PROBABILITY AND STATISTICS-2
9709
(March, June and November series 2020 – 2023 With marking scheme)
HYPOTHESIS TESTING
EXERCISE -2

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1) SP 2020 9709_6 Q6

At a certain hospital it was found that the probability that a patient did not arrive for an appointment was 0.2. The hospital carries out some publicity in the hope that this probability will be reduced. They wish to test whether the publicity has worked.

A random sample of 30 appointments is selected and the number of patients that do not arrive is noted. This figure is used to carry out a test at the 5% significance level.

(a) Explain why the test is one-tailed and state suitable null and alternative hypotheses. [2]

(b) Use a binomial distribution to find the critical region, and find the probability of a Type I error. [5]

(c) In fact 3 patients out of the 30 do not arrive.

State the conclusion of the test, explaining your answer. [2]

2) SP 2020 9709_6 Q7

The mean weight of bags of carrots is μ kilograms. An inspector wishes to test whether $\mu = 2.0$. He weighs a random sample of 200 bags and his results are summarised as follows.

$$\Sigma x = 430 \qquad \Sigma x^2 = 1290$$

Carry out the test at the 10% significance level. [7]

3) March 2020 9709_62 Q3

In the past, the mean time taken by Freda for a particular daily journey was 39.2 minutes. Following the introduction of a one-way system, Freda wishes to test whether the mean time for the journey has decreased. She notes the times, t minutes, for 40 randomly chosen journeys and summarises the results as follows.

$$n = 40 \qquad \Sigma t = 1504 \qquad \Sigma t^2 = 57\,760$$

(a) Calculate unbiased estimates of the population mean and variance of the new journey time. [3]

(b) Test, at the 5% significance level, whether the population mean time has decreased. [5]

4) March 2020 9709_62 Q7

A national survey shows that 95% of year 12 students use social media. Arvin suspects that the percentage of year 12 students at his college who use social media is less than the national percentage. He chooses a random sample of 20 students at his college and notes the number who use social media. He then carries out a test at the 2% significance level.

(a) Find the rejection region for the test. [4]

(b) Find the probability of a Type I error. [1]

(c) Jimmy believes that the true percentage at Arvin's college is 70%. Assuming that Jimmy is correct, find the probability of a Type II error. [3]

5) March 2021 9709_62 Q3

An architect wishes to investigate whether the buildings in a certain city are higher, on average, than buildings in other cities. He takes a large random sample of buildings from the city and finds the mean height of the buildings in the sample. He calculates the value of the test statistic, z, and finds that $z = 2.41$.

(a) Explain briefly whether he should use a one-tail test or a two-tail test. [1]

(b) Carry out the test at the 1% significance level. [3]

6) March 2021 9709_62 Q6

It is known that 8% of adults in a certain town own a Chantor car. After an advertising campaign, a car dealer wishes to investigate whether this proportion has increased. He chooses a random sample of 25 adults from the town and notes how many of them own a Chantor car.

(a) He finds that 4 of the 25 adults own a Chantor car.

Carry out a hypothesis test at the 5% significance level. [5]

(b) Explain which of the errors, Type I or Type II, might have been made in carrying out the test in part **(a)**. [2]

Later, the car dealer takes another random sample of 25 adults from the town and carries out a similar hypothesis test at the 5% significance level.

(c) Find the probability of a Type I error. [3]

7) March 2022 9709_62 Q2

Harry has a five-sided spinner with sectors coloured blue, green, red, yellow and black. Harry thinks the spinner may be biased. He plans to carry out a hypothesis test with the following hypotheses.

$$H_0\text{: P(the spinner lands on blue)} = \tfrac{1}{5}$$
$$H_1\text{: P(the spinner lands on blue)} \neq \tfrac{1}{5}$$

Harry spins the spinner 300 times. It lands on blue on 45 spins.

Use a suitable approximation to carry out Harry's test at the 5% significance level. [5]

8) March 2022 9709_62 Q4

In the past the time, in minutes, taken by students to complete a certain challenge had mean 25.5 and standard deviation 5.2. A new challenge is devised and it is expected that students will take, on average, less than 25.5 minutes to complete this challenge. A random sample of 40 students is chosen and their mean time for the new challenge is found to be 23.7 minutes.

(a) Assuming that the standard deviation of the time for the new challenge is 5.2 minutes, test at the 1% significance level whether the population mean time for the new challenge is less than 25.5 minutes. [5]

(b) State, with a reason, whether it is possible that a Type I error was made in the test in part **(a)**. [1]

9) March 2023 9709_62 Q4

The number of accidents per 3-month period on a certain road has the distribution $\text{Po}(\lambda)$. In the past the value of λ has been 5.7. Following some changes to the road, the council carries out a hypothesis test to determine whether the value of λ has decreased. If there are fewer than 3 accidents in a randomly chosen 3-month period, the council will conclude that the value of λ has decreased.

(a) Find the probability of a Type I error. [2]

(b) Find the probability of a Type II error if the mean number of accidents per 3-month period is now actually 0.9. [3]

10) March 2023 9709_62 Q6

Last year, the mean time taken by students at a school to complete a certain test was 25 minutes. Akash believes that the mean time taken by this year's students was less than 25 minutes. In order to test this belief, he takes a large random sample of this year's students and he notes the time taken by each student. He carries out a test, at the 2.5% significance level, for the population mean time, μ minutes. Akash uses the null hypothesis $H_0: \mu = 25$.

(a) Give a reason why Akash should use a one-tailed test. [1]

Akash finds that the value of the test statistic is $z = -2.02$.

(b) Explain what conclusion he should draw. [2]

In a different one-tailed hypothesis test the z-value was found to be 2.14.

(c) Given that this value would lead to a rejection of the null hypothesis at the $\alpha\%$ significance level, find the set of possible values of α. [3]

11) June 2020 9709_61 Q2

In the past the yield of a certain crop, in tonnes per hectare, had mean 0.56 and standard deviation 0.08. Following the introduction of a new fertilizer, the farmer intends to test at the 2.5% significance level whether the mean yield has increased. He finds that the mean yield over 10 years is 0.61 tonnes per hectare.

(a) State two assumptions that are necessary for the test. [2]

(b) Carry out the test. [5]

12) June 2020 9709_61 Q4

A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X.

(a) Show that $\text{Var}(X) = 2$. [1]

Fiona has another spinner, also with five sides numbered 1, 2, 3, 4, 5. She suspects that it is biased so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2.6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error. [4]

(c) State what is meant by a Type II error in this context. [1]

13) June 2020 9709_62 Q2

A shop obtains apples from a certain farm. It has been found that 5% of apples from this farm are Grade A. Following a change in growing conditions at the farm, the shop management plan to carry out a hypothesis test to find out whether the proportion of Grade A apples has increased. They select 25 apples at random. If the number of Grade A apples is more than 3 they will conclude that the proportion has increased.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the probability of a Type I error. [3]

In fact 2 of the 25 apples were Grade A.

(c) Which of the errors, Type I or Type II, is possible? Justify your answer. [2]

14) June 2020 9709_63 Q3

The number of customers who visit a particular shop between 9.00 am and 10.00 am has the distribution $Po(\lambda)$. In the past the value of λ was 5.2. Following some new advertising, the manager wishes to test whether the value of λ has increased. He chooses a random sample of 20 days and finds that the total number of customers who visited the shop between 9.00 am and 10.00 am on those days is 125.

Use an approximating distribution to test at the 2.5% significance level whether the value of λ has increased. [6]

15) June 2020 9709_63 Q7

A market researcher is investigating the length of time that customers spend at an information desk. He plans to choose a sample of 50 customers on a particular day.

(a) He considers choosing the first 50 customers who visit the information desk.

Explain why this method is unsuitable. [1]

The actual lengths of time, in minutes, that customers spend at the information desk may be assumed to have mean μ and variance 4.8. The researcher knows that in the past the value of μ was 6.0. He wishes to test, at the 2% significance level, whether this is still true. He chooses a random sample of 50 customers and notes how long they each spend at the information desk.

(b) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(c) Given that the mean time spent at the information desk by the 50 customers is 6.8 minutes, carry out the test. [5]

(d) Give a reason why it was necessary to use the Central Limit theorem in your answer to part **(c)**. [1]

16) June 2021 9709_62 Q1

In a game, a ball is thrown and lands in one of 4 slots, labelled A, B, C and D. Raju wishes to test whether the probability that the ball will land in slot A is $\frac{1}{4}$.

(a) State suitable null and alternative hypotheses for Raju's test. [1]

The ball is thrown 100 times and it lands in slot A 15 times.

(b) Use a suitable approximating distribution to carry out the test at the 2% significance level. [5]

17) June 2021 9709_62 Q5

The time, in minutes, spent by customers at a particular gym has the distribution $N(\mu, 38.2)$. In the past the value of μ has been 42.4. Following the installation of some new equipment the management wishes to test whether the value of μ has changed.

(a) State what is meant by a Type I error in this context. [1]

(b) The mean time for a sample of 20 customers is found to be 45.6 minutes.

Test at the 2.5% significance level whether the value of μ has changed. [5]

18) June 2021 9709_63 Q2

In the past, the time, in hours, for a particular train journey has had mean 1.40 and standard deviation 0.12. Following the introduction of some new signals, it is required to test whether the mean journey time has decreased.

(a) State what is meant by a Type II error in this context. [1]

(b) The mean time for a random sample of 50 journeys is found to be 1.36 hours.

Assuming that the standard deviation of journey times is still 0.12 hours, test at the 2.5% significance level whether the population mean journey time has decreased. [5]

(c) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(b)**. [2]

19) June 2021 9709_63 Q3

The local council claims that the average number of accidents per year on a particular road is 0.8. Jane claims that the true average is greater than 0.8. She looks at the records for a random sample of 3 recent years and finds that the total number of accidents during those 3 years was 5.

(a) Assume that the number of accidents per year follows a Poisson distribution.

(i) State null and alternative hypotheses for a test of Jane's claim. [1]

(ii) Test at the 5% significance level whether Jane's claim is justified. [4]

(b) Jane finds that the number of accidents per year has been gradually increasing over recent years.

State how this might affect the validity of the test carried out in part **(a)(ii)**. [1]

20) June 2022 9709_61 Q2

Arvind uses an ordinary fair 6-sided die to play a game. He believes he has a system to predict the score when the die is thrown. Before each throw of the die, he writes down what he thinks the score will be. He claims that he can write the correct score more often than he would if he were just guessing. His friend Laxmi tests his claim by asking him to write down the score before each of 15 throws of the die. Arvind writes the correct score on exactly 5 out of 15 throws.

Test Arvind's claim at the 10% significance level. [5]

21) June 2022 9709_61 Q7

In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

(a) State suitable null and alternative hypotheses for Jenny's test. [1]

(b) Find the probability that Jenny will make a Type I error. [3]

(c) Jenny found that the sample mean run time was 27.2 minutes.

Explain briefly whether it is possible for her to make a Type I error or a Type II error or both. [2]

22) June 2022 9709_62 Q1

(a) A javelin thrower noted the lengths of a random sample of 50 of her throws. The sample mean was 72.3 m and an unbiased estimate of the population variance was 64.3 m^2.

Find a 92% confidence interval for the population mean length of throws by this athlete. [3]

(b) A discus thrower wishes to calculate a 92% confidence interval for the population mean length of his throws. He bases his calculation on his first 50 throws in a week.

Comment on this method. [1]

23) June 2022 9709_62 Q2

In the past, the mean height of plants of a particular species has been 2.3 m. A random sample of 60 plants of this species was treated with fertiliser and the mean height of these 60 plants was found to be 2.4 m. Assume that the standard deviation of the heights of plants treated with fertiliser is 0.4 m.

Carry out a test at the 2.5% significance level of whether the mean height of plants treated with fertiliser is greater than 2.3 m. [5]

24) June 2022 9709_62 Q4

The number of cars arriving at a certain road junction on a weekday morning has a Poisson distribution with mean 4.6 per minute. Traffic lights are installed at the junction and a council officer wishes to test at the 2% significance level whether there are now fewer cars arriving. He notes the number of cars arriving during a randomly chosen 2-minute period.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the critical region for the test. [4]

The officer notes that, during a randomly chosen 2-minute period on a weekday morning, exactly 5 cars arrive at the junction.

(c) Carry out the test. [2]

(d) State, with a reason, whether it is possible that a Type I error has been made in carrying out the test in part **(c)**. [1]

The number of cars arriving at another junction on a weekday morning also has a Poisson distribution with mean 4.6 per minute.

(e) Use a suitable approximating distribution to find the probability that more than 300 cars will arrive at this junction in an hour. [3]

25) June 2022 9709_63 Q2

Anton believes that 10% of students at his college are left-handed. Aliya believes that this is an under-estimate. She plans to carry out a hypothesis test of the null hypothesis $p = 0.1$ against the alternative hypothesis $p > 0.1$, where p is the actual proportion of students at the college that are left-handed. She chooses a random sample of 20 students from the college. She will reject the null hypothesis if at least 5 of these students are left-handed.

(a) Explain what is meant by a Type I error in this context. [1]

(b) Find the probability of a Type I error in the test. [3]

(c) Given that the true value of p is 0.3, find the probability of a Type II error in the test. [2]

26) June 2022 9709_63 Q3

Batteries of type A are known to have a mean life of 150 hours. It is required to test whether a new type of battery, type B, has a shorter mean life than type A batteries.

(a) Give a reason for using a sample rather than the whole population in carrying out this test. [1]

A random sample of 120 type B batteries are tested and it is found that their mean life is 147 hours, and an unbiased estimate of the population variance is 225 hours^2.

(b) Test, at the 2% significance level, whether type B batteries have a shorter mean life than type A batteries. [5]

(c) Calculate a 94% confidence interval for the population mean life of type B batteries. [3]

27) June 2023 9709_61 Q3

In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes.

Stating a necessary assumption, carry out the test at the 1% significance level. [6]

28) June 2023 9709_61 Q7

The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1.9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the critical region for the test and state the probability of a Type I error. [6]

(c) State what is meant by a Type I error in this context. [1]

(d) During the 4 randomly chosen weeks there are a total of 3 accidents.

State the conclusion that the manager should reach. Give a reason for your answer. [2]

(e) Assuming that the mean remains 1.9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. [4]

29) June 2023 9709_62 Q3

The masses, in kilograms, of newborn babies in country A are represented by the random variable X, with mean μ and variance σ^2. The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \qquad \Sigma x = 1625 \qquad \Sigma x^2 = 5663.5$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

A researcher wishes to test whether the mean mass of newborn babies in a neighbouring country, B, is different from that in country A. He chooses a random sample of 60 newborn babies in country B and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part **(a)** are the correct values for μ and σ^2. Assume also that the variance of the masses of newborn babies in country B is the same as in country A.

(b) Carry out the test at the 1% significance level. [5]

30) June 2023 9709_62 Q6

When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $\frac{1}{3}$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $\frac{1}{3}$.

(a) Use a binomial distribution to find the probability of a Type I error. [2]

The true probability of being awarded a medal is denoted by p.

(b) Given that the probability of a Type II error is 0.8926, find the value of p. [3]

31) June 2023 9709_63 Q5

Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. [5]

(c) Under what circumstances would it **not** be necessary to use the Central Limit Theorem in answering **(b)**? [1]

32) June 2023 9709_63 Q8

A new light was installed on a certain footpath. A town councillor decided to use a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path during any 20-minute period during the evening was 1.01.

After the light was installed, the total number, n, of people using the path during 3 randomly chosen 20-minute periods during the evening was noted.

(a) Given that the value of n was 6, use a Poisson distribution to carry out the test at the 5% significance level. [6]

(b) Later a similar test, at the 5% significance level, was carried out using another 3 randomly chosen 20-minute periods during the evening.

Find the probability of a Type I error. [2]

(c) State what is meant by a Type I error in this context. [1]

(d) State, in context, what further information would be needed in order to find the probability of a Type II error. Do not carry out any further calculation. [2]

33) October 2020 9709_61 Q5

The number of absences per week by workers at a factory has the distribution Po(2.1).

(a) Find the standard deviation of the number of absences per week. [1]

(b) Find the probability that the number of absences in a 2-week period is at least 2. [3]

(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. [2]

Following a change in working conditions, the management wished to test whether the mean number of absences has decreased. They found that, in a randomly chosen 3-week period, there were exactly 2 absences.

(d) Carry out the test at the 10% significance level. [5]

(e) State, with a reason, which of the errors, Type I or Type II, might have been made in carrying out the test in part **(d)**. [2]

34) October 2020 9709_61 Q6

The time, in minutes, for Anjan's journey to work on Mondays has mean 38.4 and standard deviation 6.9.

(a) Find the probability that Anjan's mean journey time for a random sample of 30 Mondays is between 38 and 40 minutes. [5]

Anjan wishes to test whether his mean journey time is different on Tuesdays. He chooses a random sample of 30 Tuesdays and finds that his mean journey time for these 30 Tuesdays is 40.2 minutes. Assume that the standard deviation for his journey time on Tuesdays is 6.9 minutes.

(b) **(i)** State, with a reason, whether Anjan should use a one-tail or a two-tail test. [1]

(ii) Carry out the test at the 10% significance level. [5]

(iii) Explain whether it was necessary to use the Central Limit theorem in part **(b)(ii)**. [1]

35) October 2020 9709_62 Q4

The areas, X cm^2, of petals of a certain kind of flower have mean μ cm^2. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized by

$$\Sigma x = 1850, \qquad \Sigma x^2 = 17\,850.$$

Test the botanist's claim at the 2.5% significance level. [8]

36) October 2020 9709_62 Q6

A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer.

(a) Carry out the test at the 10% significance level. [5]

(b) Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance test at the 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she finds that there are 3 or fewer packets containing the prize offer.

Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. [3]

(c) Explain what is meant by a Type II error in this context. [1]

37) October 2021 9709_61 Q5

(a) The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the researcher's suspicion at the 2% significance level. [5]

38) October 2021 9709_61 Q7

The masses, in grams, of apples from a certain farm have mean μ and standard deviation 5.2. The farmer says that the value of μ is 64.6. A quality control inspector claims that the value of μ is actually less than 64.6. In order to test his claim he chooses a random sample of 100 apples from the farm.

(a) The mean mass of the 100 apples is found to be 63.5 g.

Carry out the test at the 2.5% significance level. [5]

(b) Later another test of the same hypotheses at the 2.5% significance level, with another random sample of 100 apples from the same farm, is carried out.

Given that the value of μ is in fact 62.7, calculate the probability of a Type II error. [5]

39) October 2021 9709_62 Q4

A certain kind of firework is supposed to last for 30 seconds, on average, after it is lit. An inspector suspects that the fireworks actually last a shorter time than this, on average. He takes a random sample of 100 fireworks of this kind. Each firework in the sample is lit and the time it lasts is noted.

(a) Give a reason why it is necessary to take a sample rather than testing all the fireworks of this kind. [1]

It is given that the population standard deviation of the times that fireworks of this kind last is 5 seconds.

(b) The mean time lasted by the 100 fireworks in the sample is found to be 29 seconds.

Test the inspector's suspicion at the 1% significance level. [5]

(c) State with a reason whether the Central Limit theorem was needed in the solution to part **(b)**. [1]

40) October 2021 9709_62 Q6

A machine is supposed to produce random digits. Bob thinks that the machine is not fair and that the probability of it producing the digit 0 is less than $\frac{1}{10}$. In order to test his suspicion he notes the number of times the digit 0 occurs in 30 digits produced by the machine. He carries out a test at the 10% significance level.

(a) State suitable null and alternative hypotheses. [1]

(b) Find the rejection region for the test. [4]

(c) State the probability of a Type I error. [1]

It is now given that the machine actually produces a 0 once in every 40 digits, on average.

(d) Find the probability of a Type II error. [3]

(e) Explain the meaning of a Type II error in this context. [1]

41) October 2022 9709_61 Q2

A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

42) October 2022 9709_61 Q7

In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a random sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. [5]

Later Laxmi carries out a similar test with the same hypotheses, at the 8% significance level, using another random sample of size 50.

(b) Given that the population mean is now 31.5, find the probability of a Type II error. [5]

43) October 2022 9709_62 Q2

In the past, the mean length of a particular variety of worm has been 10.3 cm, with standard deviation 2.6 cm. Following a change in the climate, it is thought that the mean length of this variety of worm has decreased. The lengths of a random sample of 100 worms of this variety are found and the mean of this sample is found to be 9.8 cm.

Assuming that the standard deviation remains at 2.6 cm, carry out a test at the 2% significance level of whether the mean length has decreased. [5]

44) October 2022 9709_62 Q4

The number of faults in cloth made on a certain machine has a Poisson distribution with mean 2.4 per 10 m^2. An adjustment is made to the machine. It is required to test at the 5% significance level whether the mean number of faults has decreased. A randomly selected 30 m^2 of cloth is checked and the number of faults is found.

(a) State suitable null and alternative hypotheses for the test. [1]

(b) Find the probability of a Type I error. [3]

Exactly 3 faults are found in the randomly selected 30 m^2 of cloth.

(c) Carry out the test at the 5% significance level. [2]

Later a similar test was carried out at the 5% significance level, using another randomly selected 30 m^2 of cloth.

(d) Given that the number of faults actually has a Poisson distribution with mean 0.5 per 10 m^2, find the probability of a Type II error. [2]

Marking Scheme

1) .

(a)	Looking for decrease (or improvement)	1	**B1**
	H_0: P(not arrive) = 0.2 H_1: P(not arrive) < 0.2	1	**B1**
		2	
(b)	P($X = 0$) and P($X = 1$) attempted	1	**M1**
	$P(X \leqslant 2) = 0.8^{30} + 30 \times 0.8^{29} \times 0.2 + {}^{30}C_2 \times 0.8^{28} \times 0.2^2$ (= 0.0442)	1	**M1**
	$P(X \leqslant 3) = 0.8^{30} + 30 \times 0.8^{29} \times 0.2 + {}^{30}C_2 \times 0.8^{28} \times 0.2^2 + {}^{30}C_3 \times 0.8^{27} \times 0.2^3 = 0.123$	1	**B1**
	Critical region (cr) is $X \leqslant 2$	1	**A1**
	P(Type I) = 0.0442 (3 sf)	1	**A1**
		5	
(c)	3 is outside cr	1	**M1**
	No evidence that p has decreased (or that publicity has worked)	1	**A1**
		2	

2)

$H_0: \mu = 2.0; \quad H_1: \mu \neq 2.0$	1	**B1**
$\bar{x} = \frac{430}{200} = 2.15$	1	**B1**
$s^2 = \frac{200}{199}\left(\frac{1290}{200} - \left(\frac{430}{200}\right)^2\right)$	1	**M1**
= 1.8366834	1	**A1**
$\frac{2.15 - 2.0}{\sqrt{\frac{1.8366834}{200}}}$ (= 1.565)	1	**M1**
$z = 1.645$	1	**M1**
No evidence that $\mu \neq 2.0$	1	**A1**

3) .

(a)	est (μ) = 37.6 or $\frac{1504}{40}$ or $\frac{188}{5}$	**B1**
	est (σ^2) = $\frac{40}{39}\left[\frac{57760}{40} - 37.6^2\right]$ = 31.0154 = $\frac{2016}{65}$	**M1**
	= 31.(0) (3 sf)	**A1**
(b)	H_0: Pop mean (or μ) = 39.2 H_1: Pop mean (or μ) < 39.2	**B1**
	$\frac{'37.6' - 39.2}{\frac{\sqrt{'31.0154'}}{\sqrt{40}}}$	**M1**
	= –1.817	**A1**
	'1.817' > 1.645 OE	**M1**
	There is evidence that mean time has decreased	**A1FT**

4) .

(a)	P($X \leqslant n$) ($n \leqslant 20$) attempted, using B(20, 0.95)	**M1**
	P($X \leqslant 17$) or P($X \leqslant 16$) attempted, using B(20, 0.95)	**M1**
	(P($X \leqslant 17$)) = 0.0755 and (P($X \leqslant 16$)) = 0.0159	**A1**
	Rej region is $X \leqslant 16$ or $X < 17$	**A1**
(b)	0.0159	**B1**
		1
(c)	Use of B(20, 0.7)	**M1**
	P($X > 16 \mid p = 0.7$)	**M1**
	= 0.107	**A1**

5) .

(a)	One-tail because investigating whether "higher"	**B1**
		1
(b)	H_0: Population mean (or μ) in city same as for others H_1: Population mean (or μ) in city greater than for others	**B1 FT**
	$2.41 > 2.326$ or $0.008 < 0.01$ or $0.992 > 0.99$	**M1**
	There is evidence that buildings are higher [on average].	**A1 FT**

6) .

(a)	H_0: population proportion = 0.08 OE H_1: population proportion > 0.08 OE	**B1**
	$P(X \geq 4) = 1 - P(X \leq 3) =$ $1 - (0.92^{25} + 25 \times 0.92^{24} \times 0.08 + {}^{25}C_2 \times 0.92^{23} \times 0.08^2 + {}^{25}C_3 \times 0.92^{22} \times 0.08^3)$	**M1**
	0.135 (3 sf)	**A1**
	$0.135 > 0.05$	**M1**
	There is no evidence that proportion owning Chantor has increased	**A1 FT**
(b)	H_0 was not rejected.	***B1 FT**
	Hence Type II might have been made.	**DB1 FT**
(c)	$P(X \geq 5) = 1 - P(X \leq 4)$ $= 1 - \left((1 - 0.1351) + {}^{25}C_4 \times 0.92^{21} \times 0.08^4\right)$ [= 0.0451]	***M1**
	$0.0451 < 0.05$	**A1**
	P(Type I error) = 0.0451 or 0.0452	**A1**

7) .

B(300, $\frac{1}{5}$) → N(60,48)	**B1**
$\frac{45.5-60}{\sqrt{48}}$	**M1**
$= -2.093$	**A1**
'2.093' > 1.96	**M1**
[Evidence to reject H_0] There is evidence that P(landing on blue) $\neq \frac{1}{5}$	**A1 FT**

8) .

(a)	$H_0: \mu = 25.5$ $H_1: \mu < 25.5$	**B1**
	$\frac{23.7-25.5}{5.2 \div \sqrt{40}}$	**M1**
	$= -2.189$	**A1**
	'2.189' < 2.326	**M1**
	[Accept H_0] No evidence that mean time has decreased	**A1 FT**
(b)	No, because H_0 was not rejected	**B1 FT**

9) .

(a)	$e^{-5.7}(1 + 5.7 + \frac{5.7^2}{2!})$ **or** $e^{-5.7}(1 + 5.7 + 16.245)$ **or** $0.003346 + 0.01907 + 0.05436$	**M1**
	$= 0.0768$ (3 sf)	**A1**
		2
(b)	$e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!})$	**M1**
	$= 1 - e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!}) = 1 - e^{-0.9}(1 + 0.9 + 0.405) = 1 - (0.4066 + 3659 + 0.1647)$	**A1**
	$= 0.0629$ (3 sf)	**A1**

10) .

(a)	He is expecting a **decrease** (in μ)	**B1**
		1
(b)	$-2.02 < -1.96$	**M1**
	(Reject H_0) There is evidence to suggest that this year's (mean) time is less than 25	**A1**
(c)	$1 - \phi(2.14)$ [= 0.0162]	**M1**
	1.62	**A1**
	$\alpha \geqslant 1.62$ (3 sf)	**A1ft**
		3
(d)	$\frac{24.8-m}{3.9\div10}$	**M1**
	$\frac{24.8-m}{3.9\div10} = -1.645$	**M1**
	$m = 25.4$ (3 sf)	**A1**

11) .

(a)	Assume standard deviation unchanged or standard deviation = 0.08	**B1**
	Assume yields normally distributed	**B1**
		2
(b)	H_0: Population mean yield (or μ) = 0.56 H_1: Population mean yield (or μ) > 0.56	**B1**
	$\frac{0.61-0.56}{\frac{0.08}{\sqrt{10}}}$	**M1**
	1.976	**A1**
	Comp 1.96	**M1**
	There is evidence that mean yield has increased	**A1**
		5

12) .

(a)	$(1^2+2^2+3^2+4^2+5^2)\div 5-3^2 \quad (=2\ \textbf{AG})$	**B1**
		1
(b)	N(3, 2)	**M1**
	$\dfrac{2.6-\text{"}3\text{"}}{\sqrt{\frac{2}{40}}}(=-1.789)$	**M1**
	$\Phi(\text{"}-1.789\text{"}) = 1-\Phi(\text{"}1.789\text{"})$	**M1**
	0.0367 to 0.0368	**A1**
		4
(c)	Concluding that spinner is unbiased when it is biased	**B1**
		1

13) .

(a)	H_0: Proportion = 0.05 H_1: Proportion > 0.05	**B1**
		1
(b)	$1-(0.95^{25}+25\times 0.95^{24}\times 0.05+{}^{25}C_2\times 0.95^{23}\times 0.05^2+{}^{25}C_3\times 0.95^{22}\times 0.05^3)$	**M1**
	Completely correct expression	**A1**
	0.0341	**A1**
		3
(c)	Type II	**B1**
	Will conclude proportion not increased	**B1**
		2

14) .

H_0: $\lambda = 104$ (or 5.2) H_1: $\lambda > 104$ (or 5.2)	**B1**
N(104, 104) stated or implied	**B1**
$\dfrac{124.5-104}{\sqrt{104}}$	**M1**
2.010	**A1**
2.010 > 1.96	**M1**
There is evidence that λ has increased	**A1**
	6

15) .

(a)	Later customers might spend times different from first ones	**B1**
		1
(b)	0.02	**B1**
	Concluding that $\mu \neq 6.0$, when actually $\mu = 6.0$	**B1**
		2

(c)	H₀: $\mu = 6.0$ H₁: $\mu \neq 6.0$	B1
	$\frac{6.8-6.0}{\sqrt{\frac{4.8}{50}}}$	M1
	2.582	A1
	comp 2.326	M1
	Evidence that $\mu \neq 6.0$	A1
		5
(d)	Population distribution unknown	B1
		1

16) .

(a)	$H_0: p = \frac{1}{4}$ $H_1: p \neq \frac{1}{4}$	B1
		1
(b)	$N\left(25, \frac{75}{4}\right)$	B1
	$\pm \frac{15.5-25}{\sqrt{\frac{75}{4}}}$ or $\frac{\frac{15.5}{100}-0.25}{\sqrt{\frac{0.25\times0.75}{100}}}$	M1
	± -2.194 (2.19)	A1
	$-2.326 < -2.194$ or $0.0141 > 0.01$ or $0.9859 < 0.99$	M1
	No evidence to reject that the probability is $\frac{1}{4}$	A1 FT

17) .

(a)	Conclude that (population) mean time has changed (or is not 42.4) although μ has not changed (or is still 42.4)	B1

(b)	H_0: population mean (or μ) = 42.4 H_1: population mean (or μ) $\neq$ 42.4	B1
	$\pm \dfrac{45.6-42.4}{\sqrt{38.2 \div 20}}$	M1
	± 2.315	A1
	2.240 < '2.315'	M1
	There is evidence that μ or mean time has changed	A1 FT

18) .

2(a)	Conclude (mean) (journey) time has not decreased when in fact it has.	**B1**
(b)	H_0: Pop mean (or μ) = 1.4 H_1: Pop mean (or μ) < 1.4	**B1**
	$\dfrac{1.36-1.4}{\frac{0.12}{\sqrt{50}}}$	**M1**
	–2.357 or – 2.36	**A1**
	–2.357 < –1.96 or 0.0092 < 0.025 or 0.9908 > 0.975 Or CV method 1.36 < 1.367	**M1**
	There is evidence that (mean) (journey) times have decreased	**A1 FT**
(c)	H_0 was rejected OE	***B1 FT**
	Type I	**DB1 FT**

19) .

(a)(i)	H_0: $\lambda = 2.4$ H_1: $\lambda > 2.4$	**B1**

3(a)(ii)	$1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!})$	M1
	0.0959 (3 sf)	A1
	$0.0959 > 0.05$	M1
	There is evidence that Jane's claim not justified or There is insufficient evidence to support Jane's claim	A1 FT
(b)	Mean not constant so Poisson model not valid	B1

20) .

H_0: P(correct) $= \frac{1}{6}$ H_1: P(correct) $> \frac{1}{6}$	B1
$1 - ({}^{15}C_4 \times (\frac{5}{6})^{11} \times (\frac{1}{6})^4 + {}^{15}C_3 \times (\frac{5}{6})^{12} \times (\frac{1}{6})^3 + {}^{15}C_2 \times (\frac{5}{6})^{13} \times (\frac{1}{6})^2 + 15 \times (\frac{5}{6})^{14} \times \frac{1}{6} + (\frac{5}{6})^{15})$	M1
0.0898 or 0.0897 (3 sf)	A1
$0.0898 < 0.1$	M1
[Reject H_0] There is evidence (at the 10% level) that Arvind can predict scores	FTA1

21) .

(a)	H_0: pop mean run time = 28.2 mins H_1: pop mean run time < 28.2 mins	B1
		1
(b)	$\frac{27-28.2}{4/\sqrt{40}}$ [= –1.897]	M1
	Φ(< '–1.897') = 1 – Φ('1.897')	M1
	0.0289 (3 sf)	A1
		3
(c)	H_0 is not rejected so…	M1
	Type II error can be made and Type I error cannot be made	A1

22) .

a)	$72.3 \pm z\sqrt{\frac{64.3}{50}}$	**M1**
	$z = 1.751$	**B1**
	CI is 70.3 to 74.3 metres (3 s.f.)	**A1**
		3
b)	Not random sample	**B1**

23) .

H_0: Pop mean height = 2.3 H_1: Pop mean height > 2.3	**B1**
$\dfrac{2.4 - 2.3}{\frac{0.4}{\sqrt{60}}}$	**M1**
1.936 or 1.937 or 1.94	**A1**
'1.936' < 1.96	**M1**
[Do not reject H_0] No evidence that (mean) height (with fertiliser) is more than without	**A1 FT**

24) .

(a)	H_0: Pop mean = 4.6 [or 9.2] H_1: Pop mean < 4.6 [or 9.2]	**B1**
(b)	Use of Poisson with $\lambda = 9.2$	**B1**
	$P(X \leqslant 3) = e^{-9.2}(1 + 9.2 + \frac{9.2^2}{2} + \frac{9.2^3}{3!}) = 0.0184$ or 0.018 [< 0.02]	**M1**
	$P(X \leqslant 4) = 0.0184 + e^{-9.2} \times \frac{9.2^4}{4!} = 0.0486$ or 0.049 [> 0.02]	***A1**
	CR is $X \leqslant 3$	**DA1**
(c)	5 is not in critical region OR $P(X \leqslant 5) = 0.104 > 0.02$ so [not reject H_0] no evidence that number of cars arriving is now fewer	**M1** **A1 FT**
(d)	No, because H_0 was not rejected	**B1 FT**

(e)	N(276, 276)	B1
	$\frac{300.5-276}{\sqrt{276}}$ [= 1.475]	M1
	$1-\phi$('1.475') = 0.0701 (3 s.f.)	A1

25) .

(a)	Conclude more than 10% of the students are left handed when this is not true	B1
		1
(b)	$1-(0.9^{20}+20\times0.9^{19}\times0.1+{}^{20}C_2\times0.9^{18}\times0.1^2+{}^{20}C_3\times0.9^{17}\times0.1^3+{}^{20}C_4\times0.9^{16}\times0.1^4)$	M2
	0.0432 (3 s.f.)	A1
(c)	$0.7^{20}+20\times0.7^{19}\times0.3+{}^{20}C_2\times0.7^{18}\times0.3^2+{}^{20}C_3\times0.7^{17}\times0.3^3+{}^{20}C_4\times0.7^{16}\times0.3^4$	M1
	0.238 or 0.237 (3 s.f.)	A1

26) .

(a)	Batteries unusable after testing or Population too big or too costly or too time consuming to use the whole population oe	B1
(b)	$H_0: \mu = 150$ $H_1: \mu < 150$	B1
	$\frac{147-150}{\sqrt{225}\div\sqrt{120}}$	M1
	–2.191	A1
	–2.191 < –2.054 [or –2.055]	M1
	[Reject Ho] There is evidence that the (mean) life of type *B* is less than type *A* (or less than 150)	A1 FT

(c)	$147 \pm z \times \frac{15}{\sqrt{120}}$	M1
	$z = 1.881$ [or 1.882]	B1
	144 to 150 (3 s.f.)	A1

27) .

Assume SD still = 5.2	B1
H_0: $\mu = 24.0$ H_1: $\mu > 24.0$	B1
$\frac{25.8-24.0}{\frac{5.2}{\sqrt{50}}}$	M1
= 2.448	A1
'2.448' > 2.326	M1
[Reject H_0] There is evidence that (mean) amount of wheat is greater.	A1FT

28) .

(a)	H_0: $\lambda = 7.6$ [or 1.9] H_1: $\lambda < 7.6$ [or 1.9]	B1
(b)	Mean = 7.6	B1
	$P(X \leqslant 2) = e^{-7.6}\left(1 + 7.6 + \frac{7.6^2}{2}\right)$ [= 0.0188 or 0.0187]	M1
	$P(X \leqslant 3) = e^{-7.6}\left(1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!}\right)$ [= 0.0554 or 0.0553]	M1
	0.0188 or 0.0187 and 0.0554 or 0.0553	A1
	Critical region is $X \leqslant 2$	A1
	P(Type I error) = $P(X \leqslant 2)$ = 0.0188 or 0.0187 (3 sf)	B1FT
(c)	Concluding that the (mean) no. of accidents has reduced when it has not.	B1

(d)	3 not in critical region.	M1
	No evidence mean number of accidents has decreased.	A1FT
(e)	N(98.8, 98.8)	B1
	$\frac{100.5-98.8}{\sqrt{98.8}}$ [= 0.171]	M1
	$1 - \Phi$('0.171')	M1
	= 0.432 (3 sf)	A1

29) .

(a)	Est (μ) = 3.25 = 13/4 or 1625/500	B1
	Est(σ^2) = $\frac{500}{499}(\frac{5663.5}{500} - \text{"}3.25\text{"}^2)$ or $\frac{1}{499}\left(5663.5 - \frac{1625^2}{500}\right)$	M1
	= 0.766 (3 sf) or 1529/1996	A1
(b)	H_0: Pop mean (or μ) = '3.25' H_1: Pop mean (or μ) $\neq$ '3.25'	B1FT
	$\frac{2.95-\text{"}3.25\text{"}}{\sqrt{\text{"}0.766\text{"}\div 60}}$	M1
	= –2.655	A1
	'2.655' > 2.576 or '–2.655' < –2.576	M1
	[Reject H_0] There is evidence that (mean) mass in (country B) is different (from country A).	A1FT

30) .

(a)	$(1 - \frac{1}{3})^{10}$	M1
	= 0.0173 (3 sf)	A1
		2
(b)	$1 - (1 - p)^{10} = 0.8926$	M1
	$1 - p = 0.1074^{0.1}$ [= 0.800]	M1
	p = 0.200 (3 sf) or 0.2	A1

31) .

(a)	$\bar{x} = 1700/50 = 34$	**B1**
	$\text{Est}(\sigma^2) = \frac{50}{49}\left(\frac{59050}{50} - 34^2\right)$ or $\frac{1}{49}\left(59050 - \frac{1700^2}{50}\right)$	**M1**
	$= 25.5$ (3 sf) or $\frac{1250}{49}$	**A1**
		3
(b)	H_0: Population mean time = 32.4 H_1: Population mean time $\neq$ 32.4	**B1**
	$\dfrac{34 - 32.4}{\frac{\sqrt{'25.5'}}{\sqrt{50}}}$	**M1**
	= 2.24 (3 sf)	**A1**
	'2.24' < 2.326	**M1**
	[Not reject H_0] Insufficient evidence that (mean) time has changed	**A1FT**
(c)	Distribution of times in the population is normal	**B1**

32) .

(a)	H_0: Pop mean no. people = 3.03 or 1.01 (per 20 min) H_1: Pop mean no. people > 3.03 or 1.01 (per 20 min)	**B1**
	Use of Po(3.03)	**M1**
	$= 1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!})$ $= 1 - e^{-3.03}(1 + 3.03 + 4.5905 + 4.6364 + 3.5120 + 2.128)$ $= 1 - (0.04832 + 0.1464 + 0.2218 + 0.2240 + 0.1697 + 0.1028)$	**M1**
	= 0.0870 (3sf) [0.0869727]	**A1**
	0.0870 > 0.05	**M1**
	(Do not reject H_0) Insufficient evidence to believe (mean) number of people has increased	**A1FT**

(b)	$"0.0869727" - e^{-3.03} \times \frac{3.03^6}{6!}$ or $0.869727 - e^{-3.03}(1.0748)$ or $0.869727 - 0.05193$ or $1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!} + \frac{3.03^6}{6!})$	M1
	0.0350 or 0.0351	A1
(c)	Concluding that the (mean) number of people (using the path per 20 mins in the evening) has increased when it has not	B1
		1
(d)	A value for the true mean	B1
	Number of people using the path per 20 mins in the evening.	B1

33) .

(a)	$\sqrt{2.1}$ or 1.45 (3 sf)	B1
		1
(b)	$\lambda = 4.2$	B1
	$1 - e^{-4.2}(1 + 4.2)$	M1
	$= 0.922$ (3 sf)	A1
		3
(c)	$\lambda = 6.3$ $e^{-6.3}\left(\frac{6.3^5}{5!} + \frac{6.3^6}{6!} + \frac{6.3^7}{7!}\right)$	M1
	$= 0.455$ (3 sf)	A1
(d)	$H_0: \lambda = 6.3$ $H_1: \lambda < 6.3$	B1
	$P(X \leqslant 2) = e^{-6.3}\left(1 + 6.3 + \frac{6.3^2}{2!}\right)$	M1
	$= 0.0498$ or 0.0499	A1
	‘0.0498’ < 0.1	M1
	There is evidence that mean number of absences has decreased.	A1 FT

5(e)	H_0 rejected	*B1 FT
	Hence Type I error possible	DB1 FT

34) .

(a)	$\dfrac{40-38.4}{\frac{6.9}{\sqrt{30}}} = 1.270$ $\dfrac{38-38.4}{\frac{6.9}{\sqrt{30}}} = -0.3175$	M1
		A1
		A1
	$\Phi(\text{'}1.270\text{'}) - (1 - \phi(\text{'}0.3175\text{'}))$	M1
	$= 0.523$ (3 sf) or 0.522	A1
(b)(i)	2-tail because looking for 'change', not decrease or increase	B1
(b)(ii)	H_0: Population mean journey time (or μ) = 38.4 H_1: Population mean journey time (or μ) $\neq$ 38.4	B1
	$\dfrac{40.2-38.4}{\frac{6.9}{\sqrt{30}}}$	M1
	$= 1.429$	A1
	$\text{'}1.429\text{'} < 1.645$	M1
	There is no evidence that mean journey time has changed.	A1 FT
	Alternative method for question 6(b)(ii) – critical values method	
	H_0: Population mean journey time (or μ) = 38.4 H_1: Population mean journey time (or μ) $\neq$ 38.4	B1
	$38 + 1.645\left(\dfrac{6.9}{\sqrt{30}}\right)$	M1
	$= 40.47$	A1
	$40.2 < 40.47$	M1
	There is no evidence that mean journey time has changed.	A1 FT
5(b)(iii)	Yes, because population distribution unknown.	B1

35) .

$\text{est}(\mu) = \frac{1850}{200}$ or 9.25	**B1**
$\text{est}(\sigma^2) = \frac{200}{199}\left(\frac{17850}{200} - \left(\frac{1850}{200}\right)^2\right)$ or $\frac{1}{199}\left(17850 - \frac{1850^2}{200}\right)$	**M1**
$= 3.71$ or 3.7060 or $\frac{1475}{398}$	**A1**
$H_0: \mu = 8.9$ $H_1: \mu \neq 8.9$	**B1**
$\dfrac{\frac{1850}{200} - 8.9}{\sqrt{\frac{\text{"3.706"}}{200}}}$	**M1**
$= 2.57$(3sf) (or using areas $0.00507 - 0.0051$)	**A1**
$2.24 < 2.57$ or $0.00507 < 0.0125$	**M1**
(Reject H_0) There is evidence that μ is not 8.9	**A1 FT**

36) .

(a)

H_0: P(contains offer) $= \frac{1}{3}$ H_1: P(contains offer) $< \frac{1}{3}$	**B1**
P(0,1 or 2 offers in 20 | H_0) $= \left(\frac{2}{3}\right)^{20} + 20\left(\frac{2}{3}\right)^{19}\left(\frac{1}{3}\right) + {}^{20}C_2\left(\frac{2}{3}\right)^{18}\left(\frac{1}{3}\right)^2$	**M1**
$= 0.0176$ (3sf)	**A1**
'0.0176' < 0.1	**M1**
(Reject H_0) No evidence (at 10% level) to support manufacturers claim	**A1 FT**

(b)	$1 - P(X \leqslant 3)$	M1
	$=1-\left[\left(\frac{6}{7}\right)^{20}+20\left(\frac{6}{7}\right)^{19}\left(\frac{1}{7}\right)+{}^{20}C_2\left(\frac{6}{7}\right)^{18}\left(\frac{1}{7}\right)^2+{}^{20}C_3\left(\frac{6}{7}\right)^{17}\left(\frac{1}{7}\right)^3\right]$	A1
	= 0.318 (3sf)	A1
		3
(c)	Concluding that prop is 1 in 3 when it is actually less(1 in 7)	B1

37) .

(a)(i)	Po(0.025)	B1
	$n = 2500 > 50$, $np = 0.025 < 5$	B1
(a)(ii)	$1 - e^{-0.025}$	M1
	0.0247 (3sf)	A1
		2
(b)	$H_0: p = 0.3$ $H_1: p < 0.3$	B1
	$0.7^{28} + 28 \times 0.7^{27} \times 0.3 + {}^{28}C_2 \times 0.7^{26} \times 0.3^2 + {}^{28}C_3 \times 0.7^{25} \times 0.3^3 + {}^{28}C_4 \times 0.7^{24} \times 0.3^4$	M1
	0.0474	A1
	$0.0474 > 0.02$ [Not reject H_0]	M1
	No evidence that suspicion is true.	A1 ft

38) .

a)	$H_0: \mu = 64.6$ $H_1: \mu < 64.6$	B1
	$[\pm]\frac{63.5-64.6}{5.2 \div \sqrt{100}}$	M1
	$[\pm]-2.115$	A1
	'2.115' > 1.96 or '–2.115' < –1.96 [do not accept H_0]	M1
	There is evidence that $\mu < 64.6$	A1 FT

(b)	$\frac{m-64.6}{5.2 \div \sqrt{100}} = -1.96$	M1
	$m = 63.5808$	A1
	$\frac{63.5808-62.7}{5.2 \div \sqrt{100}}$ [= 1.694]	M1
	$1 - \Phi(\text{'}1.694\text{'})$	M1
	0.0451	A1

39) .

(a)	Fireworks are destroyed when tested.	B1
(b)	H_0: Pop mean time lasted (or μ) = 30 H_1: Pop mean time lasted (or μ) < 30	B1
	$\pm \frac{29-30}{\frac{5}{\sqrt{100}}}$	M1
	± -2	A1
	$-2 > -2.326$ [Do not reject H_0]	M1
	There is not enough evidence that mean time lasted is less than 30 seconds OR Not enough evidence to support the inspector's suspicion	A1 FT
(c)	Yes. Because population distribution is unknown [condone not Normal].	B1

40) .

(a)	H_0: $P(0) = \frac{1}{10}$ H_1: $P(0) < \frac{1}{10}$	B1
(b)	For B(30,0.1)	M1
	$P(X = 0) = 0.9^{30}$ [= 0.0424] [<0.1]	M1
	$P(X = 0 \text{ or } 1) = 0.9^{30} + 30 \times 0.9^{29} \times 0.1 = 0.184$ [>0.1]	B1
	Rejection region is 0 zeros	A1
c)	0.0424	B1

(d)	Bin(30, $\frac{1}{40}$)	B1
	$1 - 0.975^{30}$	M1
	0.532 (3dp)	A1
(e)	Not concluding that the probability is less than $\frac{1}{10}$, when in fact it is.	B1

41) .

(a)	H_0: P(red) = 0.2 H_1: P(red) < 0.2	B1
	$P(X \le 4) = 0.8^{40} + 40\times0.8^{39}\times0.2 + {}^{40}C_2\times0.8^{38}\times0.2^2 + {}^{40}C_3\times0.8^{37}\times0.2^3 + {}^{40}C_4\times0.8^{36}\times0.2^4$	M1
	0.0759	A1
	their '0.0759' > 0.05	M1
	[Do not reject H_0]. Not enough evidence that it lands on red fewer times than if it were fair or not enough evidence to suggest that the spinner is biased	A1 FT
(b)	$P(X \leqslant 3) =$ ` 0.0759 ` $- {}^{40}C_4\times0.8^{36}\times0.2^4$	M1
	= 0.0285 or 0.0284	*A1
	Largest value of r is 3	DA1

42) .

(a)	H_0: Population mean time (or μ) = 32.5 H_1: Population mean time (or μ) < 32.5	B1
	$\pm\frac{31.8-32.5}{3.1\div\sqrt{50}}$	M1
	$= \pm -1.597$	A1
	'–1.597' < –1.406 [or '1.597' > 1.406]	M1
	[reject H_0] There is evidence that [population] [mean] **time** has **decreased**	A1 FT

(b)	$\dfrac{a-32.5}{3.1\div\sqrt{50}}=-1.406$	M1
	$a = 31.88$ or 31.9	A1
	$\dfrac{their\text{'}31.88-31.5}{3.1\div\sqrt{50}}$ [= 0.8668 to 0.8760]	M1
	1 – Φ('0.8668')	M1
	= 0.190 to 0.193 (3 sf)	A1

43) .

H_o: Population mean length = 10.3 cm H_1: Population mean length < 10.3 cm	B1
$\pm\dfrac{9.8-10.3}{2.6/\sqrt{100}}$	M1
= –1.923	A1
–1.923 > –2.054 or –2.055	M1
[Not reject H_0] No evidence that [mean] **length** has **decreased**	A1 FT

44) .

(a)	H_0: Population mean = 7.2 or 2.4 H_1: Population mean < 7.2 or 2.4	B1
		1
(b)	$\lambda = 7.2$	B1
	$[P(X\leqslant 2)]=e^{-7.2}\left(1+7.2+\dfrac{7.2^2}{2}\right)$ or $e^{-7.2}(1 + 7.2 + 25.92)$ or 0.0007465 + 0.0053754 + 0.01935 [= 0.0255] $[P(X\leqslant 3)]=\text{'}0.0255\text{'}+e^{-7.2}\times\dfrac{7.2^3}{3!}$ or '0.0255' $+e^{-7.2}$ (62.21) or '0.0255' +0.04644 [= 0.0719]	M1
	P(Type I) = 0.02547 or 0.0255 (3 sf)	B1
(c)	3 > 2 or $P(X\leqslant 3) > 0.05$ or '0.0719' > 0.05	M1
	[Not reject H_0] No evidence that [mean] **number of faults** has **decreased**	A1 FT
		2
(d)	$1 - e^{-1.5}(1 + 1.5 + 1.5^2 / 2)$ or $1 - e^{-1.5}(1 + 1.5 + 1.125)$ or 1 – (0.2231 + 0.3347 + 0.2510)	M1
	= 0.191 (3 sf)	A1