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Probability and Statistics-2

Hypothesis Testing

Exercise-2 (Solution)

Revision.

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S.2

Hypothesis testing - Notes and Revision

§ <u>Null hypothesis:</u> The claim is called the null hypothesis and is denoted by 'H_0'

§ <u>Alternate Hypothesis:</u>

When we don't accept the null hypothesis, then we have an alternate hypothesis, denoted by 'H_1'.

The null hypthesis and the alternate hypothesis both are expressed in terms of a parameter, such as probability 'p' or a mean value 'μ' (normal distribution).

§ <u>Rejection region</u> (or the critical region) and the <u>critical value</u>:

The range of values for which we reject the null hypothesis is the critical region.

The <u>value</u>, at which we change accepting the null hypothesis to rejecting, is called <u>critical value</u>.

§ <u>Type I error:</u>

The type of error, where a <u>null hypothesis is rejected despite being correct</u>, is called a Type-I error, or <u>P(reject H_0/H_0 is true)</u>

§ <u>Type II error;</u>

When the <u>null hypothesis is in fact false</u>, but is <u>accepted</u>. or P(Type II error) = P(accept H_0/H_0 is false)

§ <u>Significance Level:</u> The <u>significance level</u> is the probability of <u>rejecting a claim</u>.

The significance level is given in percentage. (generally it is 5%). "The <u>lower the percentage significance level</u>, the smaller the rejection region, and the <u>more confident</u> you can be of the <u>result</u>."

§ <u>One-tailed test:</u> When the words increased (or decreased) are used.
<u>Two-tailed test:</u> When the word - the parameter is different. $p \neq a$ or $\mu \neq a$

Example 1. At a certain hospital it was found that the probability that a patient did not arrive for an appoint was 0.2. The hospital carries out some publicity in the hope that this probability will be reduced. They wish to test whether the publicity worked.
A random sample of 30 appointments is selected and the number of patients that do not arrive is noted. This figure is used to carry out a test at the 5% significance level.

(a) Explain why the test is one-tailed and state suitable null and alternate hypothesis. --[2]

(b) Use a binomial distribution to find the critical region and find the probability of type I error. --[5]

(c) In fact 3 patient out of the 30 patients do not arrive. State the colclusion of the test, explaining your answer. --[2]

[SP-20/06/Q6]

Solution (a) Random Variable, X denotes the number of patients not coming for appointment.
Test is one-tailed as looking for decrease in number of patients noting coming for an appointment.
Null Hypothesis, H_0: P(not arrive) $p=0.2$
Alternate hypothesis, H_1: $p<0.2$ Significance level $=5\%=0.05$

(b) Binomial distribution $X \sim B(n,p)$; $n=30$, $p=0.2$, $q=0.8$

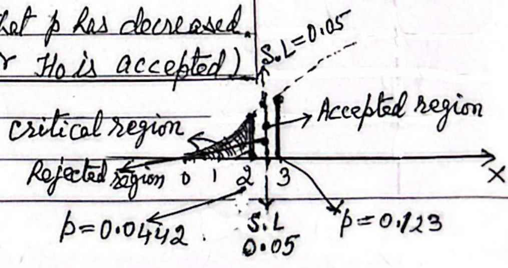
$P(X\leq 2) = 0.8^{30} + 30_{C_1} 0.2^1 \times 0.8^{29} + 20_{C_2} 0.2^2 \times 0.8^{28} = 0.0442$ --(i)

$P(X\leq 3) = - - - - - - - - + 20_{C_3} 0.2^3 \times 0.8^{27} = 0.123$ --(ii)

Here $\left.\begin{array}{l} P(X\leq 2) = 0.0442 < 0.05 \\ \text{but } P(X\leq 3) = 0.123 > 0.05 \end{array}\right\}$ ∴ Critical region is $X\leq 2$

and the critical value is $X=2$, P(Type I error) $=0.0442$ ✓

(c) $P(X\leq 3) = 0.123 > 0.05$ (3 is out-side the critical region)
∴ Hence No evidence that p has decreased.
(or H_0 is accepted)



Example 2.: The mean weight of bags of carrots is μ kilograms. An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

$$\Sigma x = 430 \quad , \quad \Sigma x^2 = 1290$$

Carry out the test at the 10% significance level. --[7]

[SP 20/06/Q 7]

Solution: Randendom variable, μ is the mean weight of bags.

Null hypothesis H_0: $\mu = 2.0$

Alternate hypothesis H_1: $\mu \neq 2.0$ (Two-tailed test)

Now estimated mean $\bar{x} = \frac{\Sigma x}{n} = \frac{430}{200} = 2.15$

estimated Variance $s^2 = \frac{200}{119}\left(\frac{1290}{200} - (2.15)^2\right) = 1.8366834$

$$P(\mu < 2.15) = P\left(z < \frac{2.15 - 2}{\sqrt{\frac{1.8366834}{200}}}\right)$$

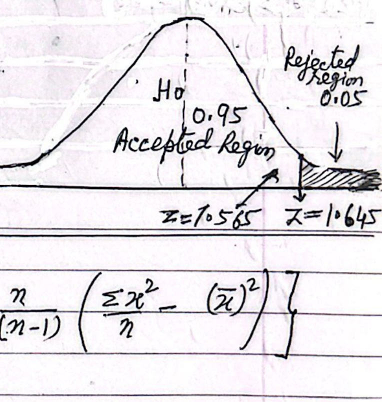
$$= P(z < 1.565) \otimes$$

$\underset{(\text{Sample})}{\text{Var}} = \frac{\sigma^2}{n}$

{ being two tailed test $\otimes$
Significance level $\otimes$
$= \frac{1}{2} \times 10\% = 0.05$
on the upper side $\Rightarrow p = 0.95$
$z = \phi^{-1}(0.95) = 1.645$ }

(H_0 is accepted) Now $1.565 < 1.645$

No evidence that $\mu \neq 2.0$



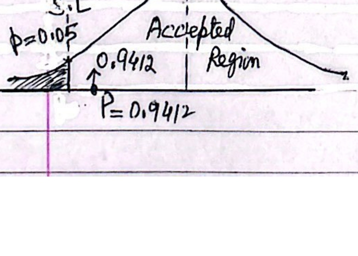
$\otimes$ Alternate method.

$P(z < 1.565)$

$= \phi(1.565)$

$= 0.9412 > 0.05$

No evidence that $\mu \neq 2.0$



$\left[\text{est}(\text{Var}\,x) = s^2 = \frac{n}{(n-1)}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right) \right]$

Example 3. In the past, the mean time taken by Freda for a particular daily journey was 39.2 minutes. Following the introduction of a one-way system. Freda wishes to test whether the time for the journey has decreased. She notes the times, t times, for randomly chosen journeys and summarises the results as follows:

$$n = 40, \quad \Sigma t = 1504, \quad \Sigma t^2 = 57760$$

(a) Calculate unbaised estimates of the population mean and variance of the new journey time. ---[3]

(b) Test, at the 5% significance level, whether the population mean time has decreased. ---[5]

M-20/62/Q3

Solution:

(a) Estimated mean $\mu = \frac{\Sigma t}{n} = \frac{15.04}{40} = 37.6$ ✓

Estimated variance:

$$s^2 = \frac{40}{39}\left[\frac{57760}{40} - 37.6^2\right] = 31.0154 \checkmark$$

$$\left\{ s^2 = \frac{n}{n-1}\left\{\frac{\Sigma x^2}{n} - (\bar{x})^2\right\}\right\}$$

(b) Significance level 5% = 0.05

Null hypthesis, H_0: $\mu = 39.2$ (population mean)

Alternate hypothesis H_1: $\mu < 39.2$

$$P(X < 37.6) = P\left(Z < \frac{37.6 - 39.2}{\sqrt{\frac{31.0154}{40}}}\right)$$

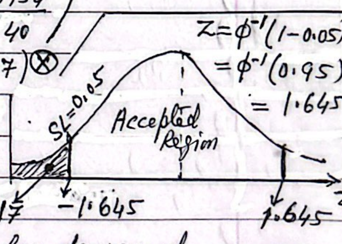
$$\text{Var}\,\bar{x} = \frac{\sigma^2}{n}$$

$$= P(Z < -1.817) \otimes$$

$$Z = \phi^{-1}(1 - 0.05) = \phi^{-1}(0.95) = 1.645$$

here $-1.817 < -1.645$

(H_0 is rejected)



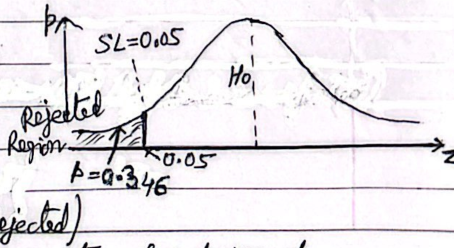
$\therefore$ There is evidence that mean time has decreased.

$\otimes$ **Alternate method:**

$P(Z < -1.817)$

$= 1 - \phi(1.817)$

$= 1 - .9654 = 0.346$



Now $0.346 < 0.05$ (H_0 is Rejected)

Hence There is evidence that the mean time has decreased.

Example 4: A national survey shows that 95% of year 12 students use social media. Arvin suspects that the percentage of year 12 students at his college who use social media is less than the national percentage. He chooses a random sample of 20 students at his college and notes the number who use social media. He then carries out a test at the 2% significance level.

(a) Find the rejection region for the test. --[4]

(b) Find the probability of a Type I error. --[1]

(C) Jimmy believes that the true percentage at Arvin's college is 70%. Assuming that Jimmy is correct, find the probability of a Type II error.

[M-20/62/Q7] --[3]

Solution: Random Variable: X is number of year 12 student to use social media.

(a) Null hypothesis: H_0 : $p = 0.95$ — Binomial distribution $B(20, 0.95)$

Alternate hypothesis H_1 : $p < 0.95$ — $\left\{ \begin{array}{l} n = 20, p = 0.95 \\ q = 0.05 \end{array} \right.$

Now $P(X \leq 17) = 1 - P(X = 18, 19, 20)$ — {Significance level = 2% = 0.02}

$= 1 - \{ {}^{20}C_{18} (.95)^{18} (.05)^{2} + {}^{20}C_{19} (.95)^{19} \times (.05) + (.95)^{20} \}$

or $P(X \leq 17) = 0.0755 > 0.02$ ---(i)

$P(X \leq 16) = 0.0755 - P(17) = 0.0755 - {}^{20}C_{17} (.95)^{17} \cdot (0.05)^{3} = 0.0159$

$\Rightarrow P(X \leq 16) = 0.0159 < 0.02$ (significance level) --(ii)

hence from (i) and (ii) Rejection region is: $X \leq 16$

(b) $P(\text{Type I error}) = 0.0159$

(C) $P(\text{Type II error}) = P(H_0$ is excepted / whereas H_0 is false, as $p = 0.7)$

Now Using $B(20, 0.7)$; $n = 20, p = 0.7, q = 0.3$

$P(\text{Type II error}) = P(X > 16) = P(X = 17, 18, 19, 20)$

$= {}^{20}C_{17} (.7)^{17} \cdot (.3)^3 + {}^{20}C_{18} (.7)^{18} \cdot (.3)^2 + {}^{20}C_{19} (.7)^{19} \cdot (.3) + (.7)^{20}$

$= 0.107$

$\therefore P(\text{Type II error}) = 0.107$

5. An architect wishes to investigate whether the buildings in a certain city are higher, on average, than buildings in other cities. He takes a large sample of buildings from the city, and find the mean height of the buildings in the sample. He calculates the value of the test statistic, z, and finds that $z = 2.41$.

(a) Explain briefly whether he should use a one-tail or a two tail test. ---[1]

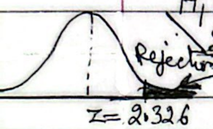
(b) Carry out the test at 1% significance level. ---[3]

M-21/62/Q3

Solution (a) One-tail because investigating whether "higher".

(b) H_0: Population mean (or μ) in city is same as for others.

H_1: Population mean (or μ) in city is greater than for others.



Significance level 1% $\Rightarrow \beta = (1-0.01) = 0.99 \Rightarrow z = \Phi^{-1}(0.99) = 2.326$

Given $z = 2.41 > 2.326$

hence there is evidence that buildings are higher (on average)

6. It is known that 8% of adults in a certain town own Chantor car. After an advertising campaign, a car dealer wishes to investigate whether this proportion has increased. He choses a random sample of 25 adults from the town and notes how many of them own a Chantor car.

(a) He finds 4 of the 25 adults own a Chantor car.
Carry out a hypothesis test at 5% significance level. ---[5]

(b) Explain which of the errors, type I or type II, might have been made in carrying out the test in part (a) ---[2]

Later the car dealer takes another random sample of 25 adults from the town and carries out a similar hypothesis test at 5% significance level.

(c) Find the probability of a type I error. ---[3]

M-21/62/Q6

Solution (a) H_0: Population proportion 8% $\Rightarrow p = 0.08$ (own a Cantor Car)

H_1: Population proportion $p > 0.08$ (increased)

$X \sim B(n, p)$; 4 out of 25 own a Cantor Car; $p = 0.08$, $q = 0.92$, $n = 25$

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - \{P(0) + P(1) + P(2) + P(3)\}$

$= 1 - \{0.92^{25} + {}^{25}C_1 (0.08)^1 (0.92)^{24} + {}^{25}C_2 (0.08)^2 (0.92)^{23} + {}^{25}C_3 (0.08)^3 (0.92)^{22}\}$

$= 0.135$ (3 sf)

$0.135 > 0.05$ (H_0 is accepted)

There is no evidence that proportion owing Cantor Car has increased (H_0 is not rejected) at 5% significance level

(continued)

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Solution:

6(b) H_0 is not rejected, hence Type II error might have been made.

(c) $P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - \{P(X \leq 3) + P(X=4)\}$

$= 1 - \{0.135 + {}^{25}C_4 \times (.08)^4 \times (.92)^{21}\} = 0.0451$

Now $0.0451 < 0.05 \rightarrow H_0$ is rejected, (when H_0 is True for $X \geq 4$)

$P(\text{Type I Error}) = \underline{0.0451}$ ✓

7. Harry has a five-sided spinner with sectors coloured blue, green, red yellow and black. Harry thinks the spinner may be baised. He plans to carry out a hypothesis test with the following hypotheses:
H_0: P(the spinner lands on blue) $= \frac{1}{5}$
H_1: P(the spinner lands on blue) $\neq \frac{1}{5}$
Harry spins the spinner 300 times. It lands on blue on 45 spins.
Use a suitable approximation to carry out Harry's test at 5% significance level. ---[5]

[M-22 | 62 | Q2]

<u>Solution</u>: $B \sim (300, \frac{1}{5}) \rightarrow N(60, 48)$ $\{B(n,p) \rightarrow N(\mu, \sigma^2)$

$n = 300,\ p = \frac{1}{5},\ q = \frac{4}{5}$ $\Rightarrow \{\mu = np = 300 \times \frac{1}{5} = 60$

and $\{$Variance $\sigma^2 = npq = 300 \times \frac{1}{5} \times \frac{4}{5}$

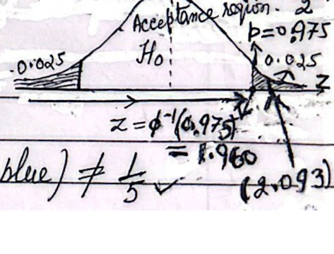
$\sigma^2 = 48$

$P(X \leq 45) = P\left(Z < \frac{45.5 - 60}{\sqrt{48}}\right)$ $\{$continuity correction $\{X \leq 45 \rightarrow 45.5$

$= P(Z < -2.093)$ [for two tailed test ⊗

$= P(Z > 2.093)$ [Significance level $= 5 = 0.05$ ⊗ $(1 - \frac{0.05}{2})$

$z = 2.093 > 1.960$



(Evidence to reject H_0)

There is evidence that P(landing on blue) $\neq \frac{1}{5}$ ✓

8. In the past, the time, in minutes, taken by students to complete a certain challenge had mean 25.5 and standard deviation 5.2. A new challenge is devised and it is expected that students will take, on the average, less than 25.5 minutes to complete this challenge. A random sample of 40 students is chosen and their mean time for the new challenge is found to be 23.7 minutes.

(a) Assuming that the standard deviation of the time for the new challenge is 5.2 minutes, test at the 1% significance level whether the population mean time for the new challenge is less the 25.5 minutes. ...[5]

(b) State, with a reason, whether it is possible that a Type I error was made in the test in part (a). ...[1]

[M-22/62/Q4]

Solution: H_0 (Null hypothesis): $\mu = 25.5$

(a) Alternate hypothesis H_1: $\mu < 25.5$

$$P(\mu < 23.7) = P\left(z < \frac{23.7 - 25.5}{5.2/\sqrt{40}}\right) \quad \left\{\begin{array}{l}\text{for sample } n = 40 \\ \text{variance} = \frac{\sigma^2}{n}\end{array}\right.$$

$$= P(z < -2.189) \otimes$$

$$= 1 - \phi(2.189)$$

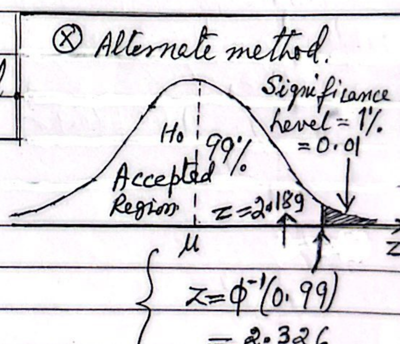
$$= 0.0142$$

Now $0.0142 > 0.01$ $\quad$ [Significance level $1\% = 0.01$]

(Accept H_0) No evidence that the mean time has decreased.

(b) No,
Because H_0 was not rejected.

⊗ Alternate method.



$\left\{\begin{array}{l} z = \phi^{-1}(0.99) \\ = 2.326 \end{array}\right.$

⊗

$$P(z < -2.189)$$
$$= P(z > 2.189)$$

Now $2.189 < 2.326$

hence Accept H_0.

9. The number of accidents per 3-months period on a certain road has the distribution $Po(\lambda)$. In the past the value λ has been 5.7. Following some changes to the road, the council carries out a hypothesis test to determine whether the value of λ has decreased. If there are fewer than 3 accidents in a randomly chosen 3-month period, the council will conclude that the value of λ has decreased.

(a) Find the probability of a Type I error. --[2]

(b) Find the probability of a Type II error if mean number of accidents per 3-months period is now actually 0.9 ---[3]

M-23/62/Q4

Solution Type I error is when H_0 is rejected [$H_0: \lambda = 5.7$]

(a) $P(X<3) = P(0)+P(1)+P(2)$

$$= e^{-5.7}\left(1+5.7+\frac{5.7^2}{2!}\right)$$

$$= 0.0768 \ (3sf)$$

$\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$, $\lambda = 5.7$

(b) Now $\lambda = 0.9$

$P(X<3) = P(0)+P(1)+P(2)$

$$= e^{-0.9}\left(1+0.9+\frac{0.9^2}{2!}\right)$$

Now the P(Type II error) $= 1 - e^{-0.9}\left(1+0.9+\frac{0.9^2}{2!}\right)$

P(H_0 is accepted / when H_0 is false) $= 1 - 0.9372$

$P(X \geq 3)$

$= \underline{0.0629} \ (3sf)$

(as $p = 0.9 > 5.7$)

10. Last year, the mean time taken by students at a school to complete a certain test was 25 minutes. Akash believes that the mean time taken by this year's students was less than 25 minutes. In order to test this belief, he takes a large random sample of this years students and he notes the time taken by each student. He carries out a test, at the 2.5% significance level, for the population mean time μ minutes. Akash uses the null hypothesis $H_0 : \mu = 25$.

(a) Give a reason why Akash should use one-tailed test ---[1]

Akash finds that the value of the test statistic is $z = -2.02$

(b) Explain what conclusion he should draw. [2]

In a different one tailed hypothesis test the z-value was found to be 2.14

(c) Given that this value would lead to a rejection of the null hypothesis at α% significance level, find the set of possible values of α. --[3]

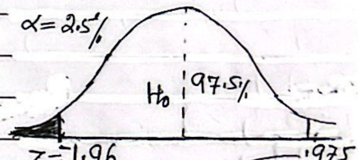
M-23/62/Q6

Solution (a) He is expecting a decrease (in μ).

(b) $z = -2.02 < -1.96$ (at 2.5%)

Rejects H_0.

There is evidence to suggest that this year's (mean) time is less than 25.



$z = 1.96$

(for $p = .975$, $z = 1.96$)

$z = \phi^{-1}(0.975)$

(c) $z = 2.14$

$p = \phi(z) = \phi(2.14) = 0.9834$

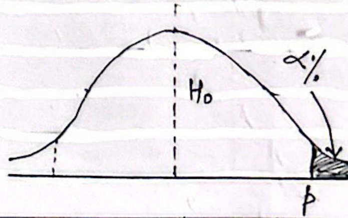
On the right of p.

$P(z > 2.14) = 1 - \phi(z) = 1 - 0.9834$

$= 0.0162$

$= 1.62\%$

$\therefore \alpha \geq 1.62$ (3 sf)



$p = \phi(z)$
$= \phi(2.14)$
$= 0.9834$

Continued for part (d) on next page.

10(d) The population mean time taken by students at another school to complete a test last year was m minutes. Sorin carries out a one-tailed test to determine whether the population mean this year is less than m, using a random sample of 100 students. He assumes that the population standard deviation of the times 3.9 minutes. The sample mean is 24.8 minutes, and this result just leads to the rejection of the null hypothesis at the 5% significance level.

(d) Find the value of m. ---[3]

$\boxed{M-23/62/Q6}$

Solution: $\mu = m$, Variance $= 3.9^2$, $n = 100$, sample var $= \frac{\sigma^2}{n} = \frac{3.9^2}{100}$

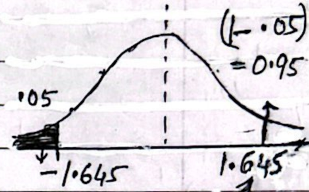
$$\Rightarrow \sigma = \frac{3.9}{10}$$

$$P(24.8 < m) = P\left(Z < \frac{24.8 - m}{\frac{3.9}{10}}\right)$$

$$\Rightarrow \frac{24.8 - m}{0.39} = -1.645$$

$$\Rightarrow m = 25.4 \ (3sf).$$

5% significance level.



$$z = \phi^{-1}(0.95) = 1.645$$

Example 11: In the past the yield of a certain crop, in tonnes per hectare, had mean 0.56 and standard deviation 0.08. Following the introduction of a new fertilizer, the farmer intends to test at the 2.5 significance level whether the mean yield has increased. He finds that the mean yield over 10 years is 0.61 tonnes per hectare.

(a) State two assumptions that are necessary for the test. --[2]

(b) Carry out the test. [S-20/61/Q2] --[5]

Solution (a) (i) Standard deviation unchanged, $\sigma = 0.08$ ✓

(ii) Yields are normally distributed, $N(\mu, \sigma^2)$

(b) Null hypothesis, $H_0: \mu = 0.56$

Alternate hypothesis, $H_1: \mu > 0.56$

$$P(\mu > 0.61) = P\left(Z > \frac{0.61 - 0.56}{\frac{0.08}{\sqrt{10}}}\right)$$

$$= P(Z > 1.976)$$

$1.976 > 1.96$

(Ho is rejected)

$\therefore$ There is evidence that mean yield has increased.

Significance level $= 0.025$ (2.5%)

$P(z > a) = 0.025$

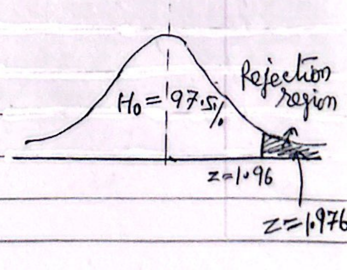
$\Rightarrow 1 - \phi(a) = 0.025$

$P(z < a) = p = \phi(a) = 0.975$

$\Rightarrow z = a = \phi^{-1}(0.975)$

$= 1.96$

or $1.976 > 1.96$



Example 12: A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X.

(a) Show that $Var(X) = 2$ ---[1]

(b) Fiona has another spinner, also with five sides numbered, 1, 2, 3, 4, 5. She suspects that is baised so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2·6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error. ---[4]

(c) State what is meant by a Type II error in this context. ---[1]

$\boxed{S\text{-}20/61/Q4}$

Solution. (a) Mean of 1, 2, 3, 4, 5 $\rightarrow \bar{x} = \frac{1+2+3+4+5}{5} = 3 \Rightarrow \bar{x} = 3$, $n = 5$

Variance $\sigma^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{(1^2+2^2+3^2+4^2+5^2)}{5} - 3^2 = 2$ ✓

$Var(X) = 2$ ✓

(b) Random variabe, X is the score on the spin → Normal distribution:

$N(\mu, \sigma^2) \sim N(3, 2)$

Null hypothesis, H_0, $\mu = 3$

Alternate hypothesis H_1, $\mu < 3$, $n = 40$

$$P(\mu < 2{\cdot}6) = P\left(z < \frac{2{\cdot}6 - 3}{\sqrt{\frac{2}{40}}}\right) \qquad \left\{ Var = \frac{\sigma^2}{n} \right.$$

$$= P(z < -1{\cdot}789)$$

$$= 1 - \phi(1{\cdot}789)$$

$$= 1 - 0{\cdot}9633$$

$P(\text{Type I error}) = 0{\cdot}0367$

(c) Concluding that spinner is unbiased when it is biased.

(H_0 is not false / when H_0 is not true)

Example 13 A shop obtains apples from a certain farm. It has been found that 5% of apples from this farm are 'Grade A'. Following a change in growing conditions at the form, the shop management plan to carry out a hypothesis test to find out whether the proportion of 'Grade A' apples has increased. The select 25 apples at random. The number of 'Grade A' apples is more than 3 they will conclude that the proportion is increased.

(a) State suitable null and alternate hypothesis for the test. --[1]

(b) Find the probability of Type I error. --[3]

In fact 2 of the 25 apples were 'Grade A'.

(c) Which of the errors, type I or type II is possible. Jusify your answer. [S-20/62/Q2] --[2]

Solution: Random variable, X is the number of 'Grade A' apples.

(a) Null hypothesis, $H_0 : p = 0.05$

Alternate hypothesis $H_1 : p > 0.05$

(b) $X \sim B(n,p) \sim B(25, 0.05)$; $n = 25,\ p = 0.05,\ q = 0.95$

$P(X > 3) = 1 - P(X = 0, 1, 2, 3)$

$= 1 - \{ (.95)^{25} + {}^{25}C_1 (.05) \times (.95)^{24} + {}^{25}C_2 (.05)^2 . (.95)^{23} + {}^{25}C_3 (.05)^3 . (.95)^{22}$

$= 1 - .9659 = 0.0341$ ✓

P(Type I Error) = 0.0341

(c) Type II error:

will conclude proportion not increased.

(Type II error: Ho is accepted / whereas Ho is false)

(Type II error: H_0 is accept / Ho is false)

$$Po(\lambda) \rightarrow N(\mu, \sigma^2) \sim N(\lambda, \lambda)$$

Example 14: The number of customers who visit a particular shop between 9.00am and 10.00am has the distribution $Po(\lambda)$. In the past the value of λ was 5.2. Following some new advertising, the manager wishes to test whether the value of λ has increased. He chooses a random sample of 20 days and finds that the total number of customers who visited the shop between 9.00am and 10.00 am on those days is 125.
Use an approximating distribution to test at 2.5% significance level whether the value of λ has increased. ---[6]

$\boxed{\text{S-20/63/Q3}}$

Solution: Random variable: X denotes the number of customers who visit a shop between 9.am and 10:00 am.

$Po(\lambda)$: $\lambda = 5.2$ each day

for 20 day $\rightarrow \lambda = 5.2 \times 20 = 104 \checkmark > 15$, $Po(104)$

$\therefore$ Poisson distribution approximates to normal distribution. $Po(104) \rightarrow N(\mu, \sigma^2) \sim N(104, 104)$ $\left\{ \begin{array}{l} \mu = \lambda = 104 \\ \sigma^2 = \lambda = 104 \end{array} \right.$

Null hypothesis, H_0: $\lambda = 104$
Alternate hypothesis, H_1: $\lambda > 104$

Now $P(X \geq 125) = P\left(z > \dfrac{124.5 - 104}{\sqrt{104}}\right)$ $\left\{ \begin{array}{l} \text{continuity correction} \\ x \geq 125 \rightarrow x > 124.5 \end{array} \right.$

$= P(z > 2.010)$ ⊛

$= 1 - \phi(2.01)$

$= 1 - 0.9778 = 0.0222$

here $0.0222 < 0.025$ (H_0 is rejected) [Significance Level $= 0.025$ (2.5%)]

There is evidence that λ has increased.

⊛ Alternate method

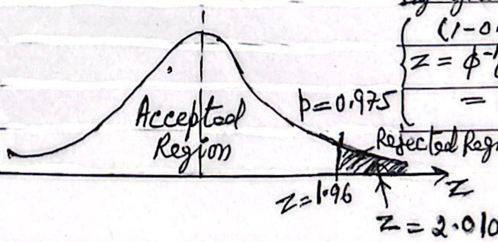
$P(z > 2.010)$

Now $2.010 > 1.96$

There is evidence that λ has increased.

Significance Level $= 0.025$

$\left\{ \begin{array}{l} (1 - 0.025) = 0.975 \\ z = \phi^{-1}(0.975) \\ \quad = 1.960 \end{array} \right.$



Example 15 A market researcher is investigating the length of time that customers spend at an information desk. He plans to choose a sample of 50 customers on a particular day.

(a) He considers choosing the first 50 customers who visit the information desk.
Explain why this method is unsuitable. --[1]

The actual lengths of time, in minutes, that customers spend at the information desk may be assumed to have mean μ and variance 4.8. The researcher knows that in the past the value of μ was 6.0. He wishes to test, at the 2% significance level, whether this is still true. He chooses a random sample of 50 customers and notes how long they each spend at the information desk.

(b) State the probability of making a Type I error and explain what is meant by a Type I error in this context. --[2]

(c) Given that the mean time spent at the information desk by the 50 customers is 6.8 minutes, carry out the test. --[5]

(d) Why to use the Central Limit theorem in your answer to part (c). --[1]

[S-20/63/Q7]

Solution: (a) Later the customers might spend times different from first ones.

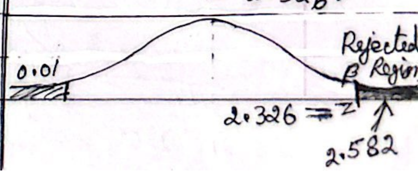
(b) P(Type I error) = 0.02
Concluding that $\mu \neq 6.0$ when actually $\mu = 6.0$, $\frac{\sigma^2}{n} = \frac{4.8}{50}$

(c) Random variable: Length of time.
Null hypothesis, H_0: $\mu = 6.0$
Alternate hypothesis, H_1: $\mu \neq 6.0$

$$P(\mu > 6) = P\left(z > \frac{6.8-6}{\sqrt{\frac{4.8}{50}}}\right)$$
$$= P(z > 2.582)$$

(Lies in the rejected region) $2.582 > 2.326$
$\therefore$ (H_0 is rejected)
or Evidence that $\mu \neq 6.0$

Two Tailed test as $\mu \neq 6.0$
Significance level 2%,
$\beta = \left(1 - \frac{0.02}{2}\right)$
$= 0.99$
$P(z < a) = p = \phi(a) = 0.99$
$z = a = \phi^{-1}(0.99)$
$= 2.326$ ✓



(d) Population distribution unknown.

16. In a game, a ball is thrown and lands in one of 4 slots, labelled A, B, C and D. Raju wishes to test whether the probability that the will land in slot A is $\frac{1}{4}$.

(a) State suitable null and alternative hypotheses for Raju's test. ---[1]

The ball is thrown 100 times and it lands in slot A 15 times.

(b) Use a suitable approximating distribution to carry out the test at the 2% significance level. ---[5]

[S-21/62/Q1]

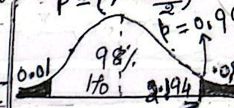
Solution (a) Null hypothesis $H_0 : p = \frac{1}{4}$

Alternative hypothesis $H_1 : p \neq \frac{1}{4}$ (Note: Two tail test)

2% significance level = 0.02

$p = \left(1 - \frac{0.02}{2}\right)$

$p = 0.99$



(b) $n = 100,\ p = \frac{1}{4},\ q = \frac{3}{4} \Rightarrow \mu = np = 100 \times \frac{1}{4} = 25$

$Var,\ \sigma^2 = npq = 100 \times \frac{1}{4} \times \frac{3}{4} = \frac{75}{4}$

$\phi^{-1}(0.99) = z = 2.326$

$X \sim N\left(25, \frac{75}{4}\right)$

$P(X > 15) = P\left(Z > \frac{15.5 - 25}{\sqrt{\frac{75}{4}}}\right) = P(Z > -2.194)$
$= P(Z < 2.194)$

Continuity Correction
$X > 15 \rightarrow X \geq 16$
$\rightarrow X \rightarrow 15.5$

(accepted region) $2.194 < 2.326$

Hence no evidence to reject that the probability is $\frac{1}{4}$ (H_0 is accepted).

17 The time, in minutes, spents by customers at a particular gym has the distribution $N(\mu, 38.2)$. In the past the value of μ has been 42.4. Following the installation of some new equipment the management wishes to test whether the value of μ has changed.

(a) State what is meant by type I error in this context. ---[1]

(b) The mean time for a sample of 20 customers is found to be 45.6 minutes. Test at 2.5% significance level whether the value of μ has changed. ---[5]

[S-21/62/Q5]

Solution (a) Conclude that (population) mean time has changed (or is not 42.4) although μ has not changed (or is still 42.4). → (Ho is rejected / when Ho is True)

(b) H_0: Population mean $\mu = 42.4$

H_1: Population mean $\mu \neq 42.4$ (Two-tail test)

; Now for Sample $n = 20$, $Var = \sqrt{\frac{38.2}{20}}$

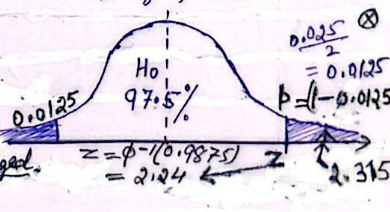
$P(X < 45.6) = P\left(Z < \frac{45.6 - 42.4}{\sqrt{\frac{38.2}{20}}}\right)$

$= P(Z < 2.315)$

2.5% significance level → 0.025

$\frac{0.025}{2} = 0.0125$

$p = (1 - 0.0125)$



$z = \phi^{-1}(0.9875) = 2.24$

(Lies in the rejected region) $2.315 > 2.24$

There is evidence that μ or mean time has changed.

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18. In the past, the time, in hours, for a particuler train journey has had mean 1.40 and standard deviation 0.12. Following the introduction of some new signals, it is required to test whether the mean journey time has decreased,

(a) State what is meant by a Type II error in this context. ---[1]

(b) The mean time for a random sample of 50 journey is found to be 1.36 hours. Assuming that the standard deviation of journey times is still 0.12 hours, test at 2.5% significance level whether the population mean journey time has decreased. ---[5]

(c) State, with a reason, which of the errors, Type I or Type II, might have been made in test in part (b). ---[2]

S-21 | 63 | Q2

Solution: (a) Conclude (mean) (Journey) time has not decreased, when in fact it has.
(Ho is True / when given Ho is false)

(b) H_0: Pop mean, $\mu = 1.4$

H_1: Pop mean, $\mu < 1.4$ — Significance level $= 2.5\%$ or 0.025

Given sample mean $= 1.36$; $n = 50$

Sample var $= \frac{\sigma^2}{n} = \frac{0.12^2}{50}$
or S.D $= \frac{0.12}{\sqrt{50}}$

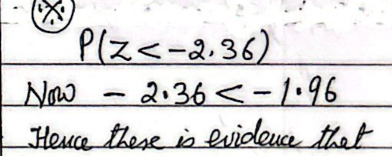
$$P(\mu \le 1.36) = P\left(z < \frac{1.36 - 1.4}{\frac{0.12}{\sqrt{50}}}\right)$$

$$= P(z < -2.36)$$
$$= 1 - \phi(2.36)$$
$$= 0.0092 < 0.025$$

Hence there is evidence that (mean) (Journey) time has decreased. (Ho is rejected).

(c) Ho is rejected.
Type I error.

Alternatively: signif level $= 0.025$



-2.36 $\quad$ $p = 0.975$
$z = \phi^{-1}(0.975) = 1.96$

$P(z < -2.36)$

Now $-2.36 < -1.96$

Hence there is evidence that (mean) (Journey) time has decreased.

19. The local council claims that the average number of accidents per year on a particular road is 0.8. Jane claims that the true average is greater than 0.8. She looks at the records for a random sample of 3 recent years and finds that the total number of accidents during those 3 years was 5.

(a) Assume that the number of accidents per year follows a Poisson distribution.

(i) State null and alternative hypotheses for test of Jane's claim. ---[1]

(ii) Test at 5% significance level whether Jane's claim is justified. ...[4]

(b) Jane finds that the number of accidents per year has been gradually increasing over recent years. [1]

State how this might affect the validity of the test carried in part (a)(ii).

[S-21/63/Q3]

Solution $\lambda = 0.8 \times 3 = 2.4$, $Po(\lambda)$.

(a) (i) Null hypothesis $H_0 : \lambda = 2.4$

Alternative hypothesis $H_1 : \lambda > 2.4$

(ii) $P(X \geq 5) = 1 - \{P(0) + P(1) + P(2) + P(3) + P(4)\}$

$$= 1 - e^{-2.4}\left(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!}\right)$$

$$= 0.0959$$

$0.0959 > 0.05$ [Significance level 5% → 0.05]

There is evidence that Jane's claim not justified.

(b) Mean not constant so Poisson model not valid.

20. Arvind uses an ordinary fair 6-sided die to play a game. He believes he has a system to predict the score when the die is thrown. Before each throw of a die, he writes down what he thinks the score will be. He claims that he can write the correct score more often than he would if he were just guessing. His friend Laxmi tests his claim by asking him to write down the score before each of 15 throws of the die. Arvind writes the corret score on exactly 5 out of 15 throws.
Test Arvind's claim at the 10% significance level. ---[5]

[S-22/61/Q2]

<u>Solution:</u> $H_0 : P(\text{Correct}) = 1/6$, $X \sim B(n, p)$, $p = \frac{1}{6}, q = \frac{5}{6}, n = 15$
$H_1 : P(\text{Correct}) > 1/6$ $B(15, \frac{1}{6})$

$P(X \geq 5) = 1 - P(X = 0, 1, 2, 3, 4)$

$$= 1 - \left\{ \left(\frac{5}{6}\right)^{15} + {}^{15}C_1 \cdot \frac{1}{6} \cdot \left(\frac{5}{6}\right)^{14} + {}^{15}C_2 \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{13} + {}^{15}C_3 \left(\frac{1}{6}\right)^3 \cdot \left(\frac{5}{6}\right)^{12} + {}^{15}C_4 \left(\frac{1}{6}\right)^4 \cdot \left(\frac{5}{6}\right)^{11} \right\}$$

$= 0.0898$

Now $0.0898 < 0.1$ (at 10% Significance level)

(Reject H_0) There is evidence (at 10% level) that Arvind can predict scores.

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21. In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

(a) State suitable null and alternative hypothesis for Jenny' test. ...[1]

(b) Find the probability that Jenny will make a Type I error. ...[3]

(C) Jenny found that the sample mean run was 27.2 minutes.
Explain briefly whether it is possible for her to make a Type-I error or Type II error or both. --[2]

|S-22| 61|Q7|

Solution (a) H_0: population mean run time = 28.2 minute
H_1: population mean run time < 28.2 min.

(b) Sample mean = 27, SD = 4.0, $n=40$, Sample Var = $\frac{\sigma^2}{n} = \frac{4^2}{40}$

$$P(X<27) = P\left(Z < \frac{27-28.2}{4/\sqrt{40}}\right)$$
$$= P(Z < -1.897)$$
$$= 1 - \phi(1.897)$$
$$= 0.0289 \text{ (3 s.f)}$$

(C) For sample mean 27.2 m.
Ho is not rejected, so
Type II error can be made and Type I error cannot be made.

22.(a) A javelin thrower noted the lengths of a random sample of 50 of her throws. The sample mean was 72.3m and an unbaised estimate of the population variance was 64.3 m². Find a 92% confidence interval for the population mean length of throws by this athele. ---[3]

(b) A discuss thrower wishes to calculate a 92% confidence interval for the population mean length of his throws. He bases his calculation on his first 50 throws in a week. Comment on this method. ---[1]

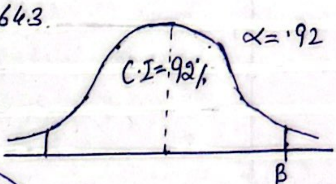
S-22/62/Q1

Solution: Confidence Interval 92% = 0.92, $\mu = 72.3$, $\sigma^2 = 64.3$

$$\mu \pm z\sqrt{\frac{\sigma^2}{n}} \quad \{n = 50$$

$$= 72.3 \pm 1.751 \cdot \sqrt{\frac{64.3}{50}}$$

CI is 70.3 to 74.3m (3sf)



$z = \phi^{-1}(\beta)$
$= \phi^{-1}(0.96)$
$z = 1.751$

$\beta = \alpha + \frac{(1-\alpha)}{2}$
$= .92 + .04$
$= 0.96$

(b) Not random sample.

23. In the past, the mean height of plants of a particular species has been 2.3m. A random sample of 60 plants of this species was treated with fertiliser and the mean height of these 60 plants was found to be 2.4m. Assume that the standard deviation of the heights of plants treated with fertiliser is 0.4 m. Carry out a test at 2.5% significance level of whether the mean height of plants treated with fertiliser is greater than 2.3m. ---[5]

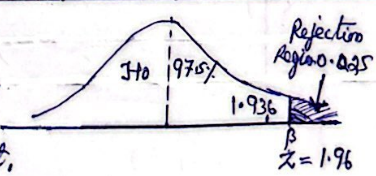
S-22/62/Q2

Solution: H_0: Pop mean height = 2.3
H_1: Pop mean heigh > 2.3 } $n = 60$, $\sigma = 0.4$

$$P(h \geq 2.4) = P\left(z > \frac{2.4 - 2.3}{0.4/\sqrt{60}}\right)$$
$$= P(z > 1.936)$$

$1.936 < 1.96$

Significance level 2.5% = 0.025
$\beta = 1 - 0.025 = 0.975$
$z = \phi^{-1}(0.975) = 1.960$ ✓



(Ho is not rejected) No evidence that (mean) height (with fertiliser) is more than without.

24. The number of cars arriving at a certain road junction on a weekday morning has a Poisson distribution with mean 4.6 per minute. Traffic lights are installed at the junction and a council officer wishes to test at the 2% significance level whether there are now fewer cars arriving. He notes the number of cars arriving during a randomly chosen 2-minute period.

(a) State suitable null and alternative hypothesis for the test. ---[1]

(b) Find the critical region for the test. ---[4]

The officer notes that, during a randomly chosen 2-minute period on a weekday morning, exactly 5 cars arrive at the junction.

(c) Carry out the test. ---[2]

(d) State, with a reason, whether it is possible that a Type I error has been made in carring out the test in part (c). ---[1]

The number of cars arriving at another junction on a weekday morning also has a Poisson distribution with mean 4.6 per minute.

(e) Use a suitable approximating distribution to find the probability that more than 300 cars will arrive at this junction in an hour. [3]

S-22/62/Q4

Solution (a) H_0: Pop. mean $= 4.6$ (or 9.2 for 2-min period)

H_1: Pop mean < 4.6 (or 9.2 for 2 min period)

(b) using $\lambda = 4.6 \times 2 = 9.2$ (for 2 min period) ; Significance Level $= 2\% = .02$

$P(X \leq 3) = e^{-9.2}\left(1 + 9.2 + \frac{9.2^2}{2} + \frac{9.2^3}{3!}\right) = 0.0184 < 0.02$

$P(X \leq 4) = 0.0184 + e^{-9.2} \times \frac{9.2^4}{4!} = 0.0486 > 0.02$

$\therefore$ The critical region is $X \leq 3$ ✓

(c) 5 is not in the critical region or $P(X \leq 5) = 0.104 > 0.02$

So [not reject H_0], no evidence that the number of cars arriving is now fewer.

(d) No, because H_0 was not rejected.

(e) value of λ for one hour (60 min) Period: $\lambda = 60 \times 4.6 = 276 = \mu$

$\sigma^2 = \lambda = 276$

Now distribution $X \sim N(276, 276)$

$P(X > 300) = P\left(Z > \frac{300.5 - 276}{\sqrt{276}}\right)$

$= P(Z > 1.475)$

$= 1 - \phi(1.475) = \underline{0.0701}$ (3 sf)

continuity correction
$X > 300 \rightarrow X \geq 301$
$X = 300.5$

25. Anton believes that 10% of students his college are left-handed. Aliya believes that this is an under-estimate. She plans to carry out a hypothesis test of the null hypothesis $p = 0.1$ against alternative hypothesis $p > 0.1$, where p is the actual proportion of students at the college that are left-handed. She chooses a random sample of 20 students from the college. She will reject the null hypothesis if atleast 5 of these students are left-handed.

(a) Explain what is meant by a Type-I error in this context. ---[1]

(b) Find the probability of a Type-I error in the test.

(c) Given that the true value of p is 0.3, find the probability of a Type II error in the test.

S-22/63/Q2

Solution (a) Conclude more than 10% of the students are left-handed when this is not true.

(b) $P[\text{Type-I error}] = P(X \geq 5) = 1 - \{0.9^{20} + 20 \times 0.9^{19} \times 0.1 + {}^{20}C_2 \times 0.9^{18} \times 0.1^2 + {}^{20}C_3 \times 0.9^{17} \times 0.1^3 + {}^{20}C_4 \times 0.9^{16} \times 0.1^4\}$

$[X \sim B(n, p)\ (n = 20, p = 0.1, q = 0.9)]$

$= 0.0432$ (3 sf)

(c) $P[\text{Type-II error}] = P(X \leq 4)$ for $B(n, p)$, $n = 20$, $p = 0.3$, $q = 0.7$

$P(X < 5) = P(0, 1, 2, 3, 4) = \{0.7^{20} + 20 \times 0.7^{19} \times 0.3 + {}^{20}C_2 \times 0.7^{18} \times 0.3^2 + {}^{20}C_3 \times 0.7^{17} \times 0.3^3 + {}^{20}C_4 \times 0.7^{16} \times 0.3^4\}$

$= 0.238$ (3 sf)

$P(\text{Type II- error}) = P(\text{accept } H_0 \mid \text{when } H_0 \text{ is false})$

$(p = 0.3 > 0.1$ Given$)$

26. Batteries of Type A are known to have a mean life of 150 hours. It is required to test whether a new type of battery, type B, has a shorter mean life than type A batteries.

(a) Give a reason for using a sample rather than the whole population in carring out this test. ---[1]

A random sample of 120 type B batteries are tested and it is found that their mean life is 147 hours, and an unbaised estimate of the population variance is 225 hours².

(b) Test, at 2% significance level, whether type B batteries have a shorter mean life than type A batteries. ---[5]

(c) Calculate a 94% confidence interval for the population mean life of type B batteries. ---[3]

[S-22/63/Q3]

Solution (a) Batteries unusable after testing (or Population too big) (or too costly) or too time consuming to use the whole population.

(b) $H_0 : \mu = 150$
$H_1 : \mu < 150$

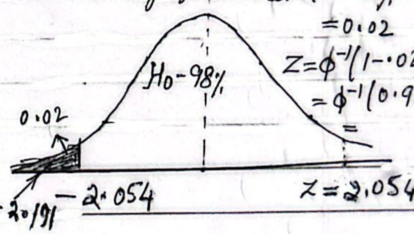
Sample mean $= 147$, $n = 120$
Est $\sigma^2 = 225$
Sample var $= \frac{\sigma^2}{n} = \frac{225}{120}$

$$P(X < 147) = P\left(Z < \frac{147-150}{\sqrt{225/120}}\right)$$
$$= P(Z < -2.191)$$

Significance Level $= 2\%$
$= 0.02$
$Z = \phi^{-1}(1 - .02)$
$= \phi^{-1}(0.98)$
$=$

Here $-2.191 < -2.054$

(Reject H_0) There is evidence that the (mean) life of Type B batteries is less than type A. (or less than 150)



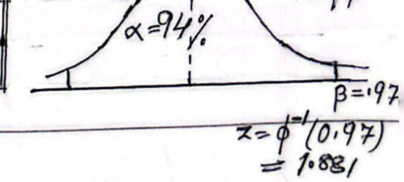
(c) C.I   Popn. Unbaised Est Var $= 225$, $n = 120$ 94% Confidence interval

$\mu \pm z \cdot \frac{\sigma}{\sqrt{n}}$ Sample Var $= \frac{225}{120}$; SD $= \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{120}}$

$147 \pm 1.881 \times \frac{15}{\sqrt{120}}$

C.I is 144 to 150 (3 sf)

$\beta = \alpha + \left(\frac{1-\alpha}{2}\right)$
$= .94 + \left(\frac{1-.94}{2}\right)$
$= 0.97$



$z = \phi^{-1}(0.97)$
$= 1.881$

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27 In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes. Stating a necessary assumption, Carry out the test at 1% significance level. ...[6]

| S-23 | 61 | Q3 |

<u>Solution:</u> Assum standard deviation $\sigma = 5.2$

$H_0 : \mu = 24.0$ and $H_1 : \mu > 2.4$; Sample $n = 50$

$\text{Var} = \frac{\sigma^2}{n} = \frac{5.2^2}{50}$

Sample $\Rightarrow SD = \frac{5.2}{\sqrt{50}}$

$P(\mu \geqslant 25.8) = P\left(z > \frac{25.8 - 24}{5.2/\sqrt{50}}\right)$

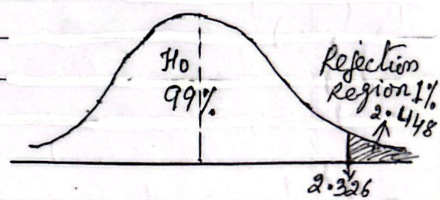
$= P(z > 2.448)$

at 1% Significance Level $= 0.1$

$z = \phi^{-1}(0.99) = 2.326$

$2.448 > 2.326$

(Reject H_0), There is no evidence that (mean) amount of wheat is greater.



28. The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1·9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. ---[1]

(b) Find the critical region for the test and state the probability of a Type I error. ---[6]

(c) State what is meant by a Type I error in this context. ---[1]

(d) During the 4 randomly chosen weeks there are a total of 3 accidents. State the conclusion that the manager should reach. Give a reason for your answer. ---[2]

(e) Assuming that the mean remain 1·9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. ---[4]

S-23|61|Q7

Solution: $\lambda = 1{\cdot}9 \times 4 = 7{\cdot}6$ (for 4-weeks Period)

(a) $H_0: \lambda = 7{\cdot}6$; $H_1: \lambda < 7{\cdot}6$

(b) Mean $\lambda = 7{\cdot}6$

$P(X \leq 2) = e^{-7.6} \times \left(1 + 7{\cdot}6 + \frac{7{\cdot}6^2}{2}\right) = 0{\cdot}0188$

$P(X \leq 3) = e^{-7.6} \times \left(1 + 7{\cdot}6 + \frac{7{\cdot}6^2}{2!} + \frac{7{\cdot}6^3}{3!}\right) = 0{\cdot}0554$

at 5% significance level, $0{\cdot}0188 < 0{\cdot}05$ and $0{\cdot}0554 > 0{\cdot}05 \Rightarrow$ Critical region is $X \leq 2$

(c) $P(\text{Type I error}) = P(X \leq 2) = 0{\cdot}0188$,

Concluding that the (mean) number of accidents has reduced when it has not.

(d) $X = 3$ is not in the critical region, No evidence that mean number of accidents has decreased.

(e) for 52 week-period $\lambda = 52 \times 1{\cdot}9$ or $\lambda = 98{\cdot}8$

for $n = 100$, $Po(\lambda)$ approches to $N(\mu, \sigma^2)$

$N(98{\cdot}8, 98{\cdot}8)$

$P(X > 100) = P\left(Z > \frac{100{\cdot}5 - 98{\cdot}8}{\sqrt{98{\cdot}8}}\right)$

$= P(Z > 0{\cdot}171)$

$= 1 - \phi(0{\cdot}171)$

$= 0{\cdot}432$ (3 sf)

continuity correction: $X > 100 \rightarrow X \geq 101$, $X \rightarrow 100{\cdot}5$

29. The masses, in kilograms, of new born babies in Country A are represented by the random variable X, with mean μ and variance σ^2. The masses of a random sample of 500 newborn babies in this country were found and the results are: $n=500$, $\Sigma x = 1625$, $\Sigma x^2 = 5663.5$

(a) Calculate unbiased estimates of μ and σ^2. ---[3]

A researcher wishes to test whether the mean mass of new born babies in a neighbouring country, B, is different from that in country A. He chooses a random sample of 60 newborn babies in Country B, and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part (a) are correct values for μ and σ^2. Assume also that the variance of the masses of newborn babies in country B is the same as in country A.

(b) Carry out the test at the 1% significance level. ---[5]

[S-23/62/Q3]

Solution (a) $n = 500$, $\Sigma x = 1625$, $\Sigma x^2 = 5663.5$

$$Est(\mu) = \frac{\Sigma x}{n} = \frac{1625}{500} = \underline{3.25} ✓$$

$$Est(\sigma^2) = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2\right) = \frac{500}{499}\left(\frac{5663.5}{500} - (3.25)^2\right)$$

$$= \underline{0.766\ (3sf)} ✓$$

(b) H_0: Population mean $\mu = 3.25$

H_1: Pop. mean $\mu \neq 3.25$ (Two-tailed test)

$n = 60$; Sample mean $= 2.95$, sample variance $= \frac{\sigma^2}{n} = \frac{0.766}{60}$

$$P(X < 2.95) = P\left(Z < \frac{2.95 - 3.25}{\sqrt{0.766/60}}\right)$$

$$= P(Z < -2.655)$$

Now $-2.655 < -2.576$

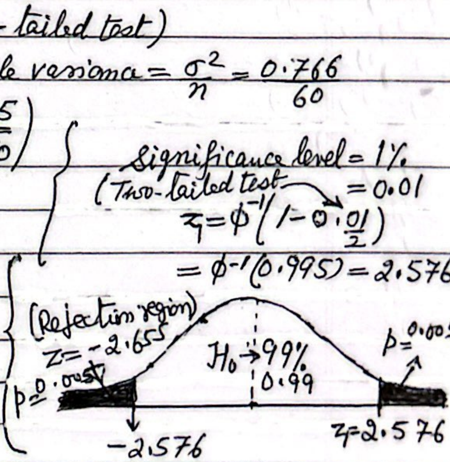
[Reject H_0]

There is evidence that (mean) mass in (Country B) is different. (from country A).

Significance level = 1% (Two-tailed test) $= 0.01$

$z = \phi^{-1}\left(1 - \frac{0.01}{2}\right)$

$= \phi^{-1}(0.995) = 2.576$



30. When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $1/3$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $1/3$.

(a) Use a binomial distribution to find the probability of a Type I error. ---[2]

The true probability of being awarded a medal is denoted by p.

(b) Given that the probability of Type II error 0.8926, find the value of p. ---[3]

S-23/62/Q6

Solution: Prob. of awarding a medal $p = 1/3$, $q = 1 - 1/3 = 2/3$, $n = 10$

(a) $P(\text{Type-I error}) = P(\text{no medal})$ { $X \sim$ Binomial distribution $B(n, p) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$= q^{n}$

$= (2/3)^{10} = \underline{0.0173}$ (3 sf)

(b) $P(\text{Type II error})$

$= P(\text{accept } H_0) = 1 - q^{n}$ $\quad B(10, p),\ q = 1 - p$

$= 1 - (1-p)^{10} = 0.8926$ (given)

$\Rightarrow (1-p)^{10} = 1 - 0.8926 = 0.1074$

$\Rightarrow 1 - p = (0.1074)^{1/10} = 0.800$ ✓

$\Rightarrow p = 1 - 0.800$

$= \underline{0.200}$ (3 sf)

31. Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as: $n=50$, $\Sigma t = 1700$, $\Sigma t^2 = 59050$

(a) Find the unbiased estimates of the population mean and variance. ...[3]

(b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. ...[5]

(c) Under what circumstances would not be necessary to use the Central Limit Theorem in answering (b)? ...[1]

S-23/63/Q5

Solution (a) Est mean $= \frac{\Sigma x}{n} = \frac{1700}{50} = \underline{34}$ ✓

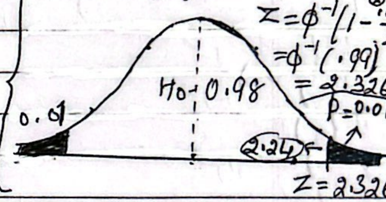
$$Est(\sigma^2) = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right) = \frac{50}{49}\left(\frac{59050}{50} - 34\right)^2 = \underline{25.5} \text{ (3 sf)}$$

(b) H_0: Population mean time $= 32.4$
H_1: Pop. mean time $\neq 32.4$ (Two-tailed test)

$$P(X < 34) = P\left(z < \frac{34 - 32.4}{\sqrt{25.5/50}}\right)$$
$$= P(z < 2.24)$$

at 2% significance level $= 0.02$

$z = \phi^{-1}\left(1 - \frac{0.02}{2}\right)$
$= \phi^{-1}(.99)$
$= 2.326$



Now $2.24 < 2.326$
(Not rejected H_0) Insufficient evidence that (mean) time has changed.

(c) Distribution of times in the population is normal.

32. A new light was installed on a certain footpath. A town councillor decided to a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path any 20-minute period during evening was 1.01.

After light was installed, the total number, n, of people using the path during 3 randomly chosen 20-min periods during the evening was noted.

(a) Given that the value of n was 6, use a Poisson distribution to carry out the test at 5% significance level. ---[6]

(b) Later a similar test, at 5% significance level, was carried out using 3 randomly chosen 20-mintes period during the eving. Find the probabity of a Type-I error. ---[2]

(c) State what is meant by a Type I error in this context. --[1]

(d) State, in content, what further information would be needed in order to find the probability of a Type-II error. Do not carry out any calculation [S-23/63/Q8] ---[2]

Solution: For 20-minutes-3 periods, $\lambda = 3 \times 1.01 = 3.03$

(a) H_0: Pop. mean no. people $\lambda = 3.03$

H_1: Pop. mean no of people $\lambda > 3.03$, using $P_0(3.03)$, $n = 6$

$$P(n \geq 6) = 1 - e^{-3.03}\left[1 + 3.03 + \frac{3.03^2}{2!} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!}\right] = 0.870 \text{ (3sf)}$$

here $0.0870 > 0.05$ [5% Significance Level]

Hence (Do not reject H_0) Insuffisient evidence to believe (mean) number of people have increased.

(b) P(Type-I error)

$$= P(X \geq 7) = 1 - e^{-3.03}\left(1 + 3.03 + \frac{3.03^2}{2!} + \dots + \frac{3.03^6}{6!}\right) = \underline{0.0350} < 0.05$$

(Reject H_0)

(c) Conclude that the (mean) number of people (using the path per 20-min period in the evening) has increased when it was not.

(d) A value for true mean.

Number of people using the path per 20-min in the evening.

33. The number of absences per week by workers at a factory has the distribution $Po(2\cdot1)$

(a) Find the standard deviation of the number of absences per week. ---[1]

(b) Find the prob. that the number of absences in a 2-week period is at least 2. --[3]

(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. ---[2]

Following a change in working conditions, the management wished to test whether the mean number of absences has decreased. They found that, in a randomly chosen 3-week period, there were exactly 2 absences.

(d) Carry out the test at the 10% significance level. ---[5]

(e) State, with a reason, which of the errors, Type I or Type II, might have been made in carrying out the test in part (d). ---[2]

W-20/61/Q5

Solution (a) $Po(2\cdot1) \Rightarrow \lambda = 2\cdot1$ $\left\{ Po(\lambda) \sim N(\mu, \sigma^2) = N(\lambda, \lambda) \Rightarrow \sigma^2 = \lambda \Rightarrow S\cdot D = \sqrt{\lambda} \right.$

Hence Standard deviation, $\sigma = \sqrt{\lambda} = \sqrt{2\cdot1} = 1\cdot45$ ✓

(b) Absence per week $\lambda = 2\cdot1 \Rightarrow$ for 2-weeks period $\lambda = 2\cdot1 \times 2 = 4\cdot2$

$P(X \geq 2) = 1 - P(X=0, X=1) = 1 - e^{-4\cdot2}[1 + 4\cdot2] = 0\cdot923$ ✓

(c) In 3-week period $\lambda = 2\cdot1 \times 3 = 6\cdot3$ $\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

$P(4 < X < 8) = P(X = 5, 6, 7)$

$$= e^{-6\cdot3}\left[\frac{6\cdot3^5}{5!} + \frac{6\cdot3^6}{6!} + \frac{6\cdot3^7}{7!}\right] = 0\cdot0455 \checkmark$$

(d) Null hypothesis, $H_0: \lambda = 6\cdot3$ (3-week period)

Alternate hypothesis $H_1: \lambda < 6\cdot3$ (Significance Level 10%)

$$P(X \leq 2) = e^{-6\cdot3}\left[1 + 6\cdot3 + \frac{6\cdot3^2}{2!}\right] = 0\cdot0498 < 0\cdot1$$

(H_0 is rejected) ⊗

hence there is evidence that the mean number of absences has decreased.

(e) ⊗ H_0 is rejected, hence Type I error posible ✓

34. The time, in minutes, for Anjan's journey to work on Mondays has mean 38.4 and standard deviation 6.9.

(a) Find the probability that Anjan's mean journey's time for a random sample of 30 Mondays is between 38 and 40 minutes.

Anjan wishes to test whether his mean journey time is different on Tuesdays. He chooses a random sample of 30 Tuesdays and finds that his mean journey time for these 30 Tuesdays is 40.2 minutes. Assume that the standard deviation for his journey time on Tuesdays is 6.9 minutes. --- [1]

(b) (i) State, with a reason, whether Anjan should use a one-tail or two-tail test.

(ii) Carry out the test at 10% significance level. --- [5]

(iii) Explain whether it was necessary to use the Central Limit Theorem in part (b)(ii).

W-20/ 61/Q6

Solution: Random sample $n = 30$, $(\sigma = 6.9)$; Variance of sample $= \frac{\sigma^2}{n} = \frac{6.9^2}{30}$

(a) $P(38 < X < 40) = P\left(\frac{38-38.4}{6.9/\sqrt{30}} < Z < \frac{40-38.4}{6.9/\sqrt{30}}\right)$

$= P(-0.3175 < Z < 1.270) = \phi(1.270) - \phi(-0.3175)$

$= \phi(1.270) - (1-\phi(0.3175)) = \underline{0.523}$ (3 sf)

(b) (i) 2-tail because looking for "change". (not decrease or increase)

(ii) H_0: Population mean journey time $\mu = 38.4$

H_1: Population mean journey time $\mu \neq 38.4$ (Two-tails test); Significance level $10\% = 0.1$

$P(X < 40.2) = P\left(Z < \frac{40.2-38.4}{6.9/\sqrt{30}}\right)$ Sample Var $= \frac{6.9^2}{30}$

$= P(Z < 1.429)$

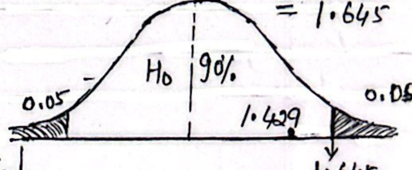
$z = \phi^{-1}\left(1 - \frac{0.1}{2}\right)$

$= \phi^{-1}(0.95)$

$= 1.645$

Now $1.429 < 1.645$

Hence there is no evidence that mean journey time has changed (H_0 is accepted)



(iii) Yes, because population distribution unknown.

35 The areas, $X\ cm^2$, of petals of a certain kind of flower have mean $\mu\ cm^2$. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized as: $\Sigma x = 1850$; $\Sigma x^2 = 17850$. Test the botanist's claim at the 2.5% significance level. ---[8]

[W-20/62/Q4]

Solution: $\Sigma x = 1850$, $\Sigma x^2 = 17850$, $n = 200$

$$est(\mu) = \frac{\Sigma x}{n} = \frac{1850}{200} = 9.25.$$

$$est(\sigma^2) = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right)$$

$$= \frac{200}{199}\left(\frac{17850}{200} - (9.25)^2\right)$$

$$= \underline{3.71} \quad (\text{or } 3.706)$$

; Sample variance $= \frac{\sigma^2}{n} = \frac{3.706}{200}$

Now $H_0: \mu = 8.9$

$H_1: \mu \neq 8.9$ (Two-tailed test)

$$P(\mu < 9.25) = P\left(z < \frac{9.25 - 8.9}{\sqrt{3.706/200}}\right)$$

$$= P(z < 2.57)$$

Significance level $2.5\% = 0.025$

$$z = \phi^{-1}\left(1 - \frac{0.025}{2}\right)$$

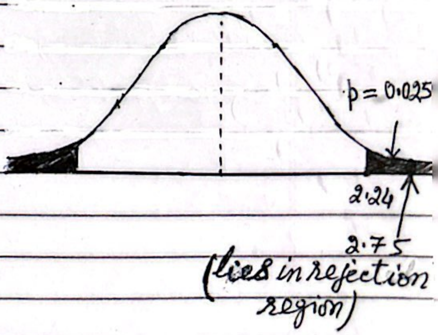
$$= \phi^{-1}(1 - 0.0125)$$

$$= \phi^{-1}(0.9875)$$

$$= 2.24$$

Now $2.57 > 2.24$

(Reject H_0) There is evidence that $\mu \neq 8.9$



36 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 these packets contain the prize offer.

(a) Carry out the test at 10% significance level. ---[5]

(b) Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance level at 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she find that there are 3 or fewer packets containing the prize offer.

Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. ---[3]

(c) Explain what is meant by a type II error in this context. ---[1]

W-20/62/Q6

Solution: H_0: P(Contain offer) $= \frac{1}{3}$

(a) H_1: P(Contains offer) $< \frac{1}{3}$

$X \sim B(n, p) \rightarrow B(20, \frac{1}{3}),\ p = \frac{1}{3},\ q = \frac{2}{3},\ n = 20$, Significance level 10% $= 0.1$

$P(X \leq 2) = P(0,1,2) = \left(\frac{2}{3}\right)^{20} + 20\left(\frac{2}{3}\right)^{19}\cdot\left(\frac{1}{3}\right) + {}^{20}C_2\left(\frac{2}{3}\right)^{18}\cdot\left(\frac{1}{3}\right)^2 = 0.0176$ (3sf)

Here $0.0176 < 0.1 \Rightarrow$ (Reject H_0) No evidence (at 10% level) to support manufacturer's claim.

(b) $p = \frac{1}{7}$; $q = \frac{6}{7}$, $n = 20$ and P(H_0 is rejected) $= P(X \leq 3)$

⊗ P(Type II error) $= 1 - P(X \leq 3) = 1 - P(X = 0,1,2,3)$
or $P(X > 3)$

$= 1 - \left\{\left(\frac{6}{7}\right)^{20} + 20\left(\frac{6}{7}\right)^{19}\cdot\left(\frac{1}{7}\right) + {}^{20}C_2\left(\frac{6}{7}\right)^{18}\cdot\left(\frac{1}{7}\right)^2 + {}^{20}C_3\left(\frac{6}{7}\right)^{17}\cdot\left(\frac{1}{7}\right)^3\right\}$

$= \underline{0.318}$ (3 sf)

(c) Concluding that proportion is 1 in 3, when it is actually less (1 in 7).

⊗ P(Type II error) = P(Accept H_0 / where H_0 is false)
(as $\frac{1}{7} < \frac{1}{3}$)

37. (a) The proportion of people having a particular medical condition is 1 in 100000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. ---[2]

(ii) Use the approximating distribution to calculate $P(X>0)$ ---[2]

(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the research's suspicion at the 2% significance level. [W-21/61/Q5] --[5]

Solution (a) $p = 1/100\,000$, $n = 2500 \Rightarrow$ mean $\lambda = np = 2500 \times \frac{1}{100\,000} = 0.025$

(i) here $n > 50$ and $np = 0.025 < 5$

$\therefore$ Poisson distribution. $Po(0.025)$ $\{ Po(\lambda)$

(ii) $P(X>0) = 1 - P(0) = 1 - e^{-\lambda} = \underline{(1 - e^{-0.025})}$ ✓

(b) Null hypothesis $H_0 : p = 0.3$ (30%)

Alternate hypothesis $H_1 : p < 0.3$

Now 4 out of 28 ; $p = 0.3, q = 0.7$

$P(X \leq 4) = P(X = 0, 1, 2, 3, 4)$

$= 0.7^{28} + 28 \times 0.3 \times 0.7^{27} + {}^{28}C_2 \times 0.3^2 \times 0.7^{26} + {}^{28}C_3 \times 0.3^3 \times 0.7^{25} + {}^{28}C_4 \times 0.3^4 \times 0.7^{24}$

$= 0.0474 > 0.02$ Significance level (2%)

$\therefore$ H_0 is not rejected.

or No evidence that suspician is true.

38. The masses, in grams, of apples from a certain farm have mean μ and standard deviation 5.2. The farmer says that the value of μ is 64.6. A quality control inspector claims that the value of μ is actually less than 64.6. In order to test his claim he chooses a random sample of 100 apples from the farm.

(a) The mean mass of 100 apples is found to be 63.5.
Carry out the test at 2.5% significance level. ---[5]

(b) Later another test of the same hypothesis at 2.5% significance level, with another random sample of 100 apples from the same farm, is carried out. Given that the value of μ is in fact 62.7, Calculate the probability of a Type II error. ---[5]

W-21/61/Q7

Solution: $H_0: \mu = 64.6$

$H_1: \mu < 64.6$

Sample mean $= 64.6$, $n = 100$

Sample Var $= \frac{\sigma^2}{n} = \frac{5.2^2}{100}$

$P(\mu \leq 63.5) = P\left(z < \frac{63.5 - 64.6}{5.2/\sqrt{100}}\right)$

$= P(z < -2.115)$

Significance level $= 2.5\% = 0.025$

$z = \phi^{-1}(1 - 0.025)$

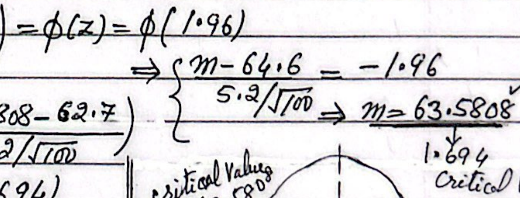
$z = \phi^{-1}(0.975) = 1.96$

Now

$-2.115 < -1.96$

(do not accept H_0)

There is evidence that $\mu < 64.6$



(b) $P(\mu < m) = P\left(z < \frac{m - 64.6}{5.2/\sqrt{100}}\right) = \phi(z) = \phi(1.96)$

$\Rightarrow \frac{m - 64.6}{5.2/\sqrt{100}} = -1.96 \Rightarrow m = 63.5808$

(Now given $\mu = 62.7$)

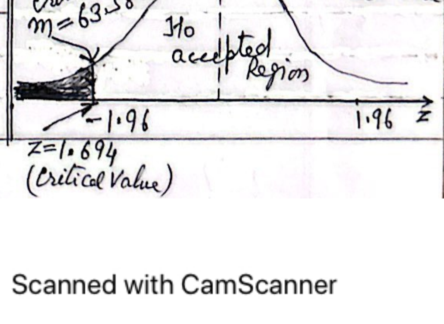
$P(z < 63.5808) = P\left(z < \frac{63.5808 - 62.7}{5.2/\sqrt{100}}\right)$

P(Rejected region) $= P(z < 1.694)$

1.694 Critical Value

Now P(Type II Error) $= 1 - P(z > 1.694)$

$= 1 - \phi(1.694) = 0.0451$



39. A certain kind of firework is supposed to last for 30 seconds, on average, after it is lit. An inspector suspects that the fireworks actually last a shorter time than this, on average. He takes a random sample of 100 fireworks of this kind. Each firework in the sample is lit and the time it lasts is noted.

(a) Give a reason why it is necessary to take a sample rather than testing all the fireworks of this kind. ---[1]

It is given that the population standard deviation of the times that fireworks of this kind last is 5 seconds.

(b) The mean time lasted by 100 fireworks in the sample is found to be 29 seconds. Test the inspector's suspicion at the 1% significance level. ---[5]

(c) State with a reason whether the Central Limit Theorem was needed in the solution to part (b) ---[1]

W-21	62	Q4

Solution (a) Fireworks are destroyed when tested.

(b) Pop. $\sigma = 5$, $n = 100$, for sample mean $= 29$, sample SD $= \frac{\sigma}{\sqrt{n}} = \frac{5}{\sqrt{100}}$

$P(t \leq 29) = P\left(z < \frac{29-30}{5/\sqrt{100}}\right)$

Significance level 1% = 0.01

$= P(z < -2)$

$z = \phi^{-1}(1-.01)$

$= \phi^{-1}(0.99)$

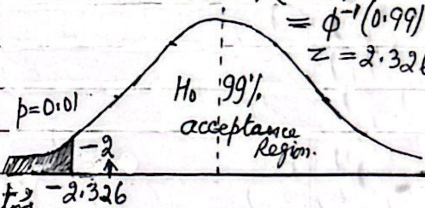
$z = 2.326$

Now $-2 > -2.326$

(Do not reject H_0)

There is not enough evidence that mean time lasted is less than 30 seconds.

(or Not enough evidence to support the inspector's suspicion.



(c) Yes, Because population distribution is unknown.

40. A machine is supposed to produced random digits. Bob thinks that the machine is not fair and that the probability of it producing the digit 0 is less than $1/10$. In order to test his suspicion he notes the number of times the digit 0 occurs in 30 digits produced by the machine. He carries out a test at the 10% significance level.

(a) State suitable null and alternative hypothesis. ---[1]

(b) Find the rejection region for the test. ---[4]

(c) State the Type I error. ---[1]

It is now given that the machine actually produces a '0' once in every 40 digits, on average.

(d) Find the probability of a type II error. ---[3]

(e) Explain the meaning of a type II error in this context. ---[1]

[W-21 / 62 / Q6]

Solution (a) $H_0: P(0) = 1/10$; $H_1: P(0) < 1/10$,

(b) For $X \sim B(n,p) \to B(30, 0.1)$ $\quad p = 1/10 = 0.1,\ q = 0.9$

$P(X=0) = (0.9)^{30} = 0.0424 < 0.1$ {Significance level $= 10\%$ or 0.1}

lies in the rejection region.

$P(X \leq 1) = P(X = 0 \text{ or } 1) = (0.9)^{30} + 30 \times 0.9^{29} \times 0.1 = 0.184 > 0.1$ Lies in Accepted Region

"Hence Rejection region is '0' zeros."

(c) $P(\text{Type I error}) = P(H_0 \text{ is rejected}) = 0.0424$ (for $X = 0$)

(d) $P(\text{Type II error}) = P(H_0 \text{ is accepted})$

$\Rightarrow P(X > 0) = 1 - P(X = 0) = 1 - (0.975)^{30}$

$= 0.532$ (3dp)

$\left\{ \text{Given } P(X=0) = \frac{1}{40} = p = 0.025,\quad q = 1 - 0.025 = 0.975 \right.$

(e) Not concluding that the probability is less than $1/10$, when in fact it is.

(wrong accept H_0 / when H_0 is false)

41. A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Sushma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at 5% significance level. ---[5]

Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. ---[3]

W-22/61/Q2

Solution: $H_0: P(\text{red}) = 0.2$ $\quad \{ X \sim B(n,p),\ B(40, 0.2);\ p = \frac{1}{5} = 0.2$

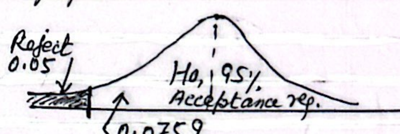
(a) $H_1: P(\text{red}) < 0.2$ $\quad q = 0.8$

$P(X \leq 4) = P(X = 0,1,2,3,4) = 0.8^{40} + 40 \times 0.8^{39} \times 0.2 + {}^{40}C_2 \times 0.8^{38} \times (0.2)^2 + {}^{40}C_3 (0.8)^{37} \times (0.2)^3 + {}^{40}C_4 (0.8)^{36} \cdot (0.2)^4 = 0.0759$

Now $0.0759 > 0.05$

(Do not reject H_0) Not enough evidence that it lands on red fewer times than if it were fair (Not biased spinner)

Significance level 5% = 0.05



(b) $P(X \leq 3) = P(0,1,2,3,4) - P(X=4)$

$= 0.0759 - {}^{40}C_4 (0.8)^{36} \times (0.2)^4$

$= 0.0285 < 0.05$

Largest value of $\underline{r = 3}$ (lies in rejection region).

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42\. In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a randomly choosen sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. ---[5]

Later Laxmi carries out a similar test with the same hypothesis, at the 8% signilificane level, using another random sample of 50.

(b) Given that the sample mean is now 31.5, find the probability of Type II error. ---[5]

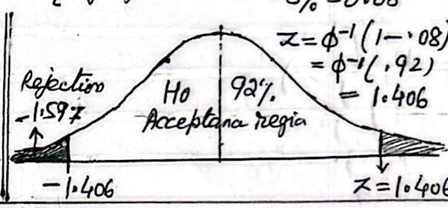
[W-22/61/07]

Solution: H_0: Population mean time $\mu = 32.5$
H_1: Population mean time $\mu < 32.5$

$N(\mu, \sigma^2) \to N(32.5, 3.1)$
Sample $n = 50$
Sample variance $= \frac{\sigma^2}{n} = \frac{3.1^2}{50}$
Significance level $\to 8\% = 0.08$

$$P(X < 31.8) = P\left(z < \left(\frac{31.8 - 32.5}{3.1/\sqrt{50}}\right)\right)$$
$$= P(z < -1.597)$$

$z = \phi^{-1}(1 - .08)$
$= \phi^{-1}(.92)$
$= 1.406$



Now $-1.597 < -1.406$
(Reject H_0)
There is evidence that (population) (mean) time has **decreased**.

(b) $$P(X < a) = P\left(z < \frac{a - 32.5}{3.1/\sqrt{50}}\right) \Rightarrow \frac{a - 32.5}{3.1/\sqrt{50}} = -1.406 \Rightarrow a = 31.88$$
(Critical value of mean)

(Now Sample mean = 31.5)

$$P(\text{Type II error}) = P(X \geq 31.88) = P\left(z > \frac{31.88 - 31.5}{3.1 \div \sqrt{50}}\right) = P(z > 0.8668)$$

P(accept Ho / whereas Ho is rejected)
(as the sample mean $31.5 < 32.5$)

$= 1 - \phi(0.8668)$
$= 0.190$ ✓ (3sf)

43. In the past, the mean length of a particular variety of worm has been 10·3 cm, with standard deviation 2·6 cm. Following a change in climate, it is thought that the mean length of this variety of worm has decreased. The lengths of a random sample of 100 worms of this variety are found and the mean of this sample is found to be 9·8 cm.

Assuming that the standard deviation remains at 2·6 cm, carry out a test at the 2% significance level of whether the mean length has decreased.

[W-22/62/Q2] --- [5]

Solution: H_0: Population mean length $\mu = 10.3$ cm

H_1: Population mean length $\mu < 10.3$ cm

Sample size $n = 100$
Sample mean $= 9.8$
Sample Variance $= \frac{2.6}{\sqrt{100}}$

$$P(\mu \leq 9.8) = P\left(z < \frac{9.8 - 10.3}{2.6/\sqrt{100}}\right)$$

$$= P(z < -1.923)$$

Significance level $2\% \to 0.02$

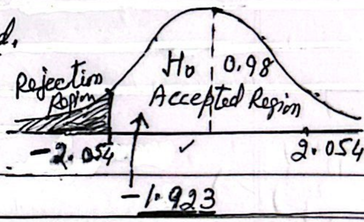
$z = \phi^{-1}(1 - 0.02)$

$z = \phi^{-1}(0.98) = 2.054$ ✓

Now $-1.923 > -2.054$

[Not rejected H_0]

No evidence that (mean) length has decreased.



44. The number of faults in cloth made on a certain machine has a Poisson distribution with mean 2.4 per $10m^2$. An adjustment is made to the machine. It is required to test at the 5% significance level whether the mean number of faults has decreased. A randomly selected $30m^2$ of cloth is checked and the number of fault is found.

(a) State suitable null and alternative hypotheses for the test. --[1]

(b) Find the probability of Type I error. --[3]

Exactly 3 faults are found in the randomly selected $30m^2$ of cloth.

(c) Carry out the test at the 5% significance level. --[2]

Later a similar test was carried out at the 5% significance level, using another randomly selected $30m^2$ of cloth.

(d) Given that the number of faults actually has a poisson distribution with mean 0.5 per $10m^2$, find the probability of a Type II error. --[2]

[W-22/62/Q4]

Solution: (a) H_0: Population mean = 7.2 [$\lambda = 2.4$ per $10m^2 \Rightarrow$ for $30m^2$, $\lambda = 7.2$]

H_1: Population mean < 7.2 Significance Level 5% = 0.05

(b) $P(X \leq 2) = e^{-7.2}\left[1 + 7.2 + \frac{7.2^2}{2}\right] = 0.0255 < 0.05$

$P(X \leq 3) = 0.0255 + e^{-7.2} \times \frac{7.2^3}{3!} = 0.0719 > 0.05$

hence P(Type I error) = 0.0255.

(c) $P(X \leq 3) = 0.0719 > 0.05$

(Not Reject H_0), No evidence that (mean) number of faults has decreased.

(d) Now mean 0.5 per $10m^2 \Rightarrow \lambda = 0.5 \times 3 = 1.5$ per $30m^2$. [1.5 < 7.2]

P(Type II error) = $P(X > 2)$ {(wrongly accepted H_0 / when H_0 is false) $= 1 - P(\text{Reject } H_0)$

$= 1 - P(X \leq 2)$

$= 1 - e^{-1.5} \times \left(1 + 1.5 + \frac{1.5^2}{2}\right)$

$= 1 - (0.2231 + 0.3347 + 0.2510)$

$= 0.191$ (3sf).