548)$
$= \underline{0.639}$ ✓

(b) Now $N(117, \sigma^2)$.

Given $P(X > 108) = 0.88 \Rightarrow P(X < 108) = 1 - 0.88 = 0.12$



$P(X < 108) = P\left(Z < \dfrac{108 - 117}{\sigma}\right)$

$\Rightarrow P\left(Z < -\dfrac{9}{\sigma}\right) = 0.12$

$\Rightarrow 1 - \phi\left(\dfrac{9}{\sigma}\right) = 0.12 \Rightarrow \phi\left(\dfrac{9}{\sigma}\right) = 0.88$ ✓

$\Rightarrow \dfrac{9}{\sigma} = \phi^{-1}(0.88) = 1.175 \Rightarrow \sigma = 9/1.175 = \underline{7.66 = S.D}$

(c) $P(-1.5 < Z < 1.5)$ for any one adult.

$= \phi(1.5) - \phi(-1.5)$
$= \phi(1.5) - (1 - \phi(1.5))$
$= 2\cdot\phi(1.5) - 1$
$= 2 \times 0.9332 - 1$
$= 0.8664$

Hence the prob. that all three adults satisfy it $= (0.8664)^3$

$= \underline{0.650}$ ✓

37. The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.
(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm. ---[4]
(b) Find the prob. that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean. --[3]
W-22/52/Q2

Solution: $N(\mu, \sigma^2) \sim N(55.6, 1.2^2)$,
(a). $P(X < 54.8) = P\left(z < \frac{54.8 - 55.6}{1.2}\right)$
$= P(z < -0.6667)$
$= 1 - \phi(0.6667)$
$p = 1 - 0.7477 = 0.2523$
Number of rods $= 400$
$\therefore$ Expected number $= np = 400 \times 0.2523 = 100.92$
$= \underline{100} \text{ or } \underline{101}$ ✓

(b) $P\left(-\frac{1}{2} < z < \frac{1}{2}\right) = \phi(0.5) - \phi(-0.5)$
$= \phi(0.5) - (1 - \phi(0.5))$
$= 2\phi(0.5) - 1$
$= 2 \times 0.6915 - 1$
$= \underline{0.383}$ ✓

38. At a company's call centre, 90% of callers are connected immediately to a representative. A random sample of 12 callers is chosen.

(a) Find the prob. that fewer than 10 of these callers are connected immediately. ---[3]

A random sample of 80 callers is chosen.

(b) Use an approximation to find the prob. that more than 69 of these callers are connected immediately. ---[5]

(c) Justify your approximation in part (b) ---[1]

[W-22/52/Q6]

Solution (a) $n = 12,\ p = 0.9\ (90\%),\ q = 1-0.9 = 0.1$

$P(x<10) = 1 - P(10, 11, 12)$

$= 1 - [{}^{12}C_{10}\,0.9^{10}\,0.1^{2} + {}^{12}C_{11}\,0.9^{11}\,0.1^{1} + {}^{12}C_{12}\,0.9^{12}\,0.1^{0}]$

$= 1 - (0.230128 + 0.376573 + 0.282430)$

$= \underline{0.111}$ ✓

(b) Now $n = 80,\ p = 0.9,\ q = 0.1,$

Approximation from $B(n,p) \to N(\mu, \sigma^2)$

$\mu = np = 80 \times 0.9 = 72$

Variance $= npq = 80 \times 0.9 \times 0.1 = 7.2$

$P(X>69) = P\left(Z > \frac{69.5 - 72}{\sqrt{7.2}}\right)$

Continuity correction; $X>69 \to X \geq 70 \to X = 69.5$

$= P(Z > -0.9317)$

$= \phi(0.9317)$

$= \underline{0.824}$ ✓

$\because$ [$P(Z>-0.9317) = P(Z<0.9317)$]

(c) $np = 72 > 5$ and $nq = 80 \times 0.1 = 8 > 5$

$\therefore$ Binomial distribution approximates to Normal distribution.

39. In a large college, 32% of the students have blue eyes. A random sample of 80 students is chosen. Use an approximation to find the prob. that fewer than 20 of these students have blue eyes. ---[5]

[W-22 / 53 / Q2]

Solution: $p = 0.32$ (32%) ; $n = 80$, $\mu = np = 0.32 \times 80 = 25.6$,
$q = 0.68$ $\quad Var\ \sigma^2 = npq = 80 \times 0.32 \times 0.68 = 17.408$

$P(X < 20) = P\left(Z < \frac{19.5 - 25.6}{\sqrt{17.408}}\right) = P(Z < -1.462)$

Continuity correction
$x < 20 \rightarrow x \leq 19$
$\rightarrow x = 19.5$

$= 1 - \phi(1.462) = 1 - 0.9282$
$= 0.0718$

40. Company A produces bags of sugar. A inspector finds that on average 10% of the bags are underweight. 10 of the bags are chosen at random.

(a) Find the prob. that fewer than 3 of these bags are under weight. ---[3]

The weights of the bags of sugar produced by Company B are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the prob. that a randomly chosen bag produced by Company B weighs more than 1.11 kg. ---[3]

81% of the bags of sugar produced by Company B weigh less than W kg.

(c) Find the value of W. --[3]

$\boxed{W\text{-}22/53/Q5}$

Solution: $p = 0.1,\ q = 0.9,\ n = 10$

(a) $P(x<3) = P(0,1,2) = {}^{10}C_0\, 0.1^0\, 0.9^{10} + {}^{10}C_1\, 0.1^1\, 0.9^9 + {}^{10}C_2\, 0.1^2\, 0.9^8$

$= 0.348678 + 0.38742 + 0.19371$

$= \underline{0.930}$ ✓

(b) $\mu = 1.04,\ \sigma = 0.06,\ N\sim(\mu,\sigma^2)$

$P(X>1.11) = P\left(z > \frac{1.11-1.04}{0.06}\right) = P(z>1.167)$

$= 1-\phi(1.167)$

$= 1-0.8784 = \underline{0.122}$ ✓

(c) $P(X<W) = P\left(z < \frac{W-1.04}{0.06}\right) = 0.81 \quad (81\%)$

$\Rightarrow \frac{W-1.04}{0.06} = \phi^{-1}(0.81) = 0.878$

$\Rightarrow W - 1.04 = 0.878\times 0.06 = 0.05268$

$\Rightarrow W = 1.04 + 0.0568 = 1.0926$

$\therefore W = \underline{1.09}$ ✓