PROBABILITY AND STATISTICS-2

9709

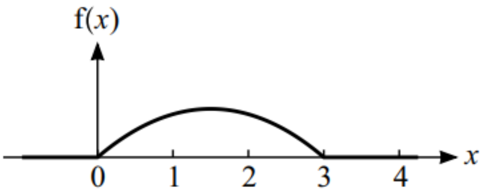
(March, June and November series 2020 – 2023 With marking scheme

CONTINUOUS RANDOM VARIABLE

EXERCISE -2

Manjula Balaji

1) SP 2020 9709_6 Q5



The diagram shows the graph of the probability density function, f, of a random variable X, where

$$f(x) = \begin{cases} \frac{2}{9}(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) State the value of $E(X)$ and find $Var(X)$. [4]

(b) State the value of $P(1.5 \leqslant X \leqslant 4)$. [1]

(c) Given that $P(1 \leqslant X \leqslant 2) = \frac{13}{27}$, find $P(X > 2)$. [2]

2) March 2020 9709_62 Q5

Bottles of Lanta contain approximately 300 ml of juice. The volume of juice, in millilitres, in a bottle is $300 + X$, where X is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{3}{4000}(100 - x^2) & -10 \leqslant x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

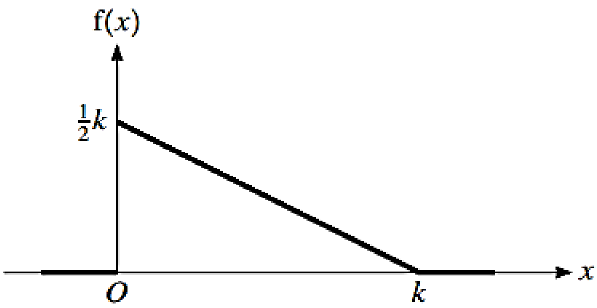
(a) Find the probability that a randomly chosen bottle of Lanta contains more than 305 ml of juice. [3]

(b) Given that 25% of bottles of Lanta contain more than $(300 + p)$ ml of juice, show that

$$p^3 - 300p + 1000 = 0.$$ [4]

(c) Given that $p = 3.47$, and that 50% of bottles of Lanta contain between $(300 - q)$ and $(300 + q)$ ml of juice, find q. Justify your answer. [2]

3) March 2021 9709_62 Q2



The diagram shows the graph of the probability density function, f, of a random variable X.

(a) Find the value of the constant k. [2]

(b) Using this value of k, find $f(x)$ for $0 \leqslant x \leqslant k$ and hence find $E(X)$. [3]

(c) Find the value of p such that $P(p < X < 1) = 0.25$. [4]

4) March 2022 9709_62 Q6

In a game a ball is rolled down a slope and along a track until it stops. The distance, in metres, travelled by the ball is modelled by the random variable X with probability density function

$$f(x) = \begin{cases} -k(x-1)(x-3) & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Without calculation, explain why $E(X) = 2$. [1]

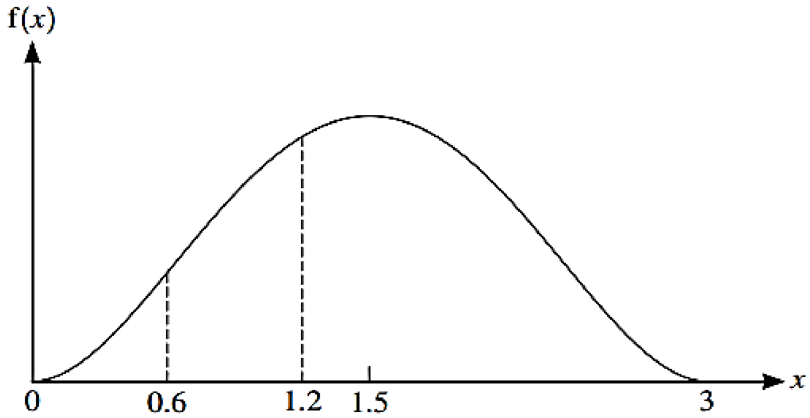
(b) Show that $k = \frac{3}{4}$. [3]

(c) Find $\text{Var}(X)$. [3]

One turn consists of rolling the ball 3 times and noting the largest value of X obtained. If this largest value is greater than 2.5, the player scores a point.

(d) Find the probability that on a particular turn the player scores a point. [4]

5) March 2023 9709_62 Q3



The diagram shows the graph of the probability density function, f, of a random variable X that takes values between $x = 0$ and $x = 3$ only. The graph is symmetrical about the line $x = 1.5$.

(a) It is given that $P(X < 0.6) = a$ and $P(0.6 < X < 1.2) = b$.

Find $P(0.6 < X < 1.8)$ in terms of a and b. [2]

(b) It is now given that the equation of the probability density function of X is

$$f(x) = \begin{cases} kx^2(3-x)^2 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{10}{81}$. [3]

(ii) Find Var(X). [3]

6) June 2020 9709_61 Q6

The length of time, T minutes, that a passenger has to wait for a bus at a certain bus stop is modelled by the probability density function given by

$$f(t) = \begin{cases} \frac{3}{4000}(20t - t^2) & 0 \leqslant t \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [1]

(b) Hence explain, without calculation, why $E(T) = 10$. [1]

(c) Find Var(T). [3]

(d) It is given that $P(T < 10 + a) = p$, where $0 < a < 10$.

Find $P(10 - a < T < 10 + a)$ in terms of p. [2]

(e) Find $P(8 < T < 12)$. [3]

(f) Give one reason why this model may be unrealistic. [1]

7) June 2020 9709_62 Q6

A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(a) Show that $k = \dfrac{a}{a-1}$. [3]

(b) Find $E(X)$ in terms of a. [3]

(c) Find the 60th percentile of X in terms of a. [4]

8) June 2020 9709_63 Q6

The length, X centimetres, of worms of a certain type is modelled by the probability density function

$$f(x) = \begin{cases} \frac{6}{125}(10 - x)(x - 5) & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(a) State the value of $E(X)$. [1]

(b) Find Var(X). [3]

(c) Two worms of this type are chosen at random.

Find the probability that exactly one of them has length less than 6 cm. [5]

9) June 2021 9709_61 Q3

The graph of the probability density function of a random variable X is symmetrical about the line $x = 4$.

Given that $P(X < 5) = \frac{20}{27}$, find $P(3 < X < 5)$. [2]

10) June 2021 9709_61 Q6

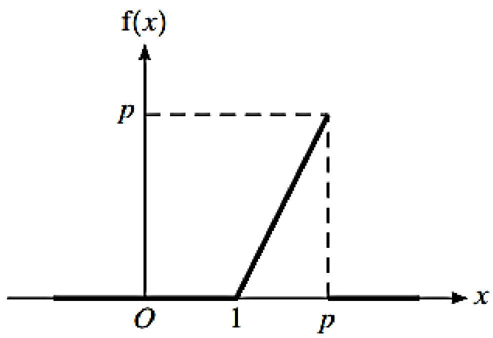
The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} k(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

State the value of $E(X)$ and show that $\text{Var}(X) = \frac{9}{5}$. [6]

11) June 2021 9709_62 Q3



The random variable X takes values in the range $1 \leqslant x \leqslant p$, where p is a constant. The graph of the probability density function of X is shown in the diagram.

(a) Show that $p = 2$. [2]

(b) Find $E(X)$. [5]

12) June 2021 9709_63 Q6

Alethia models the length of time, in minutes, by which her train is late on any day by the random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{3}{8000}(x - 20)^2 & 0 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that the train is more than 10 minutes late on each of two randomly chosen days. [4]

(b) Find $E(X)$. [4]

(c) The median of X is denoted by m.

Show that m satisfies the equation $(m - 20)^3 = -4000$, and hence find m correct to 3 significant figures. [4]

(d) State one way in which Alethia's model may be unrealistic. [1]

13) June 2022 9709_61 Q6

A random variable X has probability density function f. The graph of $f(x)$ is a straight line segment parallel to the x-axis from $x = 0$ to $x = a$, where a is a positive constant.

(a) State, in terms of a, the median of X. [1]

(b) Find $P(X > \frac{3}{4}a)$. [1]

(c) Show that $\text{Var}(X) = \frac{1}{12}a^2$. [5]

(d) Given that $P(X < b) = p$, where $0 < b < \frac{1}{2}a$, find $P(\frac{1}{3}b < X < a - \frac{1}{3}b)$ in terms of p. [2]

14) June 2022 9709_62 Q5

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{16}(4x - x^2) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $E(X) = \frac{11}{4}$. [3]

(b) Find $\text{Var}(X)$. [3]

(c) Given that the median of X is m, find $P(m < X < 3)$. [3]

15) June 2022 9709_63 Q7

The random variables X and W have probability density functions f and g defined as follows:

$$f(x) = \begin{cases} p(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

$$g(w) = \begin{cases} q(a^2 - w^2) & -a \leqslant w \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a, p and q are constants.

(a) **(i)** Write down the value of $P(X \geqslant 0)$. [1]

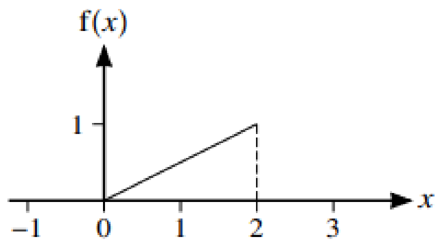
(ii) Write down the value of $P(W \geqslant 0)$. [1]

(iii) Write down an expression for q in terms of p only. [1]

(b) Given that $E(X) = 3$, find the value of a. [6]

16) June 2023 9709_61 Q2

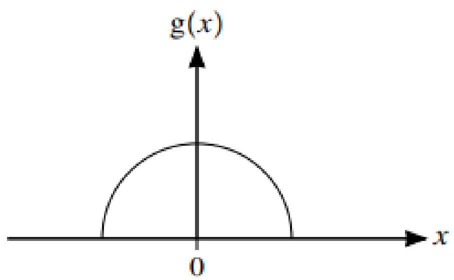
(a)



The graph of the function f is a straight line segment from (0, 0) to (2, 1).

Show that f could be a probability density function. [2]

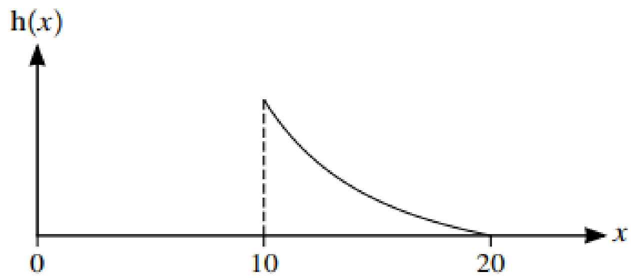
(b)



The graph of the function g is a semicircle, centre (0, 0), entirely above the x-axis.

Given that g is a probability density function, find the radius of the semicircle. [2]

(c)



The time, X minutes, taken by a large number of students to complete a test has probability density function h, as shown in the diagram.

(i) Without calculation, use the diagram to explain how you can tell that the median time is less than 15 minutes. [1]

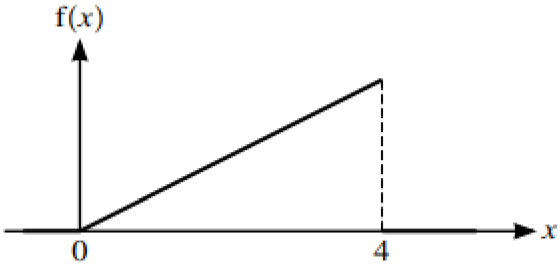
It is now given that

$$h(x) = \begin{cases} \dfrac{40}{x^2} - \dfrac{1}{10} & 10 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) Find the mean time. [3]

17) June 2023 9709_62 Q7

(a)

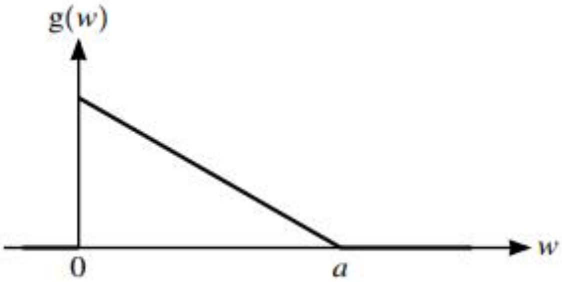


The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and 4 only. Between these two values the graph is a straight line.

(i) Show that $f(x) = kx$ for $0 \leqslant x \leqslant 4$, where k is a constant to be determined. [2]

(ii) Hence, or otherwise, find $E(X)$. [3]

(b)



The diagram shows the graph of the probability density function, g, of a random variable W which takes values between 0 and a only, where $a > 0$. Between these two values the graph is a straight line.

Given that the median of W is 1, find the value of a. [3]

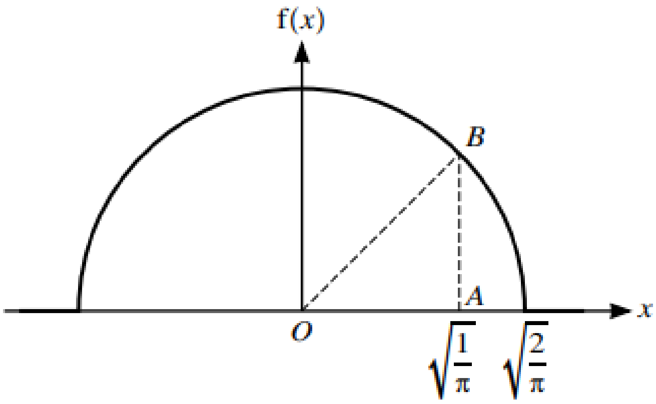
18) June 2023 9709_63 Q1

A random variable X has probability density function f, where

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$. [3]

19) June 2023 9709_63 Q7



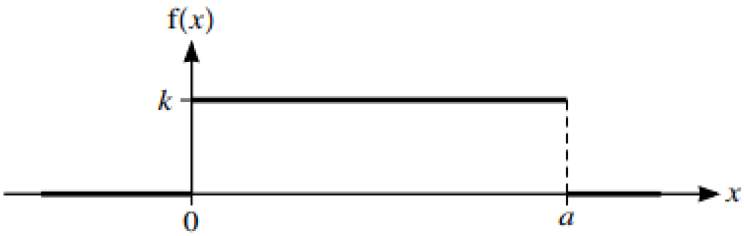
A random variable X has probability density function f, where the graph of $y = \mathrm{f}(x)$ is a semicircle with centre (0, 0) and radius $\sqrt{\frac{2}{\pi}}$, entirely above the x-axis. Elsewhere $\mathrm{f}(x) = 0$ (see diagram).

(a) Verify that f can be a probability density function. [2]

A and B are the points where the line $x = \sqrt{\frac{1}{\pi}}$ meets the x-axis and the semicircle respectively.

(b) Show that angle AOB is $\frac{1}{4}\pi$ radians and hence find $\mathrm{P}\left(X > \sqrt{\frac{1}{\pi}}\right)$. [6]

20) Oct 2020 9709_61 Q4

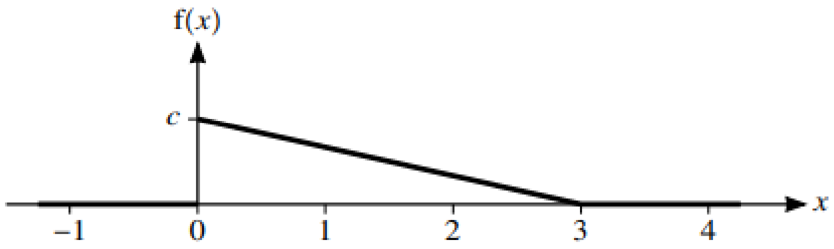


The diagram shows the probability density function, $\mathrm{f}(x)$, of a random variable X. For $0 \leqslant x \leqslant a$, $\mathrm{f}(x) = k$; elsewhere $\mathrm{f}(x) = 0$.

(a) Express k in terms of a. [1]

(b) Given that $\mathrm{Var}(X) = 3$, find a. [4]

21) Oct 2020 9709_62 Q3



A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(a) Show that $c = \frac{2}{3}$. [1]

(b) Find $P(X > 2)$. [2]

(c) Calculate $E(X)$. [4]

22) Oct 2021 9709_61 Q4

A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{18}(9 - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 1.2)$. [3]

(b) Find $E(X)$. [3]

The median of X is m.

(c) Show that $m^3 - 27m + 27 = 0$. [3]

23) Oct 2021 9709_62 Q7

(a) The probability density function of the random variable X is given by

$$f(x) = \begin{cases} kx(4 - x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{16}$. [3]

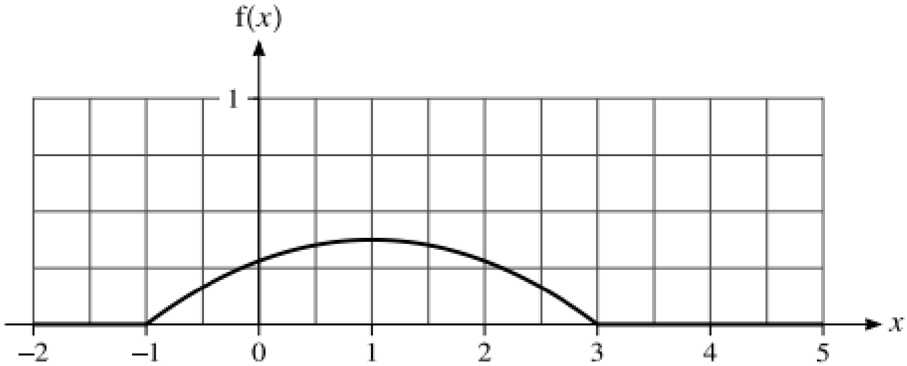
(ii) Find $E(X)$. [3]

(b) The random variable Y has the following properties.

- Y takes values between 0 and 5 only.
- The probability density function of Y is symmetrical.

Given that $P(Y < a) = 0.2$, find $P(2.5 < Y < 5 - a)$ illustrating your method with a sketch on the axes provided. [3]

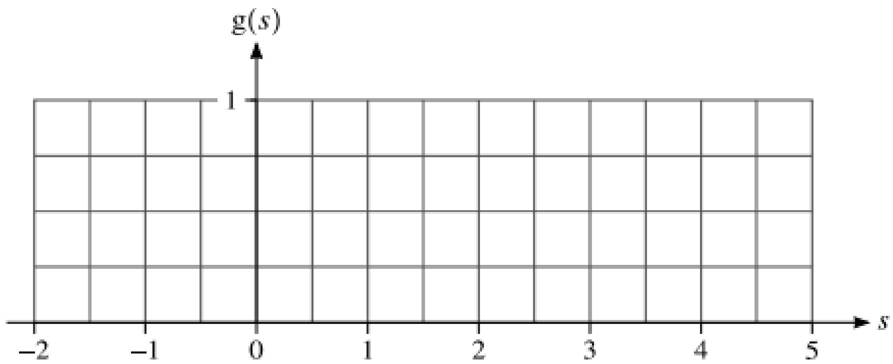
24) Oct 2022 9709_61 Q6



The diagram shows the graph of the probability density function of a random variable X that takes values between -1 and 3 only. It is given that the graph is symmetrical about the line $x = 1$. Between $x = -1$ and $x = 3$ the graph is a quadratic curve.

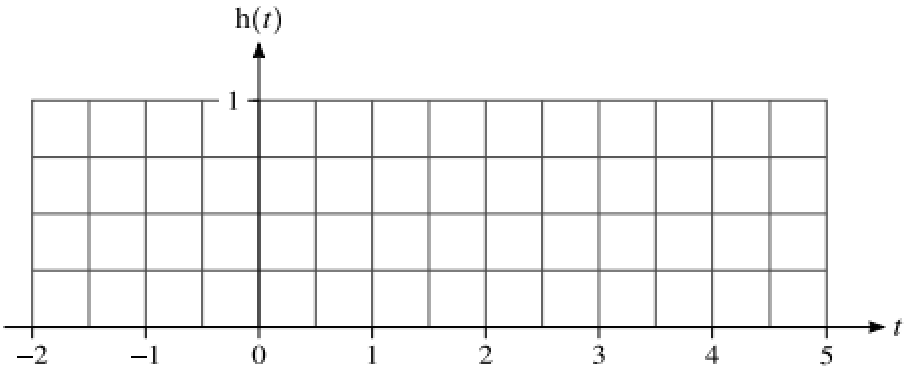
The random variable S is such that $E(S) = 2 \times E(X)$ and $Var(S) = Var(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $E(T) = E(X)$ and $Var(T) = \frac{1}{4} Var(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



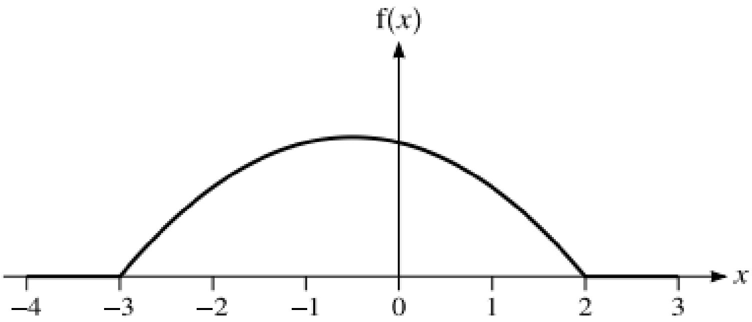
It is now given that

$$f(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $P(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

(d) Hence verify that $0.69 < a < 0.70$. [1]

25) Oct 2022 9709_62 Q7



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between -3 and 2 only.

(a) Given that the graph is symmetrical about the line $x = -0.5$ and that $P(X < 0) = p$, find $P(-1 < X < 0)$ in terms of p. [2]

(b) It is now given that the probability density function shown in the diagram is given by

$$f(x) = \begin{cases} a - b(x^2 + x) & -3 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(i) Show that $30a - 55b = 6$. [3]

(ii) By substituting a suitable value of x into $f(x)$, find another equation relating a and b and hence determine the values of a and b. [3]

Marking Scheme

1) .

(a)	$E(X) = 1.5$	1	**B1**
	$\frac{2}{9}\int_0^3 (3x^3 - x^4)\,dx$	1	**M1**
	$= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$ $= \frac{2}{9}\left[\frac{243}{4} - \frac{243}{5}\right]$ $(= 2.7)$	1	**M1**
	$\text{Var}(X)\ (= 2.7 - 1.5^2) = 0.45$	1	**A1FT**
		4	
(b)	0.5	**1**	**B1**
(c)	$\left(1 - \frac{13}{27}\right) \div 2$	1	**M1**
	$= \frac{7}{27}$ or 0.259	1	**A1**

2) .

(a)	$\frac{3}{4000}\int_5^{10} (100 - x^2)\,dx$ $= \frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_5^{10}$	**M1**
	$= \frac{3}{4000}\left(1000 - \frac{1000}{3} - 500 + \frac{125}{3}\right)$	**M1**
	$= 0.156$ (3 sf) or $\frac{5}{32}$	**A1**

(b)		
	$\frac{3}{4000}\int_{p}^{10}(100-x^2)\mathrm{d}x = \frac{1}{4}$	**M1**
	$\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_{p}^{10} = \frac{1}{4}$	**A1**
	$\frac{3}{4000}\left(1000 - \frac{1000}{3} - 100p + \frac{p^3}{3}\right) = \frac{1}{4}$	**M1**
	e.g. $\frac{2000}{3} - 100p + \frac{p^3}{3} = \frac{1000}{3}$ $p^3 - 300p + 1000 = 0$	**A1**

3) .

(a)	$\frac{1}{2}\times\frac{1}{2}k\times k = 1$	**M1**
	$k = 2$	**A1**
(b)	$\mathrm{f}(x) = -\frac{1}{2}x + 1$	**B1 FT**
	$\int_0^2 (-\frac{1}{2}x^2 + x)\mathrm{d}x = \left[-\frac{x^3}{6}+\frac{x^2}{2}\right]_0^2$	**M1**
	$\frac{2}{3}$ or 0.667 (3 sf)	**A1**

4) .

(a)	Quadratic curve, hence symmetrical	**B1**
(b)	$-k\int_1^3 (x^2-4x+3)\mathrm{d}x = 1$	**M1**
	$-k\left[\frac{x^3}{3}-2x^2+3x\right]_1^3$	**A1**
	$-k\times\left[0-\frac{4}{3}\right]=1$ or $k\times\frac{4}{3}=1$ $\left[k=\frac{3}{4}\right]$	**A1**
(c)	$-\frac{3}{4}\int_1^3\left(x^4-4x^3+3x^2\right)\mathrm{d}x$	**M1**
	$-\frac{3}{4}\times\left[\frac{x^5}{5}-x^4+x^3\right]_1^3$ $\left[=\frac{3}{4}\times\frac{28}{5}=\frac{21}{5}\right]$	**A1**
	$\left[\frac{21}{5}-2^2\right]=0.2$	**A1**
(d)	$-\frac{3}{4}\int_{2.5}^{3}\left(x^2-4x+3\right)\mathrm{d}x$	**M1**
	$=-\frac{3}{4}\times\left[\frac{x^3}{3}-2x^2+3x\right]_{2.5}^3=\frac{5}{32}$ or 0.15625	**A1**
	$1-\left(1-'\frac{5}{32}'\right)^3$	**M1**
	$= 0.399$ (3 sf)	**A1**

5) .

Alternative method for Question 3

State $2x^4 + ax^3 + 0x^2 + bx - 1 = (x^2 - x + 1)(2x^2 + Ax + B) + 3x + 2$ and form and solve equation(s) to obtain A or B	**M1**
Obtain $A = -1$, $B = -3$	**A1**
Form and solve equations for a or for b	**M1**
Obtain answer $a = -3$	**A1**
Obtain answer $b = 5$	**A1**

Alternative method for Question 3

Use remainder theorem with $x = \frac{1 \pm \sqrt{-3}}{2}$ or $x = \frac{1 \pm i\sqrt{3}}{2}$	**M1**
Obtain $-a + \frac{b}{2} \pm \frac{b\sqrt{-3}}{2} \mp \sqrt{-3} - 2 = \frac{7}{2} \pm \frac{3\sqrt{-3}}{2}$ or $-a + \frac{b}{2} \pm \frac{bi\sqrt{3}}{2} \mp i\sqrt{3} - 2 = \frac{7}{2} \pm \frac{3i\sqrt{3}}{2}$	**A1**
Solve simultaneous equations, or single equation, for a or for b	**M1**
Obtain answer $a = -3$ from exact working	**A1**
Obtain answer $b = 5$ from exact working	**A1**

6) .

(a)	'Tails down' parabola only from $x = 0$ to 20 shown	B1
		1
(b)	Symmetrical	B1
		1
(c)	$\frac{3}{4000}\int_0^{20}\left(20t^3 - t^4\right)\mathrm{d}x = \frac{3}{4000}\left[20\frac{t^4}{4} - \frac{t^5}{5}\right]_0^{20}$	M1
	$\text{Var}(T) = \frac{3}{4000} \times 160000 - 10^2$	M1
	20	A1
		3
(d)	$(p - 0.5) \times 2$ or $1 - 2(1 - p)$	M1
	$2p - 1$	A1
		2
(e)	$\frac{3}{4000}\int_8^{12}\left(20t - t^2\right)\mathrm{d}x$	M1
	$\frac{3}{4000}\left[20\frac{t^2}{2} - \frac{t^3}{3}\right]_8^{12} = \frac{3}{4000}\left(1440 - 576 - 640 + \frac{512}{3}\right)$	A1
	$\frac{37}{125}$ or 0.296	A1
		3
(f)	Does not allow times greater than 20 minutes	B1
		1

7) .

(a)	$\int_1^a \frac{k}{x^2}\mathrm{d}x = 1$	M1
	$k\left[-\frac{1}{x}\right]_1^a = 1$ $k\left[1 - \frac{1}{a}\right] = 1$	A1
	$k\left[\frac{a-1}{a}\right] = 1$ $\left(k = \frac{1}{a-1}\right)$ **AG**	A1
		3
(b)	$\frac{a}{a-1}\int_1^a \frac{1}{x}\mathrm{d}x$	M1
	$\frac{a}{a-1}\left[\ln x\right]_1^a$	A1
	$\frac{a\ln a}{a-1}$	A1
		3

(c)	$\frac{a}{a-1}\int_1^m \frac{1}{x^2}\mathrm{d}x = \frac{3}{5}$	M1
	$\frac{a}{a-1}\left[-\frac{1}{x}\right]_1^m = \frac{3}{5}$ $\frac{a}{a-1}\left[1-\frac{1}{m}\right] = \frac{3}{5}$	A1
	$\frac{1}{m} = 1 - \frac{3(a-1)}{5a}$ or $\frac{1}{m} = \frac{2a+3}{5a}$	A1
	$m = \frac{5a}{2a+3}$	A1
		4

8) .

(a)	7.5	B1
		1
(b)	$\frac{6}{125}\int_5^{10}(-x^4+15x^3-50x^2)\mathrm{d}x$	M1
	$\frac{6}{125}\left[-\frac{x^5}{5}+15\frac{x^4}{4}-50\frac{x^3}{3}\right]_5^{10} - \text{'}7.5\text{'}^2$	M1
	1.25 (3 sf)	A1
		3
(c)	$\frac{6}{125}\int_5^{6}(-x^2+15x-50)\mathrm{d}x$ $\frac{6}{125}\left[-\frac{x^3}{3}+15\frac{x^2}{2}-50x\right]_5^{6}$	M1
	$\frac{6}{125}(-102+\frac{625}{6})$ oe	M1
	0.104	A1
	$2\times(\text{'}0.104\text{'}\times(1-\text{'}0.104\text{'})$	M1
	0.186 (3 sf)	A1ft
		5

9) .

$1-\frac{20}{27}$ or $\frac{20}{27}-\frac{1}{2}$ $\frac{20}{27}-\left(1-\frac{20}{27}\right)$ or $\left(\frac{20}{27}-\frac{1}{2}\right)$	M1
$\frac{13}{27}$	A1

10) .

$E(X) = 3$	B1
$k\int_0^6 (6x - x^2)\,dx = 1$ $k\left[3x^2 - \frac{x^3}{3}\right]_0^6$ [= 1]	M1
$k\left(108 - \frac{216}{3}\right) = 1$ $k = \frac{3}{108}$ or $\frac{1}{36}$	A1
$'\frac{3}{108}'\int_0^6 (6x^3 - x^4)\,dx$ $= \frac{3}{108}\left[\frac{3x^4}{2} - \frac{x^5}{5}\right]_0^6 = 10.8$	*M1
'10.8' – '3'^{2}	DM1
$\frac{9}{5}$ or 1.8	A1

11) .

(a)	$\frac{1}{2}p(p - 1) = 1$	M1
	$p = 2$	A1

(b)	Gradient = 2 equation of line is $y = 2x + c$ line passes through (1, 0), hence $c = -2$	M1
	$y = 2x - 2$	A1
	$2\int_1^2 (x^2 - x)\mathrm{d}x$	M1
	$2\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2$	A1 FT
	$\frac{5}{3}$ or 1.67 (3 sf)	A1

12) .

(a)	$\mathrm{P}(X > 10) = \int_{10}^{20} \frac{3}{8000}(x-20)^2\,\mathrm{d}x$	M1
	$= \left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_{10}^{20}$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_{10}^{20}$ $= \frac{1}{8000}\left[0 - (-10)^3\right]$	M1
	$\frac{1}{8}$ or 0.125	A1
	$(\frac{1}{8})^2 = \frac{1}{64}$ or 0.0156 (3 sf)	B1 FT

(b)	$\int_0^{20} \frac{3}{8000}(x^3 - 40x^2 + 400x)\mathrm{d}x$	M1
	$\frac{3}{8000}\left[\frac{x^4}{4} - \frac{40x^3}{3} + \frac{400x^2}{2}\right]_0^{20}$ or $\left(\frac{3x}{8000} \times \frac{(x-20)^3}{3}\right) - \frac{1}{8000}\left(\frac{(x-20)^4}{4}\right)$	A1
	$\frac{3}{8000}\left[\frac{160000}{4} - \frac{40 \times 8000}{3} + 200 \times 400\right]$	M1
	5	A1

(c)	$\int_0^m \frac{3}{8000}(x-20)^2\,dx = 0.5$	M1
	$\left[\frac{3}{8000}\times\frac{(x-20)^3}{3}\right]_0^m = 0.5$ or $\frac{3}{8000}\left[\frac{x^3}{3}-\frac{40x^2}{2}+400x\right]_0^m = 0.5$ $\frac{1}{8000}\left[(m-20)^3-(-20)^3\right] = 0.5$	M1
	$(m-20)^3 = -4000$	A1
	$(m = 20 + \sqrt[3]{-4000})$ $m = 4.13$ (3 sf)	B1
(d)	Doesn't allow for trains > 20 mins late or Doesn't allow for trains being early	B1

13)

(a)	$\frac{a}{2}$	B1
		1
(b)	$\frac{1}{4}$	B1
		1
(c)	$f(x) = \frac{1}{a}$	B1
	$E(X) = \frac{a}{2}$	B1
	$\int_0^a \frac{1}{a}x^2\,dx$	M1
	$= \left[\frac{x^3}{3a}\right]_0^a = \frac{a^2}{3}$	A1
	$\frac{a^2}{3} - (\frac{a}{2})^2$ or $\frac{a^2}{3} - \frac{a^2}{4}$ [$= \frac{a^2}{12}$ **AG**]	A1
(d)	$P(X < \frac{b}{3}) = \frac{p}{3}$	M1
	$P(\frac{b}{3} < X < a - .\frac{b}{3}) = 1 - \frac{2p}{3}$	A1

14) .

(a)	$\frac{3}{16}\int_2^4 (4x^2 - x^3)\mathrm{d}x$	**M1**
	$= \frac{3}{16}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_2^4$	**M1**
	$= \frac{3}{16}\ (\frac{256}{3} - 64 - (\frac{32}{3} - 4)\ ) = \frac{11}{4}$ **(AG)**	**A1**
(b)	$\frac{3}{16}\int_2^4 (4x^3 - x^4)\mathrm{d}x$	***M1**
	$= \frac{3}{16}\left[x^4 - \frac{x^5}{5}\right]_2^4$ $[= \frac{39}{5}$ or 7.8] $\mathrm{Var}(X) = ‘\frac{39}{5}’ - (\frac{11}{4})^2$	**DM1**
	$= \frac{19}{80}$ or 0.2375 (or 0.238 (3 sf))	**A1**
(c)	$\frac{3}{16}\int_2^3 (4x - x^2)\mathrm{d}x$	**M1**
	$= \frac{3}{16}\left[2x^2 - \frac{x^3}{3}\right]_2^3$ $\left[= \frac{3}{16}\left(18 - 9 - \left(8 - \frac{8}{3}\right)\right)\right]$ $\left[= \frac{11}{16}\right]$ $‘\frac{11}{16}’ - \frac{1}{2}$	**M1**
	$\frac{3}{16}$ or 0.1875	**A1**

15) .

(a)(i)	1	B1
		1
(a)(ii)	$\frac{1}{2}$	B1
		1
(a)(iii)	$[q=]\ \frac{1}{2}p$	B1
(b)	$p\int_0^a (a^2-x^2)\mathrm{d}x = 1$	M1
	$\frac{2}{3}a^3p = 1$	A1
	$"\frac{3}{2a^3}"\int_0^a (a^2x-x^3)\mathrm{d}x = 3$ or $"\frac{3}{2a^3}"\int_0^a (a^2x-x^3)\mathrm{d}x = 3$	M1
	$p\times\frac{a^4}{4} = 3$	A1
	$"\frac{3}{2a^3}"\times\frac{a^4}{4} = 3$	M1
	$a = 8$	A1

16) .

(b)	$\frac{1}{2}\pi r^2 = 1$	M1
	$r=\sqrt{\frac{2}{\pi}}$ or 0.798 (3sf)	A1
(c)(i)	Area to the left of 15 is greater than 0.5	B1
(c)(ii)	$\int_{10}^{20}(\frac{40}{x}-\frac{x}{10})\mathrm{d}x$	M1
	$\left[40\ln x-\frac{x^2}{20}\right]_{10}^{20}$	A1
	$= 40\ln 2 - 15$ or 12.7 (3sf)	A1

17) .

(a)(i)	$\frac{1}{2} \times 4 \times a = 1$	**M1**
	$[a = \frac{1}{2}]$ $f(x) = \frac{1}{8}x$	**A1**
(a)(ii)	$\int_0^4 x \times \frac{1}{8}x \, dx$	**M1**
	$\left[\frac{x^3}{24}\right]_0^4$	**A1ft**
	$= \frac{8}{3}$ or 2.67 (3 sf)	**A1**
(b)	$\frac{a-1}{a} = \frac{1}{\sqrt{2}}$	**M1**
	$a\sqrt{2} - \sqrt{2} = a$	**A1**
	$a = 2 + \sqrt{2} = 3.41$	**A1**

18) .

$\frac{3}{2}\int_0^1 (x - x^3)dx$	**M1**
$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$	**A1**
$= \frac{3}{8}$	**A1**

19) .

(a)	$\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$	**M1**
	= 1, which is the area under a PDF [and $f(x) \geq 0$]	**A1**
(b)	$\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right) = \frac{\pi}{4}$	**B1**
	Area of sector = $\frac{1}{4}$	**B1**
	Area of triangle $AOB = \frac{1}{2}OA \times OB = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi} - \frac{1}{\pi}}$ or Area of triangle $AOB = \frac{1}{2}OA \times OB \times \sin(AOB) = \frac{1}{2} \times \sqrt{\frac{1}{\pi}} \times \sqrt{\frac{2}{\pi}} \sin\frac{\pi}{4}$	**M1**
	$\frac{1}{2\pi}$ or 0.1592	**A1**
	'$\frac{1}{4}$' – '$\frac{1}{2\pi}$' or '0.25' – '0.1592'	**M1**
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	**A1**
(b)	**Alternative Method for Question Q7(b): Using integration**	
	Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$	**M1**
	$y = \sqrt{\frac{2}{\pi} - x^2}$	**A1**
	Attempt to integrate (any limits)	**M1**
	Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$	**B1**
	Correct integration with correct limits	**A1**
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	**A1**

20) .

(a)	$(k =)\ \frac{1}{a}$	**B1**
		1
(b)	(Mean =) *their* $k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$	**B1 FT**
	$\frac{1}{a}\int_0^a x^2 \mathrm{d}x \left(= \frac{a^2}{3}\right)$	**M1**
	$-\left(\frac{a}{2}\right)^2 \left(= \frac{a^2}{12}\right)$	**M1**
	$\left(\frac{a^2}{12} = 3\right)\ a = 6$	**A1**

21) .

(a)	$\frac{1}{2} \times 3 \times c = 1$ $(c = \frac{2}{3}$ **AG**)	**B1**
(b)	$\left(\frac{1}{3}\right)^2$	**M1**
	$= \frac{1}{9}$ or 0.111(3sf)	**A1**

(c)	Equation of line is $y = \frac{2}{3} - \left(\frac{2}{3} \div 3\right)x$	*M1
	$E(X) = \int_0^3 \left(\frac{2}{3}x - \frac{2}{9}x^2\right) dx$	DM1
	$= \left[\frac{x^2}{3} - \frac{2x^3}{27}\right]_0^3$	A1 FT
	$= 1$	A1

22) .

(a)	$\frac{1}{18}\int_0^{1.2} (9 - x^2) dx$	M1
	$\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^{1.2}$	A1
	$\frac{71}{125}$ or 0.568	A1

(b)	$\frac{1}{18}\int_0^3 (9x - x^3) dx$	M1
	$\frac{1}{18}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$	A1
	$\frac{9}{8}$ or 1.125	A1

(c)	$\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^m = 0.5$	M1
	$\frac{1}{18}\left[9m - \frac{m^3}{3}\right] - 0.5 = 0$	A1
	$m^3 - 27m + 27 = 0$	A1

23) .

(a)(i)	$k\int_0^2 (4x - x^2)\mathrm{d}x = 1$	**M1**
	$k\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$	**A1**
	$k \times \frac{16}{3} = 1 \left[k = \frac{3}{16}\right]$	**A1**
(a)(ii)	$\frac{3}{16}\int_0^2 (4x^2 - x^3)\mathrm{d}x$	**M1**
	$\frac{3}{16}\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2$	**A1**
	$\frac{5}{4}$	**A1**
(b)	Symmetrical frequency density graph, 0 to 5, showing area 0.2 to left of a	**B1**
	Either 0.2 between $5-a$ and 5 or 0.8 between 0 and $5-a$	**B1**
	$[P(2.5<Y<5-a)] = 0.3$	**B1**

24) .

(a)	Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375)	**B1**
(b)	Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$	**B1**
	Highest point (1, 0.75)	**B1**
(c)	$\frac{3}{32}\int_{1+a}^{3} (3 + 2x - x^2)\mathrm{d}x = \frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a} (3 + 2x - x^2)\mathrm{d}x = \frac{1}{2}$	**M1**
	$\frac{3}{32}\left[3x + x^2 - \frac{x^3}{3}\right]_{1+a}^{3} = \frac{1}{4}$ or $\frac{3}{32}\left[3x + x^2 - \frac{x^3}{3}\right]_{1-a}^{1+a} = \frac{1}{2}$	**A1**
	$a^3 - 12a + 8 = 0$	**A1**
(d)	$0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) > 0 $0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) < 0 Hence $0.69 < a < 0.70$	**B1**

25) .

(a)	$1 - p$ or $p - 0.5$	**M1**
	[P($-1 < X < 0$) =] $2p - 1$	**A1**
(b)(i)	$\int_{-3}^{2} (a - b(x^2 + x))\mathrm{d}x = 1$ or $\int_{-3}^{2} (ax - b(x^3 + x^2))\mathrm{d}x = -0.5$	**M1**
	$\left[ax - b\left(\frac{x^3}{3} + \frac{x^2}{2}\right)\right]_{-3}^{2}$ (= 1) or $\left[a\frac{x^2}{2} - b\left(\frac{x^4}{4} + \frac{x^3}{3}\right)\right]_{-3}^{2}$ (= –0.5)	**A1**
	2a – 8b/3 – 2b +3a – 9b +9b / 2 = 1 or 2a – 4b – 8b / 3 – 9a / 2 + 81b / 4 – 9b = –0.5 leading to $30a - 55b = 6$ **AG**	**A1**
(b)(ii)	$a - b(9 - 3) = 0$ or $a - b(4 + 2) = 0$ [hence $a - 6b = 0$]	***M1**
	Attempt to solve $30a - 55b = 6$ and their a – 6b = 0	**DM1**
	$a = \frac{36}{125}$ or 0.288 $b = \frac{6}{125}$ or 0.048	**A1**