PROBABILITY AND STATISTICS -1

9709

(March, June and November series 2020 – 2023 With marking scheme)

Normal Distribution

EXERCISE -1

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1) SP-2020_9709_5 Q4

A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(a) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(b) Find the value of m. [3]

(c) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

2) SP-2020 _9709_5 Q5

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown 90 times.

(a) Use an approximation to find the probability that a 3 is obtained fewer than 18 times. [4]

(b) Justify your use of the approximation in part **(a)**. [1]

On another occasion, the same die is thrown repeatedly until a 3 is obtained.

(c) Find the probability that obtaining a 3 requires fewer than 7 throws. [2]

3) MARCH 2020_9709_52 Q3

The weights of apples of a certain variety are normally distributed with mean 82 grams. 22% of these apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples. [3]

(b) Find the probability that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams. [4]

4) MARCH 2020_9709_52 Q5(b)

In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

A random sample of 120 adults from Greenton is now chosen.

(b) Use an approximation to find the probability that more than 75 of them own a car. [5]

5) MARCH 2021_9709_52 Q3

The time spent by shoppers in a large shopping centre has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

(a) Find the probability that a shopper chosen at random spends between 85 and 100 minutes in the shopping centre. [3]

88% of shoppers spend more than t minutes in the shopping centre.

(b) Find the value of t. [3]

6) MARCH 2021_9709_52 Q7

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

7) MARCH 2022_9709_52 Q4

The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

(a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

(b) Find the value of σ. [3]

The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

(c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

8) MARCH 2023_9709_52 Q6

In a cycling event the times taken to complete a course are modelled by a normal distribution with mean 62.3 minutes and standard deviation 8.4 minutes.

(a) Find the probability that a randomly chosen cyclist has a time less than 74 minutes. [2]

(b) Find the probability that 4 randomly chosen cyclists all have times between 50 and 74 minutes. [4]

In a different cycling event, the times can also be modelled by a normal distribution. 23% of the cyclists have times less than 36 minutes and 10% of the cyclists have times greater than 54 minutes.

(c) Find estimates for the mean and standard deviation of this distribution. [5]

9) JUNE 2020_9709_51 Q6

The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

(a) Find the probability that a randomly chosen female snake of this species has length between 50 cm and 60 cm. [4]

The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find estimates for the mean and standard deviation of the lengths of male snakes of this species. [5]

10) JUNE 2020_9709_52 Q4

Trees in the Redian forest are classified as tall, medium or short, according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height of less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short. [3]

Of the trees that are classified as tall or medium, one third are tall and two thirds are medium.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 decimal places. [2]

(c) Find the height above which trees are classified as tall. [3]

11) JUNE 2020_9709_52 Q7(c)

On any given day, the probability that Moena messages her friend Pasha is 0.72.

(c) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

12) JUNE 2020_9709_53 Q3

In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. [2]

(b) Find the value of k such that $P(X < k) = 0.75$. [3]

13) JUNE 2020_9709_53 Q5(d)

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

14) JUNE 2021_9709_51 Q2

A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.

How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? [5]

15) JUNE 2021_9709_51 Q6(b)(c)

In Questa, 60% of the adults travel to work by car.

(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

(c) Justify the use of your approximation in part **(b)**. [1]

16) JUNE 2021_9709_52 Q2

The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.

Find the value of σ. [4]

17) JUNE 2021_9709_52 Q5(c)

Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

18) JUNE 2021_9709_53 Q5

The lengths of the leaves of a particular type of tree are modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves from this type of tree and finds that 42 are less than 4 cm long and 100 are more than 10 cm long.

(a) Find estimates for the mean and standard deviation of the lengths of leaves from this type of tree. [5]

The lengths, in cm, of the leaves of a different type of tree have the distribution $N(\mu, \sigma^2)$. The scientist takes a random sample of 800 leaves from this type of tree.

(b) Find how many of these leaves the scientist would expect to have lengths, in cm, between $\mu - 2\sigma$ and $\mu + 2\sigma$. [4]

19) JUNE 2021_9709_53 Q7b(ii)

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

20) JUNE 2022_9709_51 Q5

The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2, 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

(c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? [4]

21) JUNE 2022_9709_52 Q4

The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ. [4]

22) JUNE 2022_9709_52 Q5

In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

23) JUNE 2022_9709_53 Q5

Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 grams to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20000.

(b) Calculate an estimate for his total income from this year's apples. [3]

Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution $N(182, 20^2)$. 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

24) JUNE 2023_9709_51 Q4

A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and standard deviation 5.2 minutes.

(a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle? [4]

All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean μ and standard deviation σ. It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

(b) Find the value of μ and the value of σ. [5]

25) JUNE 2023_9709_52 Q5

The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

(a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm? [4]

The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation σ cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

(b) Find the value of σ. [3]

A random sample of 120 Eastern bluebirds is chosen.

(c) Use an approximation to find the probability that fewer than 80 of these 120 bluebirds have length greater than 17.1 cm. [5]

26) JUNE 2023_9709_53 Q2

Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.

Use an approximation to find the probability that, of the 120 voters, between 36 and 54 inclusive voted for Anil. [5]

27) JUNE 2023_9709_53 Q6

The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of the mass of grapes sold per day. [3]

The mass of grapes sold on any day is independent of the mass sold on any other day.

(b) 12 days are chosen at random.

Find the probability that less than 16 kg of grapes are sold on more than 2 of these 12 days. [3]

(c) In a random sample of 365 days, on how many days would you expect the mass of grapes sold to be within 1.3 standard deviations of the mean? [4]

28) OCT 2020_9709_51 Q5

The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

(a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

(b) On 90% of days Davin plays on his games machine for more than t hours. Find the value of t. [3]

(c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

29) OCT 2020_9709_52 Q3

Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t. [3]

(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

30) OCT 2020_9709_53 Q1

The times taken to swim 100 metres by members of a large swimming club have a normal distribution with mean 62 seconds and standard deviation 5 seconds.

(a) Find the probability that a randomly chosen member of the club takes between 56 and 66 seconds to swim 100 metres. [3]

(b) 13% of the members of the club take more than t minutes to swim 100 metres. Find the value of t. [3]

31) OCT 2020_9709_53 Q4

The 13 00 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the probability that the train arrives late on fewer than 3 days. [3]

A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [5]

32) OCT 2021_9709_51 Q7

The times, in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) **(i)** On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? [5]

(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. [3]

(b) On 90% of days, Karli spends more than t minutes on social media.

Find the value of t. [3]

33) OCT 2021_9709_52 Q6

The times taken, in minutes, to complete a particular task by employees at a large company are normally distributed with mean 32.2 and standard deviation 9.6.

(a) Find the probability that a randomly chosen employee takes more than 28.6 minutes to complete the task. [3]

(b) 20% of employees take longer than t minutes to complete the task.

Find the value of t. [3]

(c) Find the probability that the time taken to complete the task by a randomly chosen employee differs from the mean by less than 15.0 minutes. [4]

34) OCT 2021_9709_53 Q4

Raj wants to improve his fitness, so every day he goes for a run. The times, in minutes, of his runs have a normal distribution with mean 41.2 and standard deviation 3.6.

(a) Find the probability that on a randomly chosen day Raj runs for more than 43.2 minutes. [3]

(b) Find an estimate for the number of days in a year (365 days) on which Raj runs for less than 43.2 minutes. [2]

(c) On 95% of days, Raj runs for more than t minutes.

Find the value of t. [3]

35) OCT 2022_9709_51 Q2

The residents of Persham were surveyed about the reliability of their internet service. 12% rated the service as 'poor', 36% rated it as 'satisfactory' and 52% rated it as 'good'.

A random sample of 8 residents of Persham is chosen.

(a) Find the probability that more than 2 and fewer than 8 of them rate their internet service as poor or satisfactory. [3]

A random sample of 125 residents of Persham is now chosen.

(b) Use an approximation to find the probability that more than 72 of these residents rate their internet service as good. [5]

36) OCT 2022_9709_51 Q4

In a large population, the systolic blood pressure (SBP) of adults is normally distributed with mean 125.4 and standard deviation 18.6.

(a) Find the probability that the SBP of a randomly chosen adult is less than 132. [2]

The SBP of 12-year-old children in the same population is normally distributed with mean 117. Of these children 88% have SBP more than 108.

(b) Find the standard deviation of this distribution. [3]

Three adults are chosen at random from this population.

(c) Find the probability that each of these three adults has SBP within 1.5 standard deviations of the mean. [4]

37) OCT 2022_9709_52 Q2

The lengths of the rods produced by a company are normally distributed with mean 55.6 mm and standard deviation 1.2 mm.

(a) In a random sample of 400 of these rods, how many would you expect to have length less than 54.8 mm? [4]

(b) Find the probability that a randomly chosen rod produced by this company has a length that is within half a standard deviation of the mean. [3]

38) OCT 2022_9709_52 Q6

At a company's call centre, 90% of callers are connected immediately to a representative.

A random sample of 12 callers is chosen.

(a) Find the probability that fewer than 10 of these callers are connected immediately. [3]

A random sample of 80 callers is chosen.

(b) Use an approximation to find the probability that more than 69 of these callers are connected immediately. [5]

(c) Justify the use of your approximation in part **(b)**. [1]

39) OCT 2022_9709_53 Q2

In a large college, 32% of the students have blue eyes. A random sample of 80 students is chosen.

Use an approximation to find the probability that fewer than 20 of these students have blue eyes. [5]

40) OCT 2022_9709_53 Q5

Company A produces bags of sugar. An inspector finds that on average 10% of the bags are underweight.

10 of the bags are chosen at random.

(a) Find the probability that fewer than 3 of these bags are underweight. [3]

The weights of the bags of sugar produced by company B are normally distributed with mean 1.04 kg and standard deviation 0.06 kg.

(b) Find the probability that a randomly chosen bag produced by company B weighs more than 1.11 kg. [3]

81% of the bags of sugar produced by company B weigh less than w kg.

(c) Find the value of w. [3]

MARKING SCHEME

1) SP-2020_9709_5Q4

(a)	$P(X > 3900) = P\left(Z > \frac{3900-4520}{560}\right)$	1	M1	Standardising: no continuity correction, no square root, no square
	$P(Z > -1.107) = \Phi(1.107)$	1	M1	Attempt at correct area: $\Phi > 0.5$, depends on negative z
	$= 0.8657$	1	A1	Probability rounding to 0.866
	Number of days = $365 \times 0.8657 = 315$ or 316 (315.98)	1	B1FT	FT their wrong probability if previous A0, $p < 1$, FT must be accurate to 3sf
		4		
(b)	$z = 1.165$	1	B1	$\pm$ 1.165 seen
	$1.165 = \frac{8000 - m}{560}$	1	M1	Standardising equation, allow square, square root, continuity correction, must have z-value (e.g. not 0.122, 0.878, 0.549, 0.810).
	$m = 7350$ (7347.6)	1	A1	
		3		
(c)	$P(0, 1) = (0.878)^6 + {}^6C_1(0.122)^1(0.878)^5$ $(= 0.4581 + 0.3819)$	2	M1M1	M1 for Correct unsimplified expression M1 for Binomial term ${}^6C_x\,p^x(1-p)^{6-x}$ $0 < p < 1$ seen
	$= 0.840$ (accept 0.84)	1	A1	
		3		

2) SP-2020 _9709_5 Q5

(a)	$p = \frac{1}{6}$: mean $= np = 90 \times \frac{1}{6} = 15$	1	B1	Correct mean
	Variance $= npq = \frac{75}{6}$	1	B1	Correct variance
	$P(X < 18) = P\left(Z < \frac{17.5 - 15}{\sqrt{\frac{75}{6}}}\right) = P(Z < 0.7071)$	1	M1	Standardising equation, allow square, square root, continuity correction
	$= 0.760$	1	A1	
		4		
(b)	$np = 15 > 5$ and $nq = 75 > 5$, so normal justified	1	B1	Both parts needed
(c)	$1 - \left(\frac{5}{6}\right)^6$	1	M1	
	$= 0.665$	1	A1	
		2		

3) MARCH 2020_9709_52 Q3

(a)	$P(X > 87) = P\left(Z > \frac{87-82}{\sigma}\right) = 0.22$	M1	Using $\pm$ standardisation formula, not σ^2, not $\sqrt{\sigma}$, no continuity correction
	$P\left(Z < \frac{5}{\sigma}\right) = 0.78$ $\left(\frac{5}{\sigma} = \right) 0.772$	B1	AWRT ± 0.772 seen B0 for ± 0.228
	$\sigma = 6.48$	A1	
		3	
(b)	$P\left(-\frac{4}{\sigma} < Z < \frac{4}{\sigma}\right) = P(-0.6176 < Z < 0.6176)$	M1	Using ± 4 used within a standardisation formula (SOI), allow σ^2, $\sqrt{\sigma}$ and continuity correction
		M1	Standardisation formula applied to **both** *their* ± 4
	$\Phi = 0.7317$ Prob $= 2\Phi - 1 = 2(0.7317) - 1$	M1	Correct area $2\Phi - 1$ oe linked to final solution
	$= 0.463$	A1	
		4	

4) MARCH 2020_9709_52 Q5(b)

Mean = $120 \times 0.7 = 84$ Var = $120 \times 0.7 \times 0.3 = 25.2$	B1	Correct mean and variance, allow unsimplified
P(more than 75) = $P\left(z > \frac{75.5-84}{\sqrt{25.2}}\right)$	M1	Substituting *their* μ and σ into the ±standardising formula (any number), not σ^2, not $\sqrt{\sigma}$
	M1	Using continuity correction 75.5 or 74.5
$P(z > -1.693)$	M1	Appropriate area Φ, from final process, must be a probability
$= 0.955$	A1	Allow $0.9545 < p \leqslant 0.955$
	5	

5) MARCH 2021_9709_52 Q3

a)	$P\left(\left(\frac{85-96}{18}\right) < z < \left(\frac{100-96}{18}\right)\right)$	M1	Use of ±standardisation formula once with appropriate values substituted, no continuity correction, not σ^2 or $\sqrt{\sigma}$.
	$P(-0.6111 < z < 0.2222)$ $= \Phi(0.2222) + \Phi(0.6111) - 1$ $= 0.5879 + 0.7294 - 1$	M1	Appropriate area Φ, from final process, must be probability. Use of $(1 - z)$ implies M0.
	0·317	A1	Final answer which rounds to 0·317.
		3	

(b)	$z = \pm 1{\cdot}175$	B1	$1{\cdot}17 \leqslant z \leqslant 1{\cdot}18$ or $-1{\cdot}18 \leqslant z \leqslant -1{\cdot}17$
	$-1.175 = \frac{t-96}{18}$	M1	An equation using ±standardisation formula with a z-value, condone σ^2, $\sqrt{\sigma}$ or continuity correction. E.g. equating to 0·88, 0·12, 0·8106, 0·1894, 0·5478, 0·4522, ±0·175 or ±2·175 implies M0.
	74·85 or 74·9	A1	$74{\cdot}85 \leqslant t \leqslant 74{\cdot}9$
		3	

6) MARCH 2021_9709_52 Q7

(a)(i)	$\left[\frac{104+31}{400} = \right]\frac{135}{400}, \frac{27}{80}, 0{\cdot}3375$	B1	Evaluated, exact value.
		1	
(a)(ii)	**Method 1**		
	$P(M) = \frac{180}{400}, 0{\cdot}45$ $P(S) = \frac{135}{400}, 0{\cdot}3375$ $P(M \cap S) = \frac{31}{400}, 0{\cdot}0775$ $\frac{180}{400} \times \frac{135}{400} = \frac{243}{1600}, 0.151875 \neq \frac{31}{400}$ so NOT independent	M1	*Their* P(M) × *their* P(S) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
	Method 2		
	$P(M \cap S) = \frac{31}{400}$ $P(S) = \frac{135}{400}$ $P(M) = \frac{180}{400}$ $P(M\|S) = \frac{\frac{31}{400}}{\frac{135}{400}} = \frac{31}{135}, 0.2296\ldots \neq \frac{180}{400}$ so NOT independent	M1	$[P(M\|S) =]\frac{their\ P(M \cap S)}{their\ P(S)}$ (oe) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
		2	

(b)(i)	**Method 1** [1 – P(0,1,2)]		
	$= 1 - ({}^{10}C_0\, 0{\cdot}3^0\, 0{\cdot}7^{10} + {}^{10}C_1\, 0{\cdot}3^1\, 0{\cdot}7^9 + {}^{10}C_2\, 0{\cdot}3^2\, 0{\cdot}7^8)$	**M1**	${}^{10}C_x\, p^x\,(1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
	$= 1 - (0{\cdot}028248 + 0{\cdot}121061 + 0{\cdot}233474)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket, condone recovery from poor notation.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
	Method 2 [P(3,4,5,6,7,8,9,10) =]		
	${}^{10}C_3\, 0{\cdot}3^3\, 0{\cdot}7^7 + {}^{10}C_4\, 0{\cdot}3^4\, 0{\cdot}7^6 + {}^{10}C_5\, 0{\cdot}3^5\, 0{\cdot}7^5 + {}^{10}C_6\, 0{\cdot}3^6\, 0{\cdot}7^4 + {}^{10}C_7\, 0{\cdot}3^7\, 0{\cdot}7^3 + {}^{10}C_8\, 0{\cdot}3^8\, 0{\cdot}7^2 + {}^{10}C_9\, 0{\cdot}3^9\, 0{\cdot}7^1 + {}^{10}C_{10}\, 0{\cdot}3^{10}\, 0{\cdot}7^0$	**M1**	${}^{10}C_x\, p^x\,(1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
		A1	Correct unsimplified expression.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
		3	
(b)(ii)	[p = 0·3] Mean = 0·3 × 90 = 27; variance = 0·3 × 90 × 0·7 = 18·9	**B1**	Correct mean and variance, allow unsimplified. Condone $\sigma = 4{\cdot}347$ evaluated.
	$P(X < 32) = P\left(z < \frac{31.5 - 27}{\sqrt{18.9}}\right)$	**M1**	Substituting *their* μ and σ (not σ^2, $\sqrt{\sigma}$) into the ±standardising formula with a numerical value for '31·5'.
		M1	Using either 31·5 or 32·5 within a ±standardising formula with numerical values for *their* μ and σ (condone σ^2, $\sqrt{\sigma}$).
	$= \Phi(1.035)$	**M1**	Appropriate area Φ, from standardisation formula P(z<…) in final solution, must be probability.
	$= 0{\cdot}850$	**A1**	Allow $0{\cdot}8495 < p \leqslant 0{\cdot}85(0)$, final answer WWW.
		5	

7) MARCH 2022_9709_52 Q4

a)	$P(46 < X < 62) = P\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$	**M1**	46 or 62, 55 and 6 substituted into ±standardisation formula once. Condone 6^2 and continuity correction ±0.5
	$= P\left(-1.5 < Z < \frac{7}{6}\right)$	**B1**	Both standardisation values correct, accept unsimplified
	$\left[= \Phi\left(\frac{7}{6}\right) - (1 - \Phi(1.5))\right]$ $= 0.8784 + (0.9332 - 1)$	**M1**	Calculating the appropriate area from stated Φs of z-values, must be probabilities.
	0.812	**A1**	$0{\cdot}8115 < p \leqslant 0{\cdot}812$
		4	
b)	$z = \pm 0.674$	**B1**	CAO, critical z-value
	$\frac{36-42}{\sigma} = -0.674$	**M1**	36 and 42 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value
	$\sigma = 8.9[0]$	**A1**	WWW. Only dependent on M.
		3	

c)	P(male < 46) = 1−*their* 0.9332 = 0.0668	M1	FT value from part **(a)** or Correct: $1-\Phi\left(\frac{46-55}{6}\right)$,condone continuity correction, σ^2, $\sqrt{\sigma}$, and probability found. Condone unsupported correct value stated.
	P(female < 46) = $P(Z<\frac{46-42}{their\ 8.90})[=\Phi(0.449)]$ $=0.6732$	M1	46, 42 and *their* 4(b) σ (or correct σ) substituted in ±standardisation formula, condone continuity correction, σ^2, $\sqrt{\sigma}$, and probability found Condone $\frac{4}{their\ 8.90}$.
	P(both) = 0.0668 ×0.6732	M1	Product of *their* 2 probabilities (0 < both < 1) Not 0.25 or *their* final answer to **4(a)** used.
	0.0450 or 0.0449	A1	$0.0449 \leqslant p \leqslant 0.0450$
		4	

8) MARCH 2023_9709_52 Q6

(a)	$[P(X<74)=]\ P\left(Z<\frac{74-62.3}{8.4}\right)[=P(Z<1.393)]$	M1	Use of ± standardisation formula with 74, 62.3 and 8.4 substituted appropriately, not 8.4^2, not $\sqrt{8.4}$, no continuity correction.
	= 0.918	A1	$0.918 \leqslant p \leqslant 0.9185$.
		2	
(b)	$[P(50<X<74)=P]\left(\frac{50-62.3}{8.4}<Z<\frac{74-62.3}{8.4}\right)$ $[P(-1.464<Z<1.393)]$	M1	Use of ± standardisation formula with both 74 (may be seen in 6(a) if *their* value seen) & 50, 62.3 and 8.4 substituted appropriately. Condone use of $8.4^2, \sqrt{8.4}$ and continuity correction ±0.5 (73.5 or 74.5 and 49.5 or 50.5).
	$[\Phi(1.464)+\Phi(1.393)-1\ ]$ $0.9285+0.9182-1$	M1	Calculating the appropriate probability area from stated Φ of z-values (leading to *their* final answer > 0.5) but not symmetrical values.
	= 0.847	A1	$0.8465 \leqslant p < 0.8475$. **SC B1** for $0.8465 \leqslant p < 0.8475$ if M0A0 awarded.
	$(0.8467)^4=0.514$	B1 FT	Accept $0.513 \leqslant p \leqslant 0.514$. FT (*their* 4-figure p)^4, $0<p<1$.
		4	
(c)	$z_1=\frac{36-\mu}{\sigma}=-0.739$ $z_2=\frac{54-\mu}{\sigma}=1.282$	B1	$-0.740<z_1<-0.738$ or $0.738<z_1<0.740$.
		B1	$z_2=\pm1.282$ (critical value).
		M1	Use of the ±standardisation formula once with μ, σ and a z-value (not 0.23, 0.77, 0.90, 0.10, ±0.261, ±0.282…). Condone continuity correction ±0.5, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu=42.6, \quad \sigma=8.91$	M1	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	$42.58 \leqslant \mu \leqslant 42.6$, $8.90 \leqslant \sigma \leqslant 8.91$.
		5	

9) JUNE 2020_9709_51 Q6

(a)	$P\left(\frac{50-54}{6.1}<z<\frac{60-54}{6.1}\right)=P(-0.6557<Z<0.9836)$	M1
	Both values correct	A1
	$\Phi(0.9836)-\Phi(-0.6557)=\Phi(0.9836)+\Phi(0.6557)-1$ $=0.8375+0.7441-1$ (Correct area)	M1
	0.582	A1
		4

(b)	$\frac{45-\mu}{\sigma}=-0.994$	B1
	$\frac{56-\mu}{\sigma}=1.372$	B1
	One appropriate standardisation equation with μ, σ, z-value (not probability) and 45 or 56.	M1
	$11=2.366\,\sigma$ (**M1** for correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable)	M1
	$\sigma=4.65,\ \mu=49.6$	A1
		5

10) JUNE 2020_9709_52 Q4

(a)	$P(X<25)=P\left(z<\frac{25-40}{12}\right)=P(z<-1.25)$	M1
	$1-0.8944$	M1
	0.106	A1
		3
(b)	0.8944 divided by 3 (**M1** for 1 – *their* (a) divided by 3)	M1
	0.298 **AG**	A1
		2
(c)	0.2981 gives $z=0.53$	B1
	$\frac{h-40}{12}=0.53$	M1
	$h=46.4$	A1
		3

11) JUNE 2020_9709_52 Q7(c)

(c)	Mean = $100\times0.72=72$ Var = $100\times0.72\times0.28=20.16$	M1
	P(less than 64) = $P\left(z<\frac{63.5-72}{\sqrt{20.16}}\right)$ (**M1** for substituting *their μ and σ* into ±standardisation formula with a numerical value for '*63.5*')	M1
	Using either 63.5 or 64.5 within a ±standardisation formula	M1
	Appropriate area Φ, from standardisation formula P(z<…) in final solution $=P(z<-1.893)$	M1
	0.0292	A1
		5

12) JUNE 2020_9709_53 Q3

(a)	$P(X<21)=P\left(z<\frac{21-15.8}{4.2}\right)=\Phi(1.238)$	M1
	0.892	A1
		2
(b)	$z=\pm0.674$	B1
	$\frac{k-15.8}{4.2}=0.674$	M1
	18.6	A1
		3

13) JUNE 2020_9709_53 Q5(d)

(d)	Mean $= 80 \times 0.25 = 20$ Var $= 80 \times 0.25 \times 0.75 = 15$	M1
	P(more than 25) $= P\left(z > \frac{25.5-20}{\sqrt{15}}\right)$	M1
	$P(z > 1.42)$	M1
	$1 - 0.9222$	M1
	0.0778	A1
		5

14) JUNE 2021_9709_51 Q2

$\left[P\left(\left(\frac{25.2-(25.5+0.50)}{0.4}\right) < z < \left(\frac{25.2-(25.2-0.50)}{0.4}\right)\right)\right]$ $= P\left(-\frac{0.5}{0.4} < z < \frac{0.5}{0.4}\right)$	M1	Use of ± Standardisation formula once; no continuity correction, σ^2, $\sqrt{\sigma}$
$[= 2\Phi(1.25) - 1]$ $= 2 \times 0.8944 - 1$	A1	For AWRT 0.8944 SOI
	M1	Appropriate area $2\Phi - 1$ OE, from final process, must be probability
0.7888	A1	Accept AWRT 0.789
Number of rods $= 0.7888 \times 500$ $= 394$ or 395	B1FT	Correct or FT *their* 4SF (or better) probability, final answer must be positive integer, not 394.0 or 395.0, no approximation/rounding stated, only 1 answer
	5	

15) JUNE 2021_9709_51 Q6(b)(c)

(b)	[Mean =] 0.6×150 [= 90]; [Variance =] $0.6 \times 150 \times 0.4$ [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X < 81) = P\left(Z < \frac{80.5-90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833) = 1 - 0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
(c)	$np = 90$, $nq = 60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

16) JUNE 2021_9709_52 Q2

$\left[P(X > 1.1) = \frac{72}{2000} (= 0.036)\right]$ $z = \pm 1.798$	B1	$1.79 < z \leqslant 1.80$, $-1.80 \leqslant z < -1.79$ seen
$\frac{1.1-1.04}{\sigma} = 1.798$ $\left[\frac{0.06}{\sigma} = 1.798\right]$	B1	1.1 and 1.04 substituted in ±standardisation formula, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
	M1	Equate *their* ±standardisation formula to a z-value and to solve for the appropriate area leading to final answer (expect $\sigma < 0.5$). $\left(\text{Accept} \pm \frac{0.06}{\sigma} = z - \text{value}\right)$
$\sigma = 0.0334$	A1	$0.03335 \leq \sigma \leq 0.0334$. At least 3 3s.f.
	4	

17) JUNE 2021_9709_52 Q5(c)

(c)	Mean $=[60\times0.15=]9$ Variance $=[60\times0.15\times0.85=]7.65$	B1	Correct mean and variance, allow unsimplified. ($2.765 \le \sigma \le 2.77$ imply correct variance)
	$[(X\ge12)=]P\left(Z>\frac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1-\Phi(0.9039)=1-0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

18) JUNE 2021_9709_53 Q5

(a)	$z_1=\frac{4-\mu}{\sigma}=-1.378$	B1	$1.378 \leqslant z_1 \le 1.379$ or $-1.379 \leqslant z_1 \leqslant -1.378$
	$z_2=\frac{10-\mu}{\sigma}=0.842$	B1	$0.841 \leqslant z_2 \leqslant 0.842$ or $-0.842 \leqslant z_2 \leqslant -0.841$
	Solve to find at least one unknown: $\frac{4-\mu}{\sigma}=-1.378$ $\frac{10-\mu}{\sigma}=0.842$	M1	Use of ±standardisation formula once with μ, σ, a z-value and 4 or 10, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve two equations in μ and σ.
	$\sigma=2.70$ $\mu=7.72$	A1	$2.70 \leqslant \sigma \leqslant 2.71$ $7.72 \leqslant \mu \leqslant 7.73$
		5	
(b)	$\Phi(2)-\Phi(-2)=2\Phi(2)-1$	M1	Identifying 2 and −2 as the appropriate z-values
	$2\times$ *their* $0.9772-1$	B1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.9544 or 0.9545	A1	Accept AWRT 0.954
	$0.9544\times800=763.52$ 763 or 764	B1 FT	FT *their* 4SF (or better) probability, final answer must be positive integer
		4	

19) JUNE 2021_9709_53 Q7b(ii)

(b)(ii)	Mean $=120\times0.35[=42]$ Variance $=120\times0.35\times0.65[=27.3]$	B1	Correct mean and variance seen, allow unsimplified
	$P(X>32)=P(Z>\frac{32.5-42}{\sqrt{27.3}})=P(Z>-1.818)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number), condone σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 31.5 or 32.5
	$\Phi(1.818)$	M1	Appropriate area Φ, from final process, must be probability
	0.966	A1	$0.965 \leqslant p \leqslant 0.966$
		5	

20) JUNE 2022_9709_51 Q5

(a)	$P(X < 6) = P(Z < \frac{6-5.2}{1.5}) = P(Z < 0.5333)$	M1	6, 5.2, 1.5 substituted into ± standardisation formula, condone 1.5^2, continuity correction ±0.5
	0.703	A1	
		2	
(b)	$z_1 = \frac{3-\mu}{\sigma} = -1.329$	B1	$1.328 < z_1 \leqslant 1.329$ or $-1.329 \leqslant z_1 < -1.328$
	$z_2 = \frac{8-\mu}{\sigma} = 0.878$	B1	$0.877 < z_2 \leqslant 0.878$ or $-0.878 \leqslant z_2 < -0.877$
	Solve to find at least one unknown: $\frac{3-\mu}{\sigma} = -1.329$, $\frac{8-\mu}{\sigma} = 0.878$	M1	Use of the ± standardisation formula once with μ, σ, a z-value (not 0.8179, 0.7910, 0.5367, 0.5753, 0.19, 0.092 etc.) and 3 or 8, condone continuity correction but not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve their two equations in μ and σ
	$\sigma = 2.27$, $\mu = 6.01$	A1	$2.26 \leqslant \sigma \leqslant 2.27$, $6.01 \leqslant \mu \leqslant 6.02$
		5	
(c)	$[P(Z < -1) + P(Z > 1)]\ \Phi(1) - \Phi(-1) =$ $= 2 - 2\ \Phi(1)$ $= 2 - 2 \times 0.8413$	M1	Identify 1 and −1 as the appropriate z-values.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.3174	A1	Accept AWRT 0.317
	Number of leaves: $2000 \times 0.3174 = 634.8$ so 634 or 635	B1 FT	FT *their* 4 s.f. (or better) probability, final answer must be positive integer no approximation or rounding stated
		4	

21) JUNE 2022_9709_52 Q4

(a)	$[P(1.98 < X < 2.03) =]P(\frac{1.98-2.02}{0.03} < z < \frac{2.03-2.02}{0.03})$ $[= P(-1.333 < z < 0.333)]$	M1	Use of ±standardisation formula once with 2.02, 0.03 and either 1.98 or 2.03 substituted appropriately. Condone 0.03^2 and continuity correction ±0.005, not $\sqrt{0.03}$.
	$[= \Phi(0.333) - (1 - \Phi(1.333))]$ $= 0.6304 + 0.9087 - 1$	M1	Calculating the appropriate probability area from *their* z-values. (or $0.6304 - 0.09121$ or $(0.9087 - 0.5) + (0.6304 - 0.5)$ etc)
	0.539	A1	$0.539 \leqslant z < 0.5395$ Only dependent upon 2nd M mark. If M0 scored SC **B1** for $0.539 \leqslant z < 0.5395$.
		3	
(b)	$[P(X > 2.6) = \frac{134}{5000} = 0.0268]$ $[P(X<2.6) = 1 - 0.0268 =]\ 0.9732$	B1	0.9732 or $\frac{4866}{5000}$ or $\frac{2433}{2500}$ seen.
	$\frac{2.6-2.55}{\sigma} = 1.93$	M1	Use of ±standardisation formula with 2.6 and 2.55 substituted, no $\sigma^2, \sqrt{\sigma}$ or continuity correction.
		M1	*Their* standardisation formula with values substituted equated to z-value which rounds to ±1.93.
	$\sigma = 0.0259$	A1	AWRT 0.0259 or $\frac{5}{193}$. If M0 earned, SC **B1** for correct final answer.
		4	

22) JUNE 2022_9709_52 Q5

5(a)	[P(10, 11, 12) =] $^{12}C_{10}0.72^{10}0.28^2 + {}^{12}C_{11}0.72^{11}0.28^1 + {}^{12}C_{12}0.72^{12}0.28^0$	M1	One term $^{12}C_x p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
	$= 0.193725 + 0.0905726 + 0.0194084$	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
	Alternative method for question 5(a)		
	$[1 - P(0,1,2,3,4,5,6,7,8,9) =]$ $1-({}^{12}C_0 0.72^0 0.28^{12} + {}^{12}C_1 0.72^1 0.28^{11} + {}^{12}C_2 0.72^2 0.28^{10} +$ ${}^{12}C_3 0.72^3 0.28^9 + {}^{12}C_4 0.72^4 0.28^8 + {}^{12}C_5 0.72^5 0.28^7 +$ ${}^{12}C_6 0.72^6 0.28^6 + {}^{12}C_7 0.72^7 0.28^5 + {}^{12}C_8 0.72^8 0.28^4 +$ ${}^{12}C_9 0.72^9 0.28^3)$	M1	One term $^{12}C_x p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leqslant 0.304$.
		3	
5(b)	Mean $= [0.52 \times 90 =]46.8$, var $=[0.52 \times 0.48 \times 90] = 22.464$	B1	46.8 and 22.464 or 22.46 seen, allow unsimplified, ($4.739 < \sigma \leqslant 4.740$ imply correct variance).
	$[P(X < 40) =]\ P\left(z < \frac{39.5 - 46.8}{\sqrt{22.464}}\right)$	M1	Substituting *their* mean and *their* variance into ±standardisation formula (any number for 39.5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	$= [P(Z < -1.540)] = 1 - 0.9382$	M1	Appropriate area Φ, from final process, must be probability.
	0.0618	A1	$0.06175 \leqslant p \leqslant 0.0618$
		5	

23) JUNE 2022_9709_53 Q5

(a)	$[P(142 < X < 205)] = P\left(\frac{142-170}{25} < z < \frac{205-170}{25}\right)$	M1	Use of ± standardisation formula once substituting 170, 25 and either 142 or 205 appropriately.. Condone 25^2 and continuity correction ±0.5.
	$P(-1.12 < z < 1.4)$	A1	Both correct. Accept unsimplified.
	$\Phi(1.4) - (1 - \Phi(1.12)) = 0.9192 + 0.8686 - 1$	M1	Calculating the appropriate area from stated phis of z-values.
	0.788	A1	AWRT, not from wrong working
		4	
(b)	$P(X > 205) = 1 - 0.9192 = 0.0808$	B1 FT	Correct or FT from part 5(a).
	$(0.0808 \times 0.30 + \textit{their}\ 0.788 \times 0.24) \times 20000$	M1	Correct or *their* $0.0808 \times 0.30 \times k +$ *their* $0.788 \times 0.24 \times k$, k positive integer.
	[$]4266.24	A1	$4265 <$ income $\leqslant 4270$, not from wrong working
		3	
(c)	$[P(Z > \frac{w-182}{20}) = 0.72]$ $\frac{w-182}{20} = -0.583$	B1	$0.5828 \leqslant z \leqslant 0.583$ or $-0.583 \leqslant z \leqslant -0.5828$ seen.
		M1	182 and 20 substituted in ± standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$w = 170$	A1	$170 \leqslant w < 170.35$
		3	

24) JUNE 2023_9709_51 Q4

(a)	$P(Z > \frac{20-14.6}{5.2}) = P(Z > 1.03846)$	M1	Use of ± standardisation formula with 20, 14.6 and 5.2 not σ^2, not $\sqrt{\sigma}$, no continuity correction.
	$1 - 0.8504$	M1	Calculating the appropriate probability area (leading to their final answer).
	0.150	A1	0.1496, $0.149 < p \leqslant 0.15[0]$. Only dependent on the 2nd M mark so M0M1A1 possible. SC **B1** for $0.149 < p \leqslant 0.15[0]$ if M0M0A0 awarded.
	[250 × *their* 0.1496 =] 37, 38	B1 FT	Strict FT *their* at least 4-figure probability seen anywhere (give BOD if they go on to use 0.150). Final answer must be positive integer, no approximation or rounding stated.
		4	
(b)	$z_1 = \frac{14.5-\mu}{\sigma} = -0.842$ $z_2 = \frac{18.5-\mu}{\sigma} = -0.44$	B1	$-0.843 < z_1 < -0.841$ or $0.841 < z_1 < 0.843$.
		B1	$-0.441 < z_2 < -0.439$ or $0.439 < z_2 < 0.441$.
		M1	Use of the ±standardisation formula once with μ, σ and a z-value (not 0.20, 0.80, 0.67, 0.23, 0.5793, 0.7881, 0.7486, 0.591 or $1-z$ i.e. 0.158 etc.). Condone continuity correction ±0.05, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ. $\mu = 22.9$, $\sigma = 9.95$	M1	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	AWRT 22.9, 9.95.
		5	

25) JUNE 2023_9709_52 Q5

(a)	$[P(15.4 < X < 16.8) =] P(\frac{15.4-16.5}{0.6} < Z < \frac{16.8-16.5}{0.6})$ $[= P(-1.833 < Z < 0.5)]$	M1	Use of ± standardisation formula once with 16.5, 0.6 and either 15.4 or 16.8 substituted.
	$[= \Phi(0.5) + \Phi(1.833) - 1 =]$ $0.6915 + 0.9666 - 1$	M1	Calculating the appropriate probability area (leading to their final answer, expect > 0.5). $0.6915 - (1 - 0.9666)$ or $(0.6915 - 0.5) + (0.9666 - 0.5)$ OE are alternatives.
	= 0.658	A1	$0.658 \le p < 0.6585$. If A0 scored, SC **B1** for $0.658 \leqslant p < 0.6585$.
	[Expected number =] 0.6581 × 150 = 98, 99	B1 FT	FT *their* 4SF (or better) probability from a normal calculation. Must be a positive single integer answer. No approximation notation.
		4	
(b)	$\left[P\left(Z > \frac{17.1-18.4}{\sigma}\right) = 0.72\right]$ $\frac{17.1-18.4}{\sigma} = -0.583$	B1	$0.5825 < z \leqslant 0.583$ or $-0.583 \leqslant z < -0.5825$ seen.
		M1	Use of the ± standardisation formula with 17.1, 18.4, σ and a z-value (not 0.28, 0.72, 0.4175, 0.2358, 0.7642, 0.6103, 0.3897, …). Condone continuity correct ± 0.05, not $\sigma^2, \sqrt{\sigma}$.
	$\sigma = 2.23$	A1	AWRT
		3	

c)	[Mean = 120 × 0.72 =] 86.4 [Var = 120 × 0.72 × 0.28 =] 24.192	B1	86.4, $84\frac{2}{5}$ and $24\frac{24}{125}$, 24.192 to at least 3SF seen, allow unsimplified. May be seen in standardisation formula. ($4.918 \leqslant \sigma \leqslant 4.919$ implies correct variance) Incorrect notation is penalised.
	$P(X < 80) = P(Z < \frac{79.5 - 86.4}{\sqrt{24.192}})$	M1	Substituting *their* mean (not 18.4) and *their positive* 4.9185 into ± standardisation formula (any number for 79.5), condone *their* 4.918^2 and $\sqrt{their\ 4.918}$.
		M1	Using continuity correction 79.5 or 80.5 in *their* standardisation formula.
	$[P(Z < -1.4029) = 1 - \Phi(1.403)\ ]$ $1 - 0.9196$	M1	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5 . Note: correct final answer implies this M1.
	0.0804	A1	$0.0803 \leqslant p \leqslant 0.0804$
		5	

26) JUNE 2023_9709_53 Q2

Mean $= 120 \times 0.4 = 48$ Var $= 120 \times 0.4 \times 0.6 = 28.8$	B1	48 and $28\frac{4}{5}$, 28.8 seen, allow unsimplified. ($5.366 \leqslant \sigma \leqslant 5.367$ or $\frac{12\sqrt{5}}{5}$ implies correct variance).
$P(36 \leqslant X \leqslant 54) = P(\frac{35.5 - 48}{\sqrt{28.8}} < Z < \frac{54.5 - 48}{\sqrt{28.8}})$	M1	Substituting *their* μ and σ into one ±standardisation formula (any number for 35.5 or 54.5), condone σ^2 and $\sqrt{\sigma}$.
	M1	Using continuity correction 35.5, 36.5 or 53.5, 54.5 once in *their* standardisation formula. Note: $\frac{\pm 12.5}{\sqrt{28.8}}$ or $\frac{\pm 6.5}{\sqrt{28.8}}$ seen gains M2 BOD.
$[= P(-2.3292 < Z < 1.211) =]\ 0.8871 + 0.9900 - 1$	M1	Appropriate area Φ, from final process. Must be a probability. Expect final answer > 0.5 . Note: correct final answer implies this M1.
$= 0.877$	A1	$0.877 \leq p < 0.8772$.
	5	

27) JUNE 2023_9709_53 Q6

a)	$\left[P(X < 16) = P\left(Z < \frac{16 - 28}{\sigma}\right) = 0.1\right]$ $\frac{16 - 28}{\sigma} = -1.282$	B1	±1.282 seen, cao – critical value.
		M1	Use of the ±standardisation formula with 16, 28, σ and a z-value (not 0.1, 0.9, 0.282, 0.5398, 0.8159) equated to a z-value. Condone continuity correct ±0.5, not $\sigma^2, \sqrt{\sigma}$. Condone $\pm\frac{12}{\sigma} = -1.282$.
	$\sigma = 9.36$	A1	
		3	
b)	$[1 - P(0, 1, 2) =]\ 1 - ({}^{12}C_0(0.1)^0(0.9)^{12} + {}^{12}C_1(0.1)^1(0.9)^{11} + {}^{12}C_2(0.1)^2(0.9)^{10})$ $[1 - (0.2824 + 0.3766 + 0.2301)]$	M1	One term ${}^{12}C_x\ (p)^x(1-p)^{12-x}$, $0 < p < 1$. $x \neq 0,1,2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	B1	0.1108699… rounded to at least 3SF.
	Alternative Method for Question 6(b)		
	$P(3,4,5,6,7,8,9,10,11,12) = {}^{12}C_3(0.1)^3(0.9)^9 + {}^{12}C_4(0.1)^4(0.9)^8 + \ldots + {}^{12}C_{11}(0.1)^{11}(0.9)^1 + {}^{12}C_{12}(0.1)^{12}(0.9)^0$ $[0.08523 + 0.02131 + \ldots + 1.08\times10^{-10} + 1\times10^{-12}]$	M1	One term ${}^{12}C_x\ (p)^x(1-p)^{12-x}$, $0 < p < 1$. $x \neq 0,1,2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	B1	0.1108699… rounded to at least 3SF.
		3	

(c)	[$P(-1.3 < Z < 1.3)$ $= 2\Phi(1.3) - 1$] $= 2 \times 0.9032 - 1$	B1	Identifying at least one of -1.3 or 1.3 as the appropriate z-values.
		M1	Calculating the appropriate probability area from 2 symmetrical z-values (leading to their final answer, expect > 0.5).
	$= 0.806, \frac{504}{625}$	A1	0.8064, $0.806 \leqslant p < 0.8065$.
	[In 365 days 0.8064×365] $= 294$ or 295	B1 FT	Strict FT *their* at least 4-figure probability (not z-value). Final answer must be positive integer, no approximation or rounding stated.
		4	

28) OCT 2020_9709_51 Q5

a)	$P(X > 4.2) = P(z > \frac{4.2 - 3.5}{0.9})$ $= P(z > 0.7778)$	M1	Using $\pm$ standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	$1 - 0{\cdot}7818$	M1	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	$0{\cdot}218$	A1	
		3	
b)	$z = -1.282$	B1	± 1.282 seen (critical value)
	$\frac{t - 3.5}{0.9} = -1.282$	M1	An equation using $\pm$standardisation formula with a z-value, condone $\sqrt{\sigma}$, σ^2 and continuity correction
	$t = 2.35$	A1	AWRT, only dependent on M mark
		3	
(c)	$P(2.8 < X < 4.2) = 1 - 2 \times$ *their* 5(a) $\equiv 2(1 -$ *their* 5(a)$) - 1$ $\equiv 2(0{\cdot}5 -$ *their* 5(a)$)$ $= 0{\cdot}5636$	B1 FT	FT from *their* 5(a) < 0.5 or correct Accept unevaluated probability OE Accept 0·564
	Number of days $= 365 \times 0{\cdot}5636 = 205{\cdot}7$	M1	$365 \times$ *their* p
	So, 205 (days)	A1 FT	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
	Alternative method for question 5(c)		
	$P\left(\frac{2.8 - 3.5}{0.9} < z < \frac{4.2 - 3.5}{0.9}\right)$ $= \Phi(0.7778) - (1 - \Phi 0.7778)$ $= 0{\cdot}7818 - (1 - 0{\cdot}7818)$ $= 0{\cdot}5636$	B1	$0{\cdot}5635 < p \leqslant 0{\cdot}564$ OE
	Number of days $= 365 \times 0{\cdot}5636 = 205{\cdot}7$	M1	$365 \times$ *their* p
	So, 205 (days)	A1 FT	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
		3	

29) OCT 2020_9709_52 Q3

a)	$P(X > 11.3) = P(z > \frac{11.3-10.1}{1.3}) = P(z > 0\cdot9231)$	M1	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	$1 - 0.822$	M1	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	$0\cdot178$	A1	0.1779…
		3	
b)	$z = -0\cdot674$	B1	±0.674 seen (critical value)
	$\frac{t-10.1}{1.3} = -0\cdot674$	M1	An equation using ±standardisation formula with a z-value, condone $\sqrt{\sigma}$ or σ^2, continuity correction.
	$t = 9\cdot22$	A1	AWRT. Only dependent on M1
		3	
(c)	$P(8.9 < X < 11.3) = 1 - 2 \times$ *their* **3(a)** $\equiv 2(1 -$ *their* **3(a)**$) - 1$ $\equiv 2(0\cdot5 -$ *their* **3(a)**$)$ $=0.644$	B1 FT	FT from *their* **3(a)** < 0·5 or correct, accept unevaluated probability OE
	Number of days $= 90 \times 0\cdot644$ $= 57\cdot96$	M1	$90 \times$ *their p* seen, $0 < p < 1$
	So 57 (days)	A1 FT	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
	Alternative method for question 3(c)		
	$P\left(\frac{8\cdot9-10\cdot1}{1\cdot3} < z < \frac{11.3-10.1}{1\cdot3}\right)$ $= \Phi(0\cdot9231) - (1 - \Phi(0\cdot9231))$ oe $= 0\cdot822 - (1 - 0\cdot822)$ $= 0\cdot644$	B1	Accept unevaluated probability
	Number of days $= 90 \times 0\cdot644$ $= 57\cdot96$	M1	$90 \times$ *their p* seen, $0 < p < 1$
	So 57 (days)	A1 FT	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
		3	

30) OCT 2020_9709_53 Q1

a)	$P(56 < X < 66) = P\left(\frac{56-62}{5} < z < \frac{66-62}{5}\right)$ $= P(-1.2 < z < 0.8)$	M1	Using ± standardisation formula at least once, no $\sqrt{\sigma}$ or σ^2, allow continuity correction
	$\Phi(0.8) + \Phi(1.2) - 1$ $= 0.7881 + 0.8849 - 1$	M1	Appropriate area Φ, from standardisation formula in final solution
	0.673	A1	
		3	
b)	$z = 1.127$	B1	±(1.126 – 1.127) seen, 4 sf or more
	$\frac{60t-62}{5} = 1.127$ $60t = 5.635+62=67.635$	M1	z-value $= \pm\frac{(60t-62)}{5}$ condone z-value $= \pm\frac{(t-62)}{5}$ no continuity correction, condone $\sqrt{\sigma}$ or σ^2
	$t = 1.13$	A1	CAO
		3	

31) OCT 2020_9709_53 Q4

(a)	$0.65^7 + {}^7C_1\ 0.65^6\ 0.35^1 + {}^7C_2\ 0.65^5\ 0.35^2$	M1	Binomial term of form ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1$, any p, $x \neq 0, 7$
	$0.049022 + 0.184776 + 0.29848$	A1	Correct unsimplified answer
	0.532	A1	
		3	
(b)	Mean $= 142\times0.35 = 49.7$ Variance $= 142\times0.35\times0.65 = 32.305$	B1	Correct unsimplified np and npq (condone $\sigma = 5.684$ evaluated)
	$P(X > 40) = P(z > \frac{40.5 - 49.7}{\sqrt{32.305}})$	M1	Substituting *their* μ *and* σ (no $\sqrt{\sigma}$ or σ^2) into ±standardisation formula with a numerical value for '40.5'
	$P(z > -1.619)$	M1	Using either 40.5 or 39.5 within a ±standardisation formula
		M1	Appropriate area Φ, from standardisation formula $P(z > \ldots)$ in final solution, must be probability
	0.947	A1	Correct final answer
		5	

32) OCT 2021_9709_51 Q7

(a)(i)	$P(X > 142) = P\left(Z > \frac{142-125}{24}\right)$	M1	Substitution of correct values into the ±Standardisation formula, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$[= P(Z > 0.7083) =]1 - 0.7604$	M1	Appropriate numerical area Φ, from final process, must be probability, expect $p < 0.5$.
	0.2396	A1	$0.239 \leqslant p \leqslant 0.240$ to at least 3sf.
	Their 0.2396×365 [= 87.454]	M1	FT *their* 4sf (or better) probability.
	87 or 88	A1 FT	Final answer must be positive integer, no indication of approximation/rounding, only dependent on previous **M** mark. **SC B1 FT** for *their* 3sf probability $\times$ 365 = integer value, condone 0.24 used.
		5	
(a)(ii)	$P(0, 1) = 0.7604^{10} + {}^{10}C_1 \times 0.2396^1 \times 0.7604^9$ [= 0.064628 + 0.20364]	M1	One term: ${}^{10}C_x p^x(1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1 FT	Correct unsimplified expression using *their* probability to at least 3sf from **(a)(i)** or correct.
	0.268	A1	AWRT, WWW.
		3	
7(b)	$z = \pm1.282$	B1	Correct value only, critical value.
	$\frac{t-125}{24} = -1.282$	M1	Use of ± Standardisation formula with correct values substituted, allow continuity correction, σ^2, $\sqrt{\sigma}$, to form an equation with a z-value and not probability.
	$t = 94.2$	A1	AWRT, condone AWRT 94.3. Not dependent on **B** mark.
		3	

33) OCT 2021_9709_52 Q6

(a)	$\left[P(X > 28.6) =\right] P\left(Z > \frac{28.6-32.2}{9.6}\right)$ $[= P(Z > -0.375)]$	M1	28.6, 32.2 and 9.6 substituted appropriately in ± Standardisation formula once, allow continuity correction of ± 0.05, no σ^2, $\sqrt{\sigma}$.
	$[\Phi(\textit{their}\ 0.375) =]\ \textit{their}\ 0.6462$	M1	Appropriate numerical area, from final process, must be probability, expect > 0.5.
	0.646	A1	AWRT
		3	
(b)	$z = \pm 0.842$	B1	$0.841 < z \leqslant 0.842$ or $-0.842 \leqslant z < -0.841$ seen.
	$\frac{t-32.2}{9.6} = 0.842$	M1	Substituting 32.2 and 9.6 into ± standardisation formula, no continuity correction, allow σ^2, $\sqrt{\sigma}$, must be equated to a z-value.
	$t = 40.3$	A1	$40.28 \leqslant t \leqslant 40.3$ WWW
		3	
(c)	$P\left(-\frac{15}{9.6} < Z < \frac{15}{9.6}\right)$ $P(-1.5625 < Z < 1.5625)$	M1	Identifying at least one of $\frac{15}{9.6}$ and $-\frac{15}{9.6}$ as the appropriate z-values or substituting *their* (32.2 ± 15) into ± Standardisation formula once, no continuity correction, σ^2 nor $\sqrt{\sigma}$. Condone ±1.563 for **M1**.
	$[2\ \Phi(\frac{15}{9.6}) - 1]$ $= 2 \times 0.9409 - 1$	A1	$p = 0.941$ AWRT SOI
		M1	Appropriate area $2\Phi - 1$ oe, (eg $1 - 2 \times 0.0591$, $2 \times (0.9409 - 0.5)$ or $0.9409 - 0.0591$), from final process, must be probability > 0.5.
	0.882	A1	
		4	

34) OCT 2021_9709_53 Q4

(a)	$P(X > 43.2) = P\left(Z > \frac{43.2-41.2}{3.6}\right) = P(Z > 0.5556)$	M1	Use of ±Standardisation formula once, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$1 - \Phi(0.5556) = 1 - 0.7108$	M1	Appropriate area Φ, from final process, must be probability.
	0.289	A1	AWRT
		3	
(b)	Probability = 1 – *their* **(a)** = 1 – 0.2892 = 0.7108	B1FT	1 – *their* **(a)** or correct.
	$0.7108 \times 365 = 259.4$ 259, 260	B1FT	FT *their* 4SF (or better) probability, final answer must be positive integer.
		2	
(c)	$z = \pm 1.645$	B1	CAO, critical z value.
	$\frac{t-41.2}{3.6} = -1.645$	M1	Use of ±standardisation formula with μ, σ equated to a z-value, no continuity correction, allow σ^2, $\sqrt{\sigma}$.
	$t = 35.3$	A1	
		3	

35) OCT 2022_9709_51 Q2

(a)	$[P(3, 4, \ldots 7) = 1 - P(0, 1, 2, 8)]$ $= 1 - ({}^8C_0\ 0.48^0\ 0.52^8 + {}^8C_1\ 0.48^1\ 0.52^7$ $+ {}^8C_2\ 0.48^2\ 0.52^6 + {}^8C_8\ 0.48^8\ 0.52^0)$	M1	One term ${}^8C_x\ p^x(1-p)^{8-x}$, for $0 < x < 8$, $0 < p < 1$
	$= 1 - (0.00534597 + 0.039478 + 0.127544 + 0.0028179)$	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.825	B1	Mark the final answer at the most accurate value. $0.8248 < p \leqslant 0.825$ WWW.
	Alternative method for Question 2(a)		
	$[P(3, 4, 5, 6, 7) =]$ ${}^8C_3\ 0.48^3\ 0.52^5 + {}^8C_4\ 0.48^4\ 0.52^4 + {}^8C_5\ 0.48^5\ 0.52^3 + {}^8C_6$ $0.48^6\ 0.52^2 + {}^8C_7\ 0.48^7\ 0.52^1$	M1	One term ${}^8C_x\ p^x(1-p)^{8-x}$, for $0<x<8$, $0<p<1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.825	B1	Final answer $0.8248 < p \leqslant 0.825$ WWW.
		3	
(b)	$[\text{Mean} = 0.52 \times 125 =]65$, $[\text{var} = 0.52 \times 0.48 \times 125 =]31.2$	B1	65 and 31.2 seen, allow unsimplified. May be seen in standardisation formula. ($5.585 < \sigma \leqslant 5.586$ imply correct variance).
	$[P(X > 72) =]P(Z > \frac{72.5 - 65}{\sqrt{31.2}})$ $[= P(Z > 1.343)]$	M1	Substituting *their* 65 and $\sqrt{\textit{their}\ 31.2}$ into ±standardisation formula (any number for 72·5), not *their* 31.2, √ *their* 5.586 .
		M1	Using continuity correction 72·5 or 71·5 in *their* standardisation formula . Note $\frac{\pm 7.5}{\sqrt{31.2}}$ or $\frac{\pm 7.5}{5.586}$ seen gains **M2** BOD
	$= 1 - 0.9104$	M1	Appropriate area Φ, from final process, must be probability.
	0.0896	A1	$0.0896 \leqslant p \leqslant 0.0897$ WWW.
		5	

36) OCT 2022_9709_51 Q4

(a)	$P(X<132) = P(Z < \frac{132 - 125.4}{18.6}) = P(Z < 0.3548)$	M1	Use of ±standardisation formula with 132 and 125.4 substituted, condone continuity correction 132±0.5 and use of 18.6^2, $\sqrt{18.6}$
	0.639	A1	$0.6385 < p \leqslant 0.639$ If M0 scored, **SC B1** for $0.6385 < p \leqslant 0.639$
		2	
(b)	$\frac{108 - 117}{\sigma} = -1.175$	B1	$1.1749 < z \leqslant 1.175$ or $-1.175 \leqslant z < -1.1749$
		M1	108 and 117 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$\sigma = 7.66$	A1	$7.659 \leqslant \sigma \leqslant 7.66$ If M0 scored, **SC B1** for $7.659 \leqslant \sigma \leqslant 7.66$
		3	
(c)	$P(-1.5 < Z < 1.5)$ $[\Phi(1.5) - \Phi(-1.5)]$ $[= 2\Phi(1.5) - 1]$ $= 2 \times \textit{their}\ 0.9332 - 1$ **or** $\textit{their}\ 0.9332 - (1 - \textit{their}\ 0.9332)$ **or** $2 \times (\textit{their}\ 0.9332 - 0.5)$	M1	{Both 1.5 and −1.5 seen as z-values **or** appropriate use of 1.5 or −1.5} **and** {no other z-values in part}.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number. Condone *their* 0.0668 as (1 − *their* 0.9332).
	0.8664	A1	Accept answers wrt 0.866 If A0 scored **SC B1** for answers wrt 0.866
	$0.8664^3 = 0.650[36\ldots]$	B1 FT	FT *their* 4SF (or better) probability, accept final answers to 3SF.
		4	

37) OCT 2022_9709_52 Q2

(a)	[P(X<54.8)] = P($Z < \frac{54.8-55.6}{1.2}$)	M1	Use of ± standardisation formula, with 54.8, 55.6 and 1.2 substituted. condone $1.2^2, \sqrt{1.2}$ or continuity correction of 54.75 or 54.85
	[= P($Z < -0.6667$)] = 1 − 0.7477	M1	Appropriate area Φ, from final process, must be probability.
	= 0.2523	A1	$0.252 \leqslant p \leqslant 0.2525$ If A0 scored S CB1 for $0.252 \leqslant p \leqslant 0.2525$
	[Expected number =] 400 × 0.2523 = 100.92 100 or 101	B1 FT	FT *their* 4SF (or better) probability from a normal calculation. Must be a single integer answer.
		4	
(b)	[P($-\frac{1}{2} < Z < \frac{1}{2}$) = Φ(½) − Φ(−½) =] $2\Phi\left(\frac{1}{2}\right)-1$ = 2 × *their* 0.6915 − 1 or *their* 0.6915 − (1 − *their* 0.6915) or 2 × (0.6915 − 0.5)	M1	{Both ½ and −½ seen as z-values or appropriate use of +½ or −½} and {no other z-values in part}. Condone $\frac{56.2-55.6}{1.2}$ and $\frac{55[.0]-55.6}{1.2}$ seen as z-values.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number.
	0.383	A1	$0.3829 \leqslant z \leqslant 0.383$ If A0 scored SC B1 for $0.3829 \leqslant z \leqslant 0.383$
		3	

38) OCT 2022_9709_52 Q6

(a)	[1 − P(10, 11, 12) =] $1-({}^{12}C_{10}\ 0.9^{10}\ 0.1^{2} + {}^{12}C_{11}\ 0.9^{11}\ 0.1^{1} + {}^{12}C_{12}\ 0.9^{12}\ 0.1^{0})$ = 1 − (0.230128 + 0.376573 + 0.282430)	M1	One term ${}^{12}C_x\ p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.111	B1	Mark the final answer at the most accurate value, $0.1108 < p \leqslant 0.111$ WWW.
	Alternative method for Question 6(a)		
	[P(0,1,2,3,4,5,6,7,8,9) =] ${}^{12}C_0\ 0.9^0\ 0.1^{12} + {}^{12}C_1\ 0.9^1\ 0.1^{11} + {}^{12}C_2\ 0.9^2\ 0.1^{10} + {}^{12}C_3\ 0.9^3\ 0.1^9 + {}^{12}C_4\ 0.9^4\ 0.1^8 + {}^{12}C_5\ 0.9^5\ 0.1^7 + {}^{12}C_6\ 0.9^6\ 0.1^6 + {}^{12}C_7\ 0.9^7\ 0.1^5 + {}^{12}C_8\ 0.9^8\ 0.1^4 + {}^{12}C_9\ 0.9^9\ 0.1^3$)	M1	One term ${}^{12}C_x\ p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. If answer correct condone omission of any 7 of the 8 middle terms.
	0.111	B1	Final answer $0.1108 < p \leqslant 0.111$ WWW.
		3	
(b)	[Mean = 80 × 0.9 =] 72, [Variance = 80 × 0.9 × 0.1] = 7.2	B1	72 and 7.2 seen, allow unsimplified. May be seen in standardisation formula. ($2.683 \leqslant \sigma < 2.684$ imply correct variance).
	P(X > 69) = P($Z > \frac{69.5-72}{\sqrt{7.2}}$)	M1	Substituting *their* mean and $\sqrt{\text{their variance}}$ into ±standardisation formula (any number for 69·5), not *their* 7.2, not √*their* 2.683
		M1	Using continuity correction 69·5 or 68·5 in *their* standardisation formula.
	[= P($Z > -0.9317$) =] Φ(0.9317)	M1	Appropriate area Φ, from final process, must be probability.
	0.824	A1	$0.8239 \leqslant p \leqslant 0.8243$ WWW.
		5	
(c)	$np = 72$, $nq = 8$ Both greater than 5, [so approximation is valid]	B1	*np*, *nq* evaluated accurately. both *np* & *nq* referenced correctly. > 5 or greater than 5 seen.
		1	

39) OCT 2022_9709_53 Q2

Mean = $80 \times 0.32 = 25.6$, var $= 80 \times 0.32 \times 0.68 = 17.408$	B1	25.6 and 17.4[08] seen, allow unsimplified. 4.172… implies correct variance.
$P(X < 20) = P(Z < \frac{19.5 - 25.6}{\sqrt{17.408}}) = P(Z < -1.462)$	M1	Substituting *their* 25.6 and 17.408 into ±standardisation formula (any number for 19·5), not σ^2, $\sqrt{\sigma}$.
	M1	Using continuity correction 19·5 or 20·5 in *their* standardisation formula.
$= [1 - \Phi(1.462)] = 1 - 0.9282$	M1	Appropriate area Φ, from final process, must be probability. (Expect final ans < 0·5). Note: the correct final answer may imply M1 from use of calculator.
0.0718	A1	$0.0718 \leqslant p \leqslant 0.0719$
	5	

40) OCT 2022_9709_53 Q5

(a)	$[P(0, 1, 2) =]\ {}^{10}C_0\ 0.1^0 0.9^{10} + {}^{10}C_1\ 0.1^1 0.9^9 + {}^{10}C_2\ 0.1^2 0.9^8$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1, x \neq 0$
	$= 0.348678 + 0.38742 + 0.19371$	A1	Correct expression, accept unsimplified.
	0.930	B1	$0.9298 \leqslant p \leqslant 0.9303$
	Alternative method for Question 5(a)		
	$[1 - P(3, 4, 5, 6, 7, 8, 9, 10) = 1 - ({}^{10}C_3\ 0.9^7 0.1^3 + {}^{10}C_4\ 0.9^6 0.1^4 + {}^{10}C_5\ 0.9^5 0.1^5 + {}^{10}C_6\ 0.9^4 0.1^6 + {}^{10}C_7 0.9^3 0.1^7 + {}^{10}C_8\ 0.9^2 0.1^8 + {}^{10}C_9\ 0.9^1 0.1^9 + {}^{10}C_{10}\ 0.9^0 0.1^{10})$	M1	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1, x \neq 0$
		A1	Correct expression, accept unsimplified.
	0.930	B1	$0.9298 \leqslant p \leqslant 0.9303$
		3	
(b)	$[P(X > 1.11) =]P(Z > \frac{1.11 - 1.04}{0.06}) = P(Z > 1.167)$	M1	1.11, 1.04 and 0.06 substituted into ±Standardisation formula, no continuity correction not 0.06^2 or $\sqrt{0.06}$
	$= 1 - 0.8784$	M1	1 – *their* 0.8784 as final answer, must be probability. (Expect final ans < 0·5).
	0.122	A1	$0.1216 \leqslant p \leqslant 0.122$ **SC M0 M1 B1** for 0.122 with no standardisation formula.
		3	
(c)	$[P(X < w) = P(Z < \frac{w - 1.04}{0.06}) = 0.81]$	B1	$0.8775 < z \leqslant 0.878$ or $-0.878 \leqslant z < -0.8775$ seen.
	$\frac{w - 1.04}{0.06} = 0.878$	M1	1.04 and 0.06 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$w = 1.09$	A1	$1.09 \leqslant w \leqslant 1.093$
		3	