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Probability and Statistics-2

Hyothesis Testing Notes

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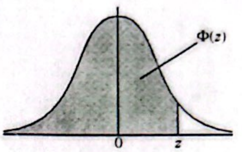
THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \le z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
															ADD				
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that

$$\mathrm{P}(Z \le z) = p.$$

$z = \phi^{-1}(p)$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

S.2

Hypothesis testing - Notes and Revision

§ Null hypothesis: The claim is called the null hypothesis and is denoted by 'H_0'

§ Alternate Hypothesis:
When we don't accept the null hypothesis, then we have an alternate hypothesis, denoted by 'H_1'.
The null hypothesis and the alternate hypothesis both are expressed in terms of a parameter, such as probability 'p' or a mean value 'μ' (normal distribution).

§ Rejection region (or the critical region) and the critical value:

The range of values for which we reject the null hypothesis is the critical region.
The value, at which we change accepting the null hypothesis to rejecting, is called critical value.

§ Type I error:
The type of error, where a null hypothesis is rejected despite being correct, is called a Type-I error.
or P(reject H_0 / H_0 is true)

§ Type II error:
When the null hypothesis is in fact false, but is accepted. or P(Type II error) = P(accept H_0 / H_0 is false)

§ Significance Level: The significance level is the probability of rejecting a claim.
The significance level is given in percentage. (generally it is 5%). "The lower the percentage significance level, the smaller the rejection region, and the more confident you can be of the result."

§ One-tailed test: when the words increased (or decreased) are used.
Two-tailed test: when the word - the parameter is different. $p \neq a$ or $\mu \neq a$

Hypothesis testing using the binomial distribution. $B(n,p)$

Example 1: Jill shoots arrows at a target. Last week, 65% of her shots hit the target. This week Jill claims that she has improved. Out of her first 20 shots this week, she hits the target with 18 shots. Assuming shots are independent, test Jill's claim at the 1% significance level. --[5]

M-16/72/Q3

Solution: Let X be the number of arrows hitting the target in 20 shots.

Null hypothesis, H_0: P(hit target) $= p = 0.65$ (65%)

Alternate hypothesis, H_1: P(hit target) ; $p > 0.65$

$X \sim B(20, 0.65)$ Binomial distribution.

$n=20, p=0.65, q=0.35$

$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

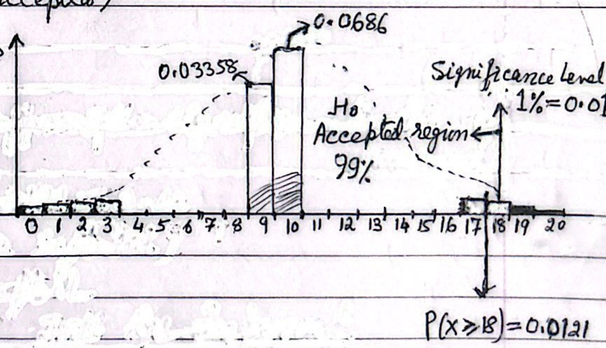
$$P(X \geq 18) = P(X = 18, 19, 20)$$
$$= {}^{20}C_{18}(0.65)^{18}\cdot(0.35)^{2} + {}^{20}C_{19}(0.65)^{19}\cdot(0.35) + {}^{20}C_{20}(0.65)^{20}$$
$$= 0.009984 + 0.001951 + 0.000181 = 0.121 \text{ (3 sf)}$$
$$= 0.0121 > 0.01 \text{ (Significance Level } 1\% = 0.01)$$

${}^{20}C_{18} = {}^{20}C_{2}$, ${}^{20}C_{19} = {}^{20}C_{1}$, ${}^{20}C_{20} = 1$

$\therefore$ There is no evidence that she has improved. (at 1% level)

(H_0 is accepted)

$P(X \geq 0) = 1$



$P(X=18) = 0.009984$
$P(X=19) = 0.001951$
$P(X=20) = 0.000181$

$P(X \geq 18) = 0.012116 > 0.01$

Note:

$P(X \geq 19) = 0.009984 + 0.001951 = 0.002132 < 0.01$

$\Rightarrow$ $X \geq 19$ is the rejection region (or critical region) (Dark Shaded) on the right side.

$X = 19$ is the critical value

Example 2: A television channel claims that 25% of its programmes are nature programmes. Kavita thinks the percentage claims is too high. To test her hypothesis, she chooses 20 programmes at random.

(a) If Kavita carries out the hypothesis test at 10% significance level, define the random variable, state its distribution, including parameters, and define the hypothesis.

(b) If there are only two nature programmes, calculate the test statistics. What conclusion does Kavita reach?

(c) If the significance level was 5%, what conclusion would Kavita reach?

Solutions (a) Let X be the number of nature programmes (random variable)

Binomial distribution: $X \sim B(20, 0.25)$ [$B(n, p) \Rightarrow n$ and p are parameters]

Null Hypothesis: $H_0: p = 0.25$

Alternate hypothesis: $H_1: p < 0.25$

$\left\{ P(X = r) = {}^nC_r\, p^r q^{n-r} \right.$; $p = 0.25$, $q = 0.75$, $n = 20$

(b) Test statistics: $P(X \leq 2) = P(X = 0, 1, 2)$

$= (0.75)^{20} + {}^{20}C_1 \times (.25)^1 (0.75)^{19} + {}^{20}C_2 \times 0.25^2 \times (0.75)^{18}$

$= 0.003171 + 0.021141 + 0.066947 = 0.0913$ (3 s.f)

$0.0913 < 0.10$ (Significance level − 10% or 0.10)

True $\therefore$ H_0 is rejected

The test result lead, Kavita to conclude that there is insufficient evidence to accept the television channel's claim.

(c) For 5% (or 0.05) significance level:

$P(X \leq 2) = 0.0913 > 0.05$ $\therefore$ H_0 is accepted.

(H_1 is rejected)

Note: (b) $P(X \leq 2) = 0.0913$

Kavita would now conclude that the proportion of nature programmes is likely to be 25%, as the television channel claims.

(Rejection region: $X \leq 2$)

Critical Value $= 2$

$\left\{ P(X \leq 3) = 0.0913 + 0.13389 = 0.225 \text{ (3sf)} \right.$

$0.225 > 0.01$

Example 3: World-wide 25% of men are colour-blind but it is believed that the condition is less widespread among a group of remote hill tribe. An anthropologist plans to test this by sending field workers to visit villages in that area. In each village 30 men are to be tested for colour-blindness. Find the rejection region for the test at the 5% level of significance, find the critical value also.

Solution: Random Variable: X is the number of colour-blind men.

The distribution $X \sim B(n,p) = B(30, 0.25)$ $\{25\% = 0.25\}$

Null hypothesis $H_0 : p = 0.25$

Alternate hypothesis $H_1 : p < 0.25$

Significance level $= 5\%$ (0.05)

The rejection region is $X \leq k$, where

$P(X \leq k) \leq 0.05$ and $P(X \leq k+1) > 0.05$ (Significance level)

Consider

$$P(X \leq 3) = P(X = 0, 1, 2, 3)$$

$$P(X = r) = {}^{n}C_r\, p^r \cdot q^{n-r}$$

$$\left\{ \begin{array}{l} B(30, 0.25) \\ p = 0.25,\ q = 0.75 \\ n = 30 \end{array} \right.$$

$$= (0.75)^{30} + {}^{30}C_1 (0.25)^1 \cdot (0.75)^{29} + {}^{30}C_2 (0.25)^2 \cdot (0.75)^{28} + {}^{30}C_3 (0.25)^3 (.75)^{27}$$

$$= 0.000178 + 0.001785 + 0.008631 + 0.026853 = 0.0375$$

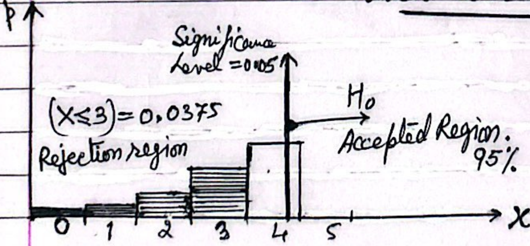
$$\therefore P(X \leq 3) = 0.0375 \leq 0.05 \quad \text{--- (i)}$$

and $P(X \leq 4) = 0.0375 + \underbrace{{}^{30}C_4 (.25)^4 (.75)^{26}}_{(0.05531)} = 0.0929 > 0.05$ --- (ii)

from (i) and (ii)

The rejection region is $X \leq 3$. ✓

and critical value $= 3$ ✓



Example 4: A darts player claims he can hit the bullseye 60% of the time. A fan thinks the player is better than that. Define the random variable and hypothesis. Calculate the test statistic based on the darts player hitting the bullseye 15 times in 17 darts thrown. At what integer percentage significance level is, 15 the critical value?

Solution: Random variable: X be the number of times the dart player hits the bulls eye. The $X \sim B(17, 0.6)$

$\{ 60\% = 0.6 = p$, $n = 17$, $q = 0.4$

Null hypothesis H_0: $p = 0.6$

Alternate hypothesis H_1: $p > 0.6$

$P(X=r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$$P(X \geq 15) = P(X = 15, 16, 17)$$

$$= {}^{17}C_{15} \times (0.6)^{15} \times (0.4)^{2} + {}^{17}C_{16} \times (0.6)^{16} \times (0.4)^{1} + (0.6)^{17}$$

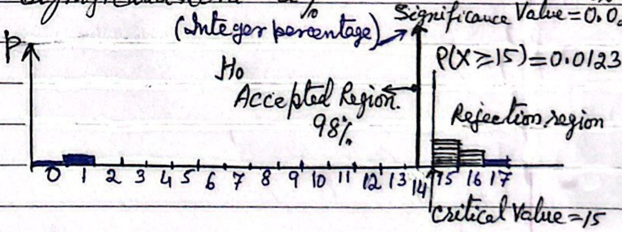
$$= 0.01023 + 0.00191 + 0.000169 = 0.0123$$

$$= 0.0123$$

Given the critical value is 15.

Hence at the smallest integer significance level $1.23\% < 2\%$

$\therefore$ Significance level $= 2\%$



One tailed / Two tailed test.

§ **One tailed test:**

If the parameter probability, p / or mean μ is increase or decrease than it is one tailed test. ($p < a$ or $p > a$) or $\mu < a$ or $\mu > a$)

§ **Two tailed test:**

For a two tailed test the parameter p or μ, mean is different. ($p \neq a$ or $\mu \neq a$)

§ In most two tailed testing the critical testing is shared equally between the two tails.

However in some problems the unequal region is allocated between the two tails.

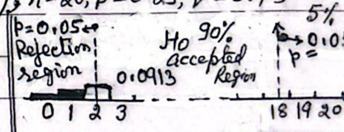
Example 5: A television channel claims that 25% of its programmes are nature programmes. Xavier thinks this claim is incorrect. To test his hypothesis, he chooses 20 programmes at random and carries out a hypothesis test at 10% significance level.

(a) Define random variable, stating its parameters and write down the two hypothesis.

(b) If there are only two nature programmes, calculate the test statistic. What conclusion does Xavier reach?

Solution: Random Variable: Let X be the number of nature programmes.

(a) $n = 20$, $p = 0.25$ (25%) $X \sim B(20, 0.25)$; $n = 20$, $p = 0.25$, $q = 0.75$



Null hypothesis, H_0 : $p = 0.25$

Alternate hypothesis, H_1 : $p \neq 0.25$ (Two-tailed)

(b) The test statistic is:

$$P(X \leq 2) = (0.75)^{20} + {}^{20}C_1 \cdot (.25)^1 \cdot (.75)^{19} + {}^{20}C_2 \cdot (.25)^2 \cdot (.75)^{18} = 0.0913$$

The critical testing is (shared) $\frac{1}{2} \times 0.1 = 0.05$ ($\frac{1}{2}$ of 10%, as a two tailed test is required)

Now $0.0913 > 0.05$. Hence H_0 is accepted.

Xavier concludes that the proportion of nature programmes is likely to be 25% as claimed.

Two tailed test / one tailed test

Example 6: A manufacturer sells bags of 20 marbles in mixed colours. It claims that 30% of the marbles are red. Ginny thinks this is incorrect and tests the claim by opening a bag of marbles and counting how many are red.

(a) Carry out a hypothesis test at the 10% level of significance given that Ginny finds three red marbles. Find the critical value for this test.

(b) Suppose Ginny thinks the percentage should be lower than 30%. State the hypothesis. Will Ginny's conclusion change?

Solution: Random varible, X be the number of red marbles, then $X \sim B(20, 0.3)$

(a) Null hypothesis, $H_0: p = 0.3$

Alternate hypothesis, $H_1: p \neq 0.3$ (Two-tailed)

$\left\{ \begin{array}{l} n = 20 \\ p = 0.3 \ (30\%) \\ q = 0.7 \end{array} \right.$

$P(X \leq 3) = P(X = 0, 1, 2, 3)$

$= (0.7)^{20} + {}^{20}C_1 \times 0.3^1 \times (0.7)^{19} + {}^{20}C_2 \times 0.3^2 \times 0.7^{18} + {}^{20}C_3 \times 0.3^3 \times (0.7)^{17}$

Significance level 10% being two tail test, Significance level $= \frac{1}{2} \times 10\%$ is shared. $= 5\% \ (0.05)$

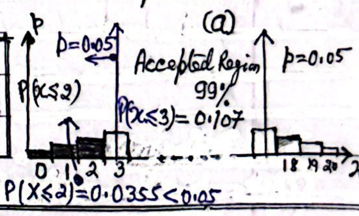
$\therefore P(X \leq 3) = 0.107 > 0.05$ hence H_0 is accepted.

There is sufficient evidence to accept the manufacturer that 30% of the marbles are red.

Now $P(X \leq 2) = 0.7^{20} + {}^{20}C_1 0.3^1 \times 0.7^{19} + {}^{20}C_2 \, 0.3^2 \times 0.7^{18} = 0.0355 < 0.05$

$\therefore H_0$ is rejected at $X = 2$, but accepted at $X = 3$

$\therefore$ The critical value is $X = 2$.

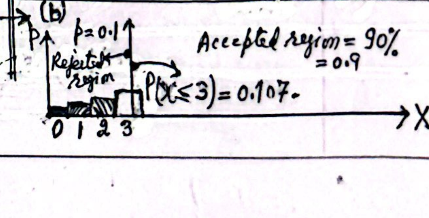


(b) Now H_0, $p = 0.3$ and H_1, $p < 0.3$ $\Big\}$ one tailed, Significance level $= 10\% = 0.1$

and $P(X \leq 3) = 0.107 > 0.10$

So H_0 is accepted.

Hence Ginny's conclusion will not change.



$P(\text{Type-I error}) = P(\text{reject } H_0 / H_0 \text{ is true})$

Example 7. An archer claims that her skill at firing arrows is such that she can hit the bullseye 40% of the time. A hypothesis test at the 5% significance level is carried out to test if the archer is as good as she claims to be. The archer fires 12 arrows at a target.

(a) Define the random variable and its parameters.
(b) State the null and alternate hypothesis.
(c) Find the rejection region.
(d) Show that a type I error occurs if the archer hits the bullseye only once.

Solution: Random Variable: X be the number of arrows that hits the bullseye.
Then X has binomial distribution with $n=12$ and $p=0.4$, $(q=0.6)$
i.e $X \sim B(12, 0.4)$

(b) Null hypothesis, $H_0: p = 0.4$
Alternate hypothesis, $H_1: p < 0.4$ Significance level $= 0.05$

(c) For rejection region, To find the smallest value of n, such that, $P(X \leq n) < 0.05$

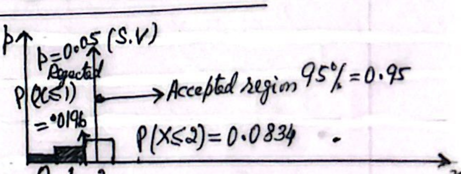
$P(X \leq 2) = 0.6^{12} + {}^{12}C_1 \times 0.4^1 \times 0.6^{11} + {}^{4}C_2 \times 0.4^2 \times 0.6^{10}$
$= 0.0834 > 0.05$ --- (i) (H_0 is accepted)

$P(X \leq 1) = 0.6^{12} + {}^{12}C_1 \times 0.4^1 \times 0.6^{11}$
$= 0.0196 < 0.05$ - (ii) (H_0 is Rejected)

from (i) and (ii) The rejection region is $X \leq 1$.

(d) If $H_0 = 0.4$ is true, then $X=1$ lies in the rejection region as $P(X \leq 1) = 0.0196$; That is the archer's claim that $p = 0.4$ will be be rejected, thus a type I error occurs.

Note: $P(\text{Type I error}) = 0.0196$.



Example 8.: Sumitra has a six-sided die. She suspect that it is biased so that it shows a six less often than it would, if it were fair. She decides to test the die by throwing it 30 times and noting the number of throws on which it shows a six.

(i) It shows a six on exactly 2 throws. Use a binomial distribution to carry out the test at the 5% significance level. ---[5]

(ii) Later, Sumitra repeats the test at the 5% significance level by throwing the die 30 times again.
Find the probability of a type I error in this second test. ---[2]

[S-19/71/Q3]

Solution: Let X be the number of sixes, when a six-sided die is thrown 30 times. Then $X \sim B(30, \frac{1}{6})$

Null Hypothesis H_0: $P(6) = p = \frac{1}{6}$

Alternate hypothesis H_1: $P(6) = p < \frac{1}{6}$

$\left\{ \begin{array}{l} n = 30, p = \frac{1}{6} \\ q = 5/6 \end{array} \right.$

(i) Significance level = 5% or 0.05 ✓

Test statistics is $P(X \leq 2) = P(0) + P(1) + P(2)$ $\quad [P(X = r) = {}^nC_r\, p^r q^{n-r}]$

$$= \left(\frac{5}{6}\right)^{30} + {}^{30}C_1 \cdot \frac{1}{6} \times \left(\frac{5}{6}\right)^{29} + {}^{30}C_2 \left(\frac{1}{6}\right)^2 \cdot \left(\frac{5}{6}\right)^{28}$$

$= 0.103 > 0.05$ the significance level

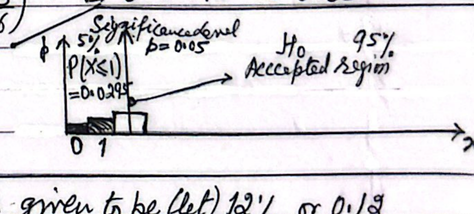
$\therefore$ No evidence that the die is biased. ($\therefore$ H_0 is accepted.)

(ii) For type I error the null hypthesis is rejected: $p < 0.05$

$$p = P(0) + P(1)$$

$$= \left(\frac{5}{6}\right)^{30} + {}^{30}C_1 \cdot \frac{1}{6} \times \left(\frac{5}{6}\right)^{29} = 0.0295 < 0.05$$

$\therefore$ P(Type I error) $= 0.0295$ ✓



Note: If the significance level is given to be (let) 12% or 0.12
Then in part (i) $0.103 < 0.12$ (The significance level)
we will say H_0 is rejected.
The die is biased towards the number 6. (H_1 is accepted)

Example 9 A ten sided spinner has edges numbered 1,2,3,4,5,6,7,8,9,10. Sanjeev claims that the spinner is biased so that it lands on the 10 more often than it would if were unbiased. In an experiment, the spinner landed on the 10 in 3 out of 9 spins.

(i) Test at 1% significance level whether Sanjeev's claim is justified: -[5]

(ii) Explain why a Type I error can not be made. --[1]

In fact the spinner is biased so that the probability that it will land on the 10 on any spin is 0.5.

(iii) Another test at 1% significance level, also based on 9 spins, is carried out. Calculate the probability of a Type II error. --[6]

[S-18/72/Q7]

Solution: Let X be the number of 10's, when a ten-sided spinner spins 9 times.

then $X \sim B(9, 0.1)$

Null Hypothesis $H_0: P(10) = p = 0.1$

Alternate hypothesis $H_1: P(10) = p > 0.1$

(i) Significance level 1% or 0.01

$B(9, 0.1)$ $n = 9$, $p = 0.1$, $q = 0.9$

$P(X \geq 3) = 1 - P(x = 0, 1, 2)$

$= 1 - \{(0.9)^9 + {}^9C_1 \times 0.1 \times (0.9)^8 + {}^9C_2 (0.1)^2 \cdot (0.9)^7\} = 0.05297$

$= 1 - (0.9470) = 0.053 > 0.01$

$\therefore$ No evidence (at 1%) to reject H_0.

Claim is not justified.

(ii) Type I error cannot be made as H_0 is not rejected

(Here H_0 is accepted)

Note:

P(Type-I error) = P(Reject H_0 / H_0 is accepted)

(iii) $P(X \geq 4) = 0.05297 - P(3)$

$= 0.05293 - {}^9C_3 (0.9)^6 (0.1)^3$

$= 0.00833 < 0.01$

$\therefore$ Critical value is 4.

$B(9, 0.5)$, $p = 0.5$, $q = 0.5$, $n = 9$

$P(X < 4) = P(x = 0, 1, 2, 3)$

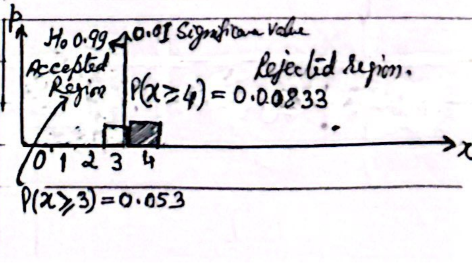
$= (0.5)^9 + {}^9C_1 (.5)(.5)^8 + {}^9C_2 (.5)^2 (.5)^7 + {}^9C_3 (.5)^3 \cdot (.5)^6$

$= 0.254$

P(Type II) = 0.254 ✓

(null hypothesis is false as $p = 0.5$ but is accepted)

P(Type II error) = P(accept H_0 / when H_0 is rejected)



Hypothesis testing using Poisson distribution Po(λ)

Example 10: A hospital patient's white blood cell count has a Poisson distribution. Before undergoing treatment the patient had a mean white blood cell count 5.2. After the treatment a random measurement of the patient's white blood cell count is made, and is used to test at the 10% significance level whether the mean white blood cell count has decreased.

(i) State what is meant by a Type I error in the context of the question, and find the probability that the test results in a Type I error. --- [4]

(ii) Given that the measured value of the white blood cell count after the treatment is 2, carry out the test. -[3]

(iii) Find the probability of a Type II error if the mean white blood cell count after the treatment is actually 4.1 [3]

[S-10/71/Q7]

Solution (i) Type I error occurs when we conclude that the patient's white blood cell count has decreased, when in fact it has not.

we need to find the largest n for which $P(X \leq n) < 0.10$

$P(X \leq 0) = e^{-5.2} = 0.005516 < 0.10$ (Significance level 10%)

$P(X \leq 1) = e^{-5.2}(1 + 5.2) = 0.03420 < 0.10$ ✓ (Ho is rejected)

$P(X \leq 2) = e^{-5.2}\left(1 + 5.2 + \frac{5.2^2}{2!}\right) = 0.10878 > 0.10$ (Ho is True)

∴ Largest value for Type I error $n = 1$, P(Type I error) $= 0.0342$ ✓

(ii) $H_0: \lambda = 5.2$ and $H_1: \lambda < 5.2$

from Part I, $P(X \leq 2) = 0.10878 > 0.10$ ⇒ Ho is accepted. ✓

hence there is insufficient evidence that the white blood cell count has decreased.

(iii) P(Type II error) $= P(X > 1 \mid \lambda = 4.1)$ [Ho is rejected for $x \leq 1$]

$= 1 - (P(0) + P(1)) = 1 - e^{-4.1}(1 + 4.1) = \underline{0.915}$ ✓

Note: P(Type II error) = P(accept Ho / whereas Ho is rejected)

as $(\lambda = 4.1 \neq 5.2)$

Example 11: In the past, the number of house sales completed per week by a building company modelled by a random variable which has the distribution $Po(0.8)$. Following a publicity campaign, the builders hope that the mean number of sales per week will increase. In order to test at the 5% significance level whether this test is the case, the total number of sales during the first 3 weeks after the campaign is noted. It is assumed that a Posisson model is still appropriate.

(i) Given that the total number of sales during the 3 weeks is 5, carry out the test. --[6]

(ii) During the following 3 weeks the same test is carried out again, using the same significance level. Find the prob. of a Type I error. --[3]

(iii) Explain what is meant by a Type I error in this context. --[1]

(iv) State what further information would be required in order to find the probability of a type II error. [W-10/73/Q7] --[1]

Solution (i) For a 3 week period $\lambda = 3 \times 0.8 = 2.4$

$H_0 : \lambda = 2.4$ and $H_1 : \lambda > 2.4$

$P(X \geq 5) = 1 - P(X \leq 4)$

$= 1 - e^{-2.4}\left[1 + 2.4 + \frac{2.4^2}{2!} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!}\right]$

$= 0.0959 > 0.05$ $\therefore$ H_0 is accepted.

$\therefore$ There is insufficient evidence to suggest that the mean number of sales per week has increased.

(ii) P(Type I error) = P(H_0 reject / H_0 is true)

Now $P(X \geq 6) = 1 - P(X \leq 5)$

$= 1 - e^{-2.4}\left[1 + 2.4 + \frac{2.4^2}{2!} + \dots + \frac{2.4^5}{5!}\right]$

$= 0.0357 < 0.05$

The smallest number $n = 6$, so that

P(type I error) = 0.0357 ✓

(iv) The actual mean number of sales per week (or an alternate to 2.4) is given.

Note:

P(Type II error)

= P(H_0 is accepted / H_0 is false)

(iii) A type I error would be made if we conclude that the mean number of sales per week had increased, when it had not, (or that the publicity campaign was a success, when it was not)

Hypothesis testing using Normal distribution.

(Sample mean, $Var = \sigma^2/n$)

12. In the past, the mean annual crop yield from a particular field has been 8.2 tonnes. During the last 16 years, a new fertiliser has been used on the field. The mean yield for these 16 years is 8.7 tonnes. Assume that the yields are normally distributed with standard deviation 1.2 tonnes. Carry out a test at the 5% significance level of whether the mean yield has increased. ---[5]

S-16 | 73 | Q2

Solution: $X \sim N(\mu, \frac{\sigma^2}{n}) = N(8.2, \frac{1.2^2}{16})$ Sample for $n = 16$ years

Sample Var $= \frac{\sigma^2}{n} = \frac{1.2^2}{16}$

$\sigma = 1.2/\sqrt{16}$

Null hypothesis H_0: Population mean yield $\mu = 8.2$

Alternate hypothesis H_1: Population mean yield $\mu > 8.2$

Significance Value 5% = 0.05

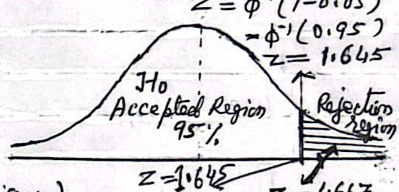
$z = \phi^{-1}(1 - 0.05)$

$= \phi^{-1}(0.95)$

$z = 1.645$

$$P(X > 8.7) = P\left(z > \frac{8.7 - 8.2}{1.2/\sqrt{16}}\right)$$

$$= P(z > 1.667)$$



Now $z = 1.667 > 1.645$ (for 5% Significance Value)

Hence reject H_0.

Evidence that the mean yield has increased. (H_1 is true)

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Example 13: A geologist suspects that rocks in another area have a mean mass which is less than 14.2 kg. A random sample of 100 rocks in this area has sample mean 13.5 kg. Assuming that the standard deviation for rocks in this area is also 3.1 kg, test at the 2% significance level whether the geologist is correct.
[S-18 / 71 / Q5(iii)] - [5]

$X \sim N(\mu, \sigma^2)$, $\sigma^2 = 3.1$

Solution: Random variable $\bar{x}$ denotes the mean mass; $N(14.2, \frac{\sigma^2}{n})$

Mean mass, $H_0 : \mu = 14.2$
Mean Mass, $H_1 : \mu < 14.2$

Sample $n = 100$

$Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{3.1^2}{100}$

$SD = \frac{\sigma}{\sqrt{n}} = \frac{3.1}{\sqrt{100}}$

$$P(\bar{X} < 13.5) = P\left(z < \frac{13.5 - 14.2}{\frac{3.1}{\sqrt{100}}}\right)$$

$$= P(z < -2.258)$$

$$= 1 - \phi(2.258)$$

$$= 1 - 0.9881$$ (Significance level 2%)

$$= 0.0119 < 0.02$$ (H_0 is rejected)

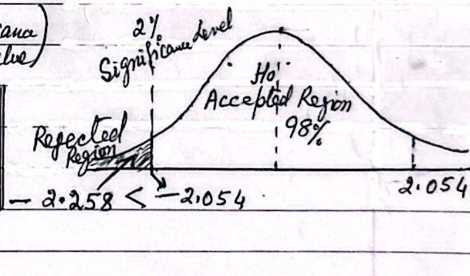
$\therefore$ There is evidence that mean mass is < 14.2

Alternate method:

Significance level 2% = 0.02
$z = \phi^{-1}(1 - 0.02)$
$= \phi^{-1}(0.98) = 2.054$

$z = -2.258 < -2.054$ (at 2% Significance Value)

(H_0 is rejected)
There is evidence (at 2% level) that mean mass in this area is less than 14.2 (or < 14.2)



Example 14: In Europe the diameters of women's ring have mean 18.5 mm. Researchers claim that women in Jakarta have smaller fingers than women in Europe. The researchers took a random sample of 20 women in Jakarta and measured the diameters of their rings. The mean diameters was found to be 18.1 mm. Assuming that the diameters of women's ring in Jakarta have a normal distribution with standard deviation 1.1 mm, carry out a hypothesis test at 2.5% level to determine whether the researcher's claim is justified. --[5]

[S-09/71/Q1]

Solution: Random variable: X denotes the mean diameter of ring

Null hypothesis: H_0: $\mu = 18.5$ mm

Alternate hypothesis H_1: $\mu < 18.5$ mm

Significance level = 2.5% (or 0.025)

Now the mean diameter of 20 women's ring be $\bar{x}$ $\quad n = 20$

Then $\bar{X} \sim N\left(18.5, \frac{1.1^2}{20}\right)$ $\left\{N\left(\mu, \frac{\sigma^2}{n}\right)\right\}$

Now $P(\bar{X} \leq 18.1) = P\left(Z \leq \frac{18.1 - 18.5}{\frac{1.1}{\sqrt{20}}}\right)$ $\quad SD = \frac{\sigma}{\sqrt{n}} = \frac{1.1}{\sqrt{20}}$

$= P(Z \leq -1.626)$ ⊗

$= 1 - \phi(1.626) = 1 - 0.9474$

$= 0.052$ ✓

Now $0.052 > 0.025$ (Significance level)

so H_0 is accepted.

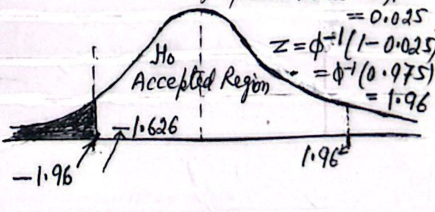
⊗⊗ Hence the researcher's claim that women in Jakarta have smaller fingers (rings with smaller diameters) is not justified.

Alternate method: ⊗

$z = -1.626 > -1.96$

(H_0 is accepted)

⊗⊗ Hence ----



P(Type II - error) = P(accept H_0 / H_0 is false)

Example 15: A gardener knows from experience that the height, X, of his sunflowers is normally distributed with mean 1.8 m and standard deviation 0.32 m. A friend claims that she can make sunflowers taller by singing to them daily. They decide to test the claim. One plant, selected at random, is sung to each day while it grows.

(a) State the null and alternate hypothesis.

(b) The gardener will be convinced that singing increasing sunflower height if $X > 2.25$ m. Calculate the probability of a Type I error.

(c) The friend says that the mean height of sunflowers sung to is 2.4 m. Assuming the standard deviation is the same, calculate the probability of a Type II error.

Solution: Random Variable, X denote the height of sunflowers.

(a) Null hypothesis H_0: Pop mean height = 1.8 $\quad X \sim N(\mu, \sigma^2)$

Alternate hypothesis H_1: Pop mean height > 1.8 $\quad X \sim N(1.8, 0.32^2)$

(b) $P(\text{Type I error}) = P(X > 2.25 / \mu = 1.8)$

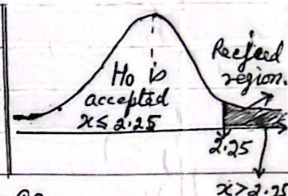
P(H_0 is false / whereas H_0 is True)

$= P\left(Z > \frac{2.25 - 1.8}{0.32}\right)$

$= P(Z > 1.406)$

$= 1 - \phi(1.406) = 1 - 0.92$

$= \underline{0.0800}$



(H_0 is false for $x > 2.25$)

(c) $P(\text{Type II error}) = P(X \leq 2.25 / \mu = 2.4)$

or P(H_0 is True / whereas H_0 is false)

$x \leq 2.25 \quad (\mu = 2.4 \neq 1.8)$

$= P\left(Z \leq \frac{2.25 - 2.4}{0.32}\right)$

$= P(Z \leq -0.469)$

$= 1 - \phi(0.469)$

$= 1 - 0.6804$

$= \underline{0.320}$

$B(n,p) \sim N(\mu,\sigma^2)$

Example 16: A farmer finds that 30% of his sheep are deficient in a particular mineral. He changes their feed and test 80 sheep to find out if the number has decreased.

(a) Define the random variable and hypothesis. Using a suitable approximating distribution, carry out a hypothesis test at 10% significance level given that 19 of the sheep are mineral deficient.

(b) Work out the critical value.

Solutions: Random variabe: Let X is the number of sheep deficient in mineral.

Then distribution $X \sim B(80, 0.3)$ $\left[\because n=80,\ p=0.3 \text{ (30\% are deficient)},\ q=0.7\right]$

Null hypothesis $H_0: p=0.3$

Alternate hypothesis $H_1: p<0.3$.

Approximating the distribution from Binomial to Normal distribution

$X \sim N(\mu,\sigma^2) = N(24, 16.8)$ $\left[\begin{array}{l} np>5 \\ nq>5 \end{array}\right.$ $\left[\begin{array}{l} \mu = np = 80\times0.3 = 24 \\ \sigma^2 = npq = 80\times0.3\times0.7 = 16.8 \end{array}\right.$

Now $P(X\leq 19) \approx P\left(Z \leq \frac{19.5-24}{\sqrt{16.8}}\right)$

$= P(Z \leq -1.098)$

$= 1-\phi(1.098)$

$= 0.136 > 0.10$ [$\because$ Significance level 10%, (or 0.10)]

[Continuity Correction; $P(X\leq 19) \to X = 19.5$ for Normal distribution]

$\therefore$ H_0 is accepted.

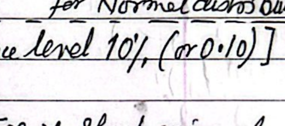
$\therefore$ There is no evidence to suggest that the percentage of sheep mineral deficient has decreased. (H_1 is rejected).

(b) To find the critical value 'C': with continuity correction to find the largest integer C: $P(X \leq C+0.5) < 0.1$

$\Rightarrow P\left(Z < \frac{(C+0.5)-24}{\sqrt{16.28}}\right) < 0.1$

$\phi\left[\frac{(C+0.5)-24}{\sqrt{16.28}}\right) < 0.1$

$\Rightarrow \frac{(C+0.5)-24}{\sqrt{16.28}} < -1.282 \Rightarrow C < 18.245$



$\phi(-z) = 1-\phi(z) = 0.1$

$\Rightarrow \phi(z) = 0.9$

$z = \phi(0.9) = 1.282$

$\therefore$ The critical Value $= 18$ ✓

17. A survey taken last year showed that the mean number of computers per household in Branley was 1·66. This year a random sample of 50 households in Branley answered a questionnaire with the following results:

Number of Computers	0	1	2	3	4	>4
Number of household	5	12	18	10	5	0

(i) Calculate the unbiased estimates for the population mean and Variance of the number of computers per household in Branley this year. --- [3]

(ii) Test at the 5% Significance level whether the mean number of Computers per household has changed since last year. --[5]

(iii) Explain whether it is possible that a type I error may have been made in the test in part (ii) --[1]

(iv) State what is meant by a type II error in the context of the test in part (ii) and give the set of values of the test statistic that could lead to a Type II error being made. --[2]

[S-12/71/Q6]

Solution: (i) $\bar{x} = \frac{\Sigma f_i x_i}{n} = \frac{0+12+36+30+20}{50} = \frac{98}{50}$

$\bar{x} = 1.96$ ✓

$\Sigma f x^2 = 254$

Variance $s^2 = \frac{n}{(n-1)} \times \left(\frac{\Sigma f x^2}{n} - (\bar{x})^2\right)$

(Estimates σ^2 unbiased)

$= \frac{50}{49}\left(\frac{254}{50} - 1.96^2\right)$

$s^2 = \frac{1548}{1225} = 1.2637$

(ii) H_0: Pop Mean $= 1.66$

H_1: Pop Mean $\neq 1.66$ (Two-tailed)

Test Statistic: $P(X > 1.96) = P\left(Z > \frac{1.96 - 1.66}{\sqrt{\frac{1.2637}{50}}}\right) = 1.887$

$1.887 < 1.96$ [Significance level 5%, $z = 1.96$]

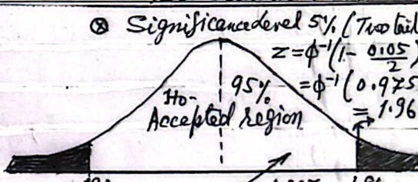
No evidence that mean has changed.

(iv) mean not changed when it has.

$-1.96 <$ test Statistic < 1.96 ✓

Significance level 5% (Two tailed)

$z = \phi^{-1}\left(1 - \frac{0.05}{2}\right) = \phi^{-1}(0.975) = 1.96$



The test statistic:

$z = \left(\frac{\bar{x} - \mu}{\sqrt{\frac{s^2}{n}}}\right)$

(iii) No, as H_0 is not rejected.

(A Type I error occurs when a true null hypothesis is rejected)

(or H_0 is rejected / H_0 is true)

$B(n,p) \sim N(\mu, \sigma^2)$

(To find Critical value)

Example 18: It is claimed that the proportion of people worldwide with red hair is 1.2%. Margaret thinks the proportion of people in Scotland with red hair is higher. At her school there are 850 students, 15 of the students have red hair.

(a) Test Margaret's claim at 2% significance level. ---[5]

(b) Find the critical value [3]

Solution: Random Variable: Let X be the number of red hair, and $X \sim B(850, 0.012)$

(a)

$p = 0.012,\ q = 0.988$ [$1.2\% = 0.012$]

Approximating distribution from Binomial to Normal distribution $N(\mu, \sigma^2)$ $\left[\begin{matrix} np>5 \\ nq>5 \end{matrix}\right.$

$X \sim N(10.2, 10.0776)$

$\left\{\begin{matrix} \mu = np = 850 \times 0.012 = 10.2 > 5 \\ \sigma^2 = npq = 850 \times 0.012 \times 0.988 \\ \sigma^2 = 10.0776 \end{matrix}\right.$

Null hypothesis, $H_0: \mu = 10.2$

Alternate hypothesis, $H_1: \mu > 10.2$

Now $P(X \geq 15) \sim P\left(z \geq \dfrac{14.5 - 10.2}{\sqrt{10.0776}}\right)$ $\left\{\begin{matrix} \text{Continuity correction} \\ P(X \geq 15) \rightarrow P(X \geq 14.5) \end{matrix}\right.$

$= P(z \geq 1.355)$

$= 1 - \phi(1.355) = 1 - 0.9123 = 0.0877$ ✓

here $0.0877 > 0.02$, [Significance Value $2\% = 0.02$]

So H_0 is accepted, or

There is insufficient evidence to support the claim that the proportion of red-haired people in Scotland is greater that 1.2%

(b) Now let the critical value be 'x' with a continuit correction be $(x - 0.5)$.

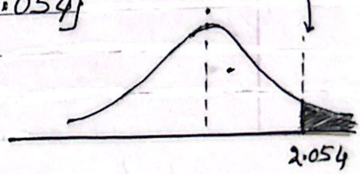
$P\left(z \geq \left(\dfrac{(x-0.5) - 10.2}{\sqrt{10.0776}}\right)\right) < 0.02$ (Significance Level 0.02)

$\Rightarrow P\left(z \leq \left(\dfrac{(x-0.5) - 10.2}{\sqrt{10.0776}}\right)\right) > 0.98$ $\left\{\begin{matrix} z = \phi^{-1}(1 - 0.02) \\ = \phi^{-1}(0.98) \\ = 2.054 \end{matrix}\right.$

$\Rightarrow \dfrac{x - 10.7}{\sqrt{10.0776}} > \phi^{-1}(0.98) = 2.054$

$\Rightarrow x > 17.22$

$\therefore$ Critical Value is 18. ✓



$$Po(\lambda) \sim N(\lambda, \lambda)$$
$$(\lambda = \mu = \sigma^2)$$

§ Hypothesis testing with the Poisson distribution to Normal distribution

Example 19: The number of calls to consumer hotline can be modelled by a Poisson diseribution with mean 62 calls every 5 minutes. Salina believes that average is too low and observes the number of calls recorded during a randomly chosen 5 minute interval to be 70. Stating the null and alternate hypothesis, test Salina's belief at the 10% significance level.

Solution: Random Varicble, X denotes the number calls recorded 5 minutes int-erval

Null hypothes is, H_0 mean $\lambda = 62$

Alternate hypothesis H_1: mean $\lambda > 62$ calls per 5 minutes

$X \sim Po(62) \rightarrow$ approximated to (as $\lambda > 15$) $N(\mu, \sigma^2) = N(\lambda, \lambda) = N(62, 62)$

$$P(X \geq 70) = P\left(Z \geq \frac{69.5 - 62}{\sqrt{62}}\right)$$

{ Continuity correction
$X \geq 70 \rightarrow Z \rightarrow 69.5$

$$= P(Z \geq 0.953)$$
$$= 1 - \phi(0.953) = 1 - 0.8297$$
$$= 0.1703$$

here $0.1703 > 0.10$ (Significance level 0.1 (10%))

$\therefore$ H_0 is accepted

There is insufficient evidence to support Salina's belief that the mean number of calls is greater than 62 per 5 min.

2Q A shop sells 2Kg bags of potatoes. A quality control inspection checks the masses of 80 randomly chosen bags. Their masses, x, are summarised as follows:

$$\Sigma x = 158.14 \quad \text{and} \quad \Sigma x^2 = 314.094$$

Assuming the masses of the bags of potatoes are normally distributed, investigate at 5% level of significance whether there is any evidence that the bags are underweight.

Solution: $\Sigma x = 158.14$ and $\Sigma x^2 = 314.094$, $n = 80$ (large)

Sample mean $\bar{x} = \frac{158.14}{80} = 1.97675$, (unknown variance)

Unbiased estimated variance $s^2 = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$

$$= \frac{1}{79}\left(314.094 - \frac{158.14^2}{80}\right)$$

$$s^2 = 0.018870316$$

Null Hypothesis H_0: Population mean mass = 2 Kg
Alternate Hypothesis H_1: Population mean mass < 2 Kg

$SD = \sqrt{\frac{s^2}{n}}$

$$P(\bar{x} \leq 1.97675) = P\left(Z \leq \frac{1.97675 - 2}{\sqrt{\frac{0.018870316}{80}}}\right) = P(Z \leq -1.514)$$

$$= 1 - \phi(1.514)$$
$$= 1 - 0.9350$$
$$= 0.0650$$

$0.0650 > 0.05$ (significance level 5%)

$\therefore$ H_0 is accepted.

There is no evidence to suggest that the bags of potatoes are underweight.

21. The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm. A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of n plants of this variety from this region and finds that their mean height is 73.1 cm. He then carries out an appropriate hypothesis test.

(i) He finds that the value of the test statistic $z = -1.563$, correct to three decimal places. Calculate the value of n. State an assumption necessary for your calculation. ---[4]

(ii) Use this value of test statistic to carry out the hypothesis test at the 6% significance level. ---[3]

[S-13/72/Q3]

Solution:
(i) $P(X \leq 73.1) = P\left(z \leqslant \dfrac{73.1 - 75.2}{5.7/\sqrt{n}}\right)$ $\left\{ X \sim N\left(\bar{x}, \dfrac{\sigma^2}{n}\right) \right.$ Assume S.d for the region $= 5.7$

$\Rightarrow \dfrac{73.1 - 75.2}{5.7/\sqrt{n}} = -1.563$ (Given)

$\Rightarrow \sqrt{n} = \dfrac{-1.563 \times 5.7}{73.1 - 75.2} = \dfrac{\cancel{-}1.563 \times 5.7}{\cancel{-}2.1}$

$\Rightarrow n = \left(\dfrac{1.563 \times 5.7}{2.1}\right)^2 = 18 \checkmark \Rightarrow \underline{n = 18} \checkmark$

(ii) $\left.\begin{array}{l} H_0: \text{Pop mean } \mu = 75.2 \\ H_1: \text{Pop mean } \mu < 75.2 \end{array}\right\}$

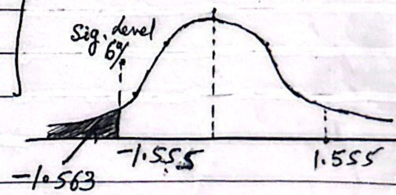
Significance level at 6% $= 0.06$

$z = \phi^{-1}(1 - 0.06) = \phi^{-1}(0.94)$

$z = 1.555$

$P(X \leq 73.1) = P\left(z \leq \dfrac{73.1 - 75.2}{5.7/\sqrt{20}}\right)$

$= P(z \leq -1.563)$



Now z: $-1.563 < -1.555$
(H_0 is not True)

There is evidence that plants are shorter.

Scanned with CamScanner

22. In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a randomly chosen sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. ---[5]

Later Laxmi carries out a similar test with the same hypothesis, at the 8% significance level, using another random sample of 50.

(b) Given that the sample mean is now 31.5, find the probability of Type II error. ---[5]

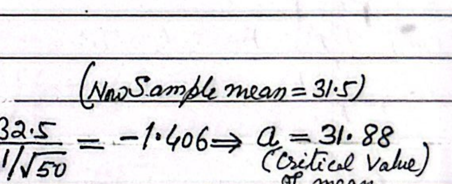
[W-22/61/07]

Solution: H_0: Population mean time $\mu = 32.5$
H_1: Population mean time $\mu < 32.5$

$N(\mu, \sigma^2) \rightarrow N(32.5, 3.1)$
Sample $n = 50$
Sample variance $= \frac{\sigma^2}{n} = \frac{3.1^2}{50}$
Significance level $\rightarrow 8\% = 0.08$

$$P(X < 31.8) = P\left(z < \left(\frac{31.8 - 32.5}{3.1/\sqrt{50}}\right)\right)$$
$$= P(z < -1.597)$$

$z = \phi^{-1}(1 - .08)$
$= \phi^{-1}(.92)$
$= 1.406$



Now $-1.597 < -1.406$
(Reject H_0)
There is evidence that (population) (mean) time has decreased.

(b) Let the critical value $= a$

$$P(X < a) = P\left(z < \frac{a - 32.5}{3.1/\sqrt{50}}\right)$$

(Now Sample mean = 31.5)

$$\Rightarrow \frac{a - 32.5}{3.1/\sqrt{50}} = -1.406 \Rightarrow a = 31.88$$

(Critical Value of mean)

(Now Sample mean = 31.5)

$$P(\text{Type II error}) = P(X \geq 31.88) = P\left(z > \frac{31.88 - 31.5}{3.1 \div \sqrt{50}}\right) = P(z > 0.8668)$$

P(accept H_0 / whereas H_0 is rejected)
(as the sample mean $31.5 < 32.5$)

$$= 1 - \phi(0.8668)$$
$$= 0.190 \checkmark \text{ (3 sf)}$$