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S-2

Probability and Statistics-2

The Poisson Distribution

(Solution Ex-2) Revision

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Example 1. The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30000 letters and these errors occur randomly. A randomly chosen booklet from this publisher contains 12500 letters.
Use a suitable approximating distribution to find the probability that this booklet contains atleast 2 errors. --- [3]

[M-20/ 62/ Q1]

Solution: $\lambda = np = 12500 \times \frac{1}{30,000} = \frac{5}{12} = 0.417 < 5$ $\left\{ \begin{array}{l} p = 1/30000 \\ n = 12,500 \end{array} \right.$

$\boxed{B(n,p) \sim Po(\lambda)}$

$P(X \geq 2) = 1 - P(X=0 \text{ or } X=1)$

$= 1 - e^{-0.417}(1 + 0.417) = 0.0661$ ✓ $\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

2. The number of accidents on a certain road has a Poisson distribution with mean 0.4 per 50 day period.

(a) Find the probability that there will be fewer than 3 accidents during a year (365 days). ---[3]

(b) The probability that there will be no accidents during a period of n days is greater than 0.95. Find the largest possible value of n. --[4]

[M-20/ 62/ Q4]

Solution (a) For 365 days $\lambda = \frac{0.4}{50} \times 365 = 2.92$ $\left[P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

$P(X<3) = P(0) + P(1) + P(2) = e^{-\lambda}\left(1 + \lambda + \frac{\lambda^2}{2}\right) = e^{-2.92}\left(1 + 2.92 + \frac{2.92^2}{2}\right)$

$= 0.441$ (3 sf)

(b) $P(X=0) = e^{-\lambda} > 0.95$

$\Rightarrow \ln e^{-\lambda} > \ln 0.95 \Rightarrow -\lambda > -0.051293$

$\Rightarrow \lambda < 0.051293$ --- ①

$\left\{ \begin{array}{l} \text{for } \lambda = 0.4 \to 50 \text{ days} \\ \lambda = 1 \Rightarrow \frac{50}{0.4} \text{ day} \end{array} \right.$

from ① and ②

$n < 6.411$ days

$\therefore \underline{n = 6}$ ✓

$\left\{ \lambda = 0.051293 \to \text{day} = \frac{50}{0.4} \times 0.051293 = 6.411 \text{ days} \right.$ -- ②

§ Poisson distribution as an approximation to binomial distribution:

$X \sim B(n, p)$, when the value of 'n' is large and 'p' is small, such that np is moderate ($n > 50$ and $np < 5$ (or $p < 0.1$)), the Poisson distribution with mean np can be used as an approximation for the binomial distribution.

$$X \sim B(n,p) \longrightarrow P_o(\lambda) \text{ where } \lambda = np.$$

Example 3. On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of X, giving the values of any parameters. --- [1]

(b) State an approximating distribution for X, giving the values of parameters. - [2]

(c) Use this approximating distribution to find each of the following.

(i) $P(X = 4)$ (ii) $P(2 \leqslant X \leqslant 4)$ --- [2] + [2]

(d) Use a suitable approximating distribution to find the probability that, in a random sample of 700 microchips, there will be at least 1 faulty one.

[M-21/62/Q4] --- [3]

Solution (a) $p = \frac{1}{400}$, $n = 1000 \Rightarrow$ Binomial distribution: $X \sim B\left(1000, \frac{1}{400}\right)$.

(b) n is very large and p is small hence Binomial distribution approximates to Poisson: $\lambda = np = 1000 \times \frac{1}{400} = 2.5 < 5$

or $P_o(2.5)$

(c) (i) $P(X = 4) = e^{-2.5} \times \frac{2.5^4}{4!} = 0.134$ (3 s.f.) $\left\{ P(X = r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right.$

(ii) $P(2 \leqslant X \leqslant 4) = P(X=2) + P(X=3) + P(X=4)$

$$= e^{-2.5}\left[\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!}\right] = 0.604 \text{ (3 s.f.)}$$

(d) Now $\lambda = \frac{1}{400} \times 700 = 1.75$ $[\lambda = np]$

$P(X \geqslant 1) = 1 - P(X = 0)$ $P(0) = e^{-\lambda}$

$= 1 - e^{-1.75} = 0.826$

4.(a) Two ponds, A and B, each contain a large number of fish. It is known that 2.4% of fish in pond A are carp and 1.8% of fish in pond B are carp. Random samples of 50 fish from pond A and 60 fish from pond B are selected.
Use appropriate poisson approximation to find the following probabilities:
(i) The sample contains at least 2 carp from pond A and at least 2 carp from pond B. ---[3]
(ii) The samples contain at least 4 carp altogether. ---[3]
(b) The random variables X and Y have the distributions $Po(\lambda)$ and $Po(\mu)$ respectively. It is given that:
- $P(X=0) = [P(Y=0)]^2$,
- $P(X=2) = k[P(Y=1)]^2$, where k is a non-zero constant. Find the value of k.

M-22/62/Q7 ---[4]

Solution (a) $p_A = 0.024$, $n_A = 50 \Rightarrow \lambda_A = np = 50 \times 0.024 = 1.2$
(i) and $p_B = 0.018$, $n_B = 60 \Rightarrow \mu_B = np = 60 \times 0.018 = 1.08$

$\left\{ P(2) = e^{-\lambda} \cdot \frac{\lambda^2}{2!},\ P(0) = e^{-\lambda},\ P(1) = e^{-\lambda} \times \lambda \right.$

$$P(A \geq 2) \cdot P(B \geq 2) = (1 - P(A=0,1))(1 - P(B=0,1)) = (1 - e^{-1.2}(1+1.2)) \times (1 - e^{-1.08}(1+1.08))$$
$$= \underline{0.0991}\ (3\ s.f)$$

(ii) $\lambda = 0.024 \times 50 + 0.018 \times 60 = 2.28$
$$P(A+B \geq 4) = 1 - P(A+B \leq 3) = 1 - e^{-2.28}\left(1 + 2.28 + \frac{2.28^2}{2!} + \frac{2.28^3}{3!}\right) = \underline{0.197}\ (3s.f)$$

(b) $P(X=0) = [P(Y=0)]^2 \Rightarrow e^{-\lambda} = (e^{-\mu})^2 \Rightarrow e^{-\lambda} = e^{-2\mu}$ --- (1)
and $P(X=2) = k[P(Y=1)]^2 \Rightarrow e^{-\lambda} \cdot \frac{\lambda^2}{2} = k[e^{-\mu} \times \mu]^2 \Rightarrow e^{-\lambda} \cdot \frac{\lambda^2}{2} = ke^{-2\mu} \cdot \mu^2$ --- (2)
from (1) and (2)
$$e^{-2\mu} \cdot \frac{(2\mu)^2}{2} = k \cdot e^{-2\mu} \cdot \mu^2 \qquad [\because \text{from (1) } e^{-\lambda} = e^{-2\mu} \Rightarrow \lambda = 2\mu]$$
$$\Rightarrow \cancel{e^{-2\mu}} \cdot 2\cancel{\mu^2} = k\cancel{e^{-2\mu}} \times \cancel{\mu^2}$$
$$\Rightarrow \underline{k = 2}$$

5. The number of orders arriving at a shop during an 8-hours working day is modelled by a random variable X with distribution Po(25.2).

(a) State two assumptions that are required for the Poisson model to be valid in this context. --- [2]

(b)(i) Find the probability that the number of orders that arrive in a randomly choosen 3-hour period is between 3 and 5 inclusive. --- [3]

(ii) Find the probability that, in two randomly chosen 1-hour period, exactly 1 order will arrive in one of the 1 hour periods, and at least 2 orders will arive in the other 1-hour period. -- [4]

(c) The shop can only deal with a maximum of 120 orders during any 36-hour period.

Use a suitable approximating distribution to find the probability that, in a randomly chosen 36-hour period, there will be too many orders for the shop to deal with. --- [4]

M-23/62/Q2

Solution 8 Hr period, $\lambda = 25.2$ $\quad P(X=r) = \dfrac{e^{-\lambda} \times \lambda^r}{r!}$

(a) Orders arrive at a constant mean rate.
Orders arrive at random.
Orders arrive independently
Orders arrive singly.

(b)(i) for 3 Hours period $\lambda = \frac{3}{8} \times 25.2 = 9.45$

$P(3 \leq X \leq 5) = P(3) + P(4) + P(5)$

$= e^{-9.45}\left[\dfrac{9.45^3}{3!} + \dfrac{9.45^4}{4!} + \dfrac{9.45^5}{5!}\right]$

$= 0.0866$ (3 sf)

(ii) for 1 hr period; $\lambda = \frac{25.2}{8} = 3.15$

P(1 order in one period & at least 2 in other)

$= [P(X=1) \times P(X \geq 2)] \times 2$

$= [P(X=1) \times \{1 - (P(0) + P(1))\}] \times 2$

$= 2 \times e^{-3.15} \times 3.15 \times \{1 - e^{-3.15}(1 + 3.15)$

$= 0.26996 \times 0.8221$

$= 0.222$ (3 s.f)

(c) for 36 hr period; $\lambda = \frac{25.2}{8} \times 36 = 113.4$

$Po(\lambda) \sim N(\mu, \sigma^2) = N(\lambda, \lambda)$, $\lambda > 15$

Hence $N(113.4, 113.4)$

$P(X > 120) = P\left(Z > \dfrac{120.5 - 113.4}{\sqrt{113.4}}\right)$

$= P(Z > 0.667)$

$= 1 - \phi(0.667)$

$= 1 - 0.7477$

$= 0.252$ (3 sf)

$= 0.252$

Continuity Correction
$X > 120 \Rightarrow X \geq 121$
$\rightarrow X \geq 120.5$

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6. Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of each other.

(a) A week is chosen at random. --[2]

(i) Find the prob. that the team scores a total of 4 goals in their two matches.

(ii) Find the prob. that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. ---[3]

(b) Use suitable approximating distribution to find the prob. that the team scores fewer than 25 goals in 10 randomly choosen weeks. ---[4]

(c) Justify the use of the approximating distribution used in part (b) --[1]

S-20 | 61 | Q5

Solution: Home match, $\lambda = 2.1$, per match

Away match, $\lambda = 0.8$, per match

$\left\{ P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!} \right.$

(a)(i) In a week → one home match and one away match ⇒ $\lambda = 2.1 + 0.8 = 2.9$ ✓

$P(X=4) = \frac{e^{-\lambda}\lambda^4}{4!} = \frac{e^{-2.9} \times 2.9^4}{4!}$

$= \underline{0.162}$ ✓ (3sf)

(ii) (In 4 goals)

$P(4H, 0\text{ away}) + P(3H, 1\text{ away})$

$= \left(\frac{e^{-2.1} \times 2.1^4}{4!}\right) \times e^{-0.8} + \left(\frac{e^{-2.1} \times 2.1^3}{3!}\right)\left(e^{-0.8} \times 0.8\right)$

$= \underline{0.113}$ (3 S.f)

(b) for 10 randomly chosen weeks

$\lambda = 10 \times 2.9 = 29$

$Po(\lambda) \sim N(\mu, \sigma^2) = N(29, 29)$

$P(X<25) = P\left(Z < \frac{24.5 - 29}{\sqrt{29}}\right)$ $\Big\{$ Continuity Correction:

$x < 25 \Rightarrow x \leq 24 \rightarrow x = 24.5$

$= P(Z < -0.83563)$

$= 1 - \phi(0.836)$

$= \underline{0.202}$ ✓ (3 Sf)

(c) $\lambda = 29 > 15$

Example 7: In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution X, including the values of any parameters. --[1]

(ii) State an appropriate approximating distribution for X, including the values of parameters. Justify your choice of approx. distribution. --[3]

(iii) Use your approximating distribution to find $P(X>2)$ --[2]

(b) Another large random sample of 'n' data items is chosen. The prob. that the sample contains no data items that are entered incorrectly is more than 0.1. Find the largest value of n. [S-20/62/Q3] --[3]

Solution (a) (i) $B(3600, 0.0012)$ $\{p = 0.12\%\}$

(ii) $Po(4.32)$; $\left.\begin{matrix} n = 3600 \\ p = .0012 \end{matrix}\right\} \Rightarrow \lambda = np = 4.32 < 5$

$\boxed{B(n,p) \to Po(\lambda)}$

(iii) $P(X>2) = 1 - P(0,1,2)$

$= 1 - e^{-4.32}\left[1 + 4.32 + \frac{4.32^2}{2}\right]$

$= \underline{0.805}$ (3 s.f.)

(b) $P(X=0) = e^{-\lambda} > 0.1$ (Given)

$\Rightarrow -\lambda > \ln 0.1$

$\Rightarrow \lambda < -\ln 0.1$

$\Rightarrow \lambda < \ln 10$

$\Rightarrow 0.0012n < \ln 10$

$\Rightarrow n < \frac{\ln 10}{0.0012}$

$n < 1918.8$

$\therefore$ Largest value of $n = \underline{1918}$

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8. The random variable X has the distribution $Po(\lambda)$

(a) (i) State the values that X can take. ---[1]

It is given that $P(X=1) = 3 \cdot P(X=0)$

(ii) Find λ. ---[1]

(iii) Find $P(4 \leq X \leq 6)$ ---[2]

(b) The random variable Y has distribution $Po(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $P(Y < 46) = 0.0668$, correct to 4 decimal places. ---[5]

Find the value of μ.

[S-20/62/Q5]

Solution (a) (i) $0, 1, 2, 3, \ldots$

(ii) $P(X=1) = 3 \cdot P(0)$ $\left\{ P_0(X=\lambda) = \frac{e^{-\lambda} \cdot \lambda^{\lambda}}{\lambda!} \right.$

$$\Rightarrow \frac{e^{-\lambda} \times \lambda^1}{1!} = 3e^{-\lambda} \Rightarrow \underline{\lambda = 3} \checkmark$$

(iii) $P(4 \leq X \leq 6) = e^{-3}\left(\frac{3^4}{4!} + \frac{3^5}{5!} + \frac{3^6}{6!}\right) = \underline{0.319} \checkmark$ (3 sf)

(b) Given μ is large $\Rightarrow P_0(\mu) \sim N(\mu, \sigma^2) = N(\mu, \mu)$

Given $P(Y < 46) = 0.0668$ $\left\{ \begin{array}{l} \text{Continuity correction} \\ Y < 46 \sim Y \leq 45 \\ Y \to 45.5 \end{array} \right.$

$$P(Y<46) = P\left(Z < \frac{45.5 - \mu}{\sqrt{\mu}}\right) = 0.0668 \quad < 0.5$$

Let $\frac{45.5 - \mu}{\sqrt{\mu}} = a < 0$ ①

Put $a = -b$; $b > 0$

$$\Rightarrow P(Z < a) = 0.0668$$

$$\Rightarrow P(Z < -b) = 0.0668$$

$$\Rightarrow 1 - \phi(b) = 0.0668$$

$$\Rightarrow \phi(b) = 1 - 0.0668 \Rightarrow \phi(b) = 0.9332 \Rightarrow b = \phi^{-1}(0.9332)$$

from ① $\Rightarrow \frac{45.5 - \mu}{\sqrt{\mu}} = a = -b = -1.5$ $b = 1.5 \checkmark$

$\Rightarrow 45.5 - \mu = -1.5\sqrt{\mu}$ --- ②

from ② $\Rightarrow \mu - 1.5\sqrt{\mu} - 45.5 = 0$

$$\Rightarrow \sqrt{\mu} = \frac{1.5 \pm \sqrt{184.25}}{2} = \frac{1.5 \pm 13.5738}{2}$$

$$\Rightarrow \sqrt{\mu} = \frac{15.0738}{2} \text{ or } \cancel{\frac{-12.0738}{2}}$$

$$\sqrt{\mu} = 7.5369 \Rightarrow \underline{\mu = 56.8} \checkmark$$

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9. Accidents at factories occur randomly and independently. On average, the number of accidents per month are 3.1 at factory A and 1.7 at factory B.
Find the probability that the total number of accident in two factories during a 2-month period is more than 3. --[4]
[S-21/61/Q1]

Solution: for one month the total accidents in month $= 3.1+1.7=4.8$

For two months $\lambda = 2\times 4.8 = 9.6$

$$P(X>3) = 1-\{P(0)+P(1)+P(2)+P(3)\}$$

$$= 1-e^{-9.6}\left\{1+9.6+\frac{9.6^2}{2}+\frac{9.6^3}{3!}\right\} \qquad \left\{P(X=x)=\frac{e^{-\lambda}\cdot\lambda^x}{x!}\right.$$

$$= \underline{0.986} \ (3\text{ sf}).$$

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Example 10: On average, 1 in 75 000 adults has a certain genetic disorder.

(a) Use a suitable approximating distribution to find the probability that, in a random sample of 10000 people, at least 1 has a genetic disorder. ---[3]

(b) In a random sample of n people, where n is large, the probability that no-one has the genetic disorder is more than 0·9. Find the largest possible value of n. ---[4]

$\boxed{S\text{-}21/61/Q5}$

Solution (a) $n = 10\,000,\ p = \frac{1}{75\,000}$ — n is large and p is small

$np = 10\,000 \times \frac{1}{75\,000} = \frac{2}{15} < 5$

hence it is Poisson distribution. $\lambda = \frac{2}{15}$

$Po\left(\frac{2}{15}\right)$ $\boxed{B(n,p) \rightarrow Po(\lambda)}$

$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-2/15} ✓$ $\{ P(0) = e^{-\lambda}$

$= \underline{0{\cdot}125}$ (3 sf)

(b) $\lambda = \frac{n \times 1}{75\,000}$ $\{ \lambda = np = n \times \frac{1}{75{,}000}$

$P(X=0) = e^{-\frac{n}{75000}} > 0{\cdot}9$ (Given)

$\Rightarrow -\frac{n}{75000} > \ln 0{\cdot}9$

$\Rightarrow -n > -0{\cdot}10536 \times 75000$

$\Rightarrow n < 7902{\cdot}04$

$\therefore$ Largest value of $\underline{n = 7902}$ ✓

Example 11: Customers arrive at a particular shop at random times. It has been found that the mean number of customers who arrive during a 5-minute interval is 2.1.

(a) Find the probability that exactly 4 customers arrive during a 10-minute interval. ---[2]

(b) Find the probability that at least 4 customers arrive during 20-minute interval. ---[2]

(c) Use a suitable approximating distribution to find the probability that fewer than 40 customers arrive during a 2-hour interval. [S-21/62/Q7] ---[4]

Solution (a) Random Variable, X denotes the number of customers.

During a 5-minute interval - mean number of customers $\lambda = 2.1$; $Po(2.1)$

$\therefore$ During 10-minute interval - mean number of Customers $\lambda = \frac{10}{5} \times 2.1 = 4.2$

$$P(X=4) = e^{-4.2} \times \frac{4.2^4}{4!} \qquad \left[P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right]$$

$$= 0.194 \ (3 sf)$$

(b) During 20-minute interval: $\lambda = \frac{20}{5} \times 2.1 = 8.4$ $\quad Po(8.4)$

$$P(X \geq 4) = 1 - P(X = 0, 1, 2, 3) = 1 - e^{-8.4} \left\{ 1 + 8.4 + \frac{8.4^2}{2!} + \frac{8.4^3}{3!} \right\}$$

$$= 0.968 \ (3sf)$$

(c) During 2 Hr (120 minutes) interval $\lambda = \frac{120}{5} \times 2.1 = 50.4 > 15$

$\therefore$ approximating distribution is normal distribution. $X \sim N(\mu, \sigma^2) = N(\lambda, \lambda) = N(50.4, 50.4)$

$$P(X < 40) = P\left(Z < \frac{39.5 - 50.4}{\sqrt{50.4}}\right) \qquad \left[\begin{array}{l} \text{Continuity Correction.} \\ X < 40 \sim X < 39.5 \\ \text{or } X \leq 39 \end{array} \right]$$

$$= P(Z < -1.535)$$

$$= 1 - \phi(1.535)$$

$$= 1 - 0.9376$$

$$= 0.0624$$

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12. Most plants of a certain type have three leaves. However, it is known that, on average, 1 in 10000 of these plants have four leaves and plants with four leaves are called "lucky". The number of lucky plants in a random sample of 25000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. ---[2]

Use your approximating distribution to answer parts (b) and (c)

(b) Find $P(X \leq 3)$ ---[2]

(c) Given that $P(X=k) = 2P(X=k+1)$, find k. ---[2]

The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geq 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. --[3]

[S-21/63/Q5]

Solution: $n = 25000$, $p = \frac{1}{10,000}$

(a) $\lambda = np = 25000 \times \frac{1}{10,000} = 2.5 < 5$

$n > 50$ and $p = 0.0001 < 0.1$

$\therefore$ Poission distribution $Po(\lambda)$

(as an approx. to Binomial dis.) $= \underline{Po(2.5)}$ ✓

(b) $P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$

$P(X \leq 3) = P(X = 0, 1, 2, 3)$

$= e^{-2.5} \times \left(1 + 2.5 + \frac{2.5^2}{2!} + \frac{2.5^3}{3!}\right)$

$= \underline{0.758}$ (3sf)

(c) $P(X=k) = 2P(X=k+1)$

$\Rightarrow e^{-2.5} \times \frac{2.5^k}{k!} = 2 \times e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$

$\Rightarrow k+1 = 5$

$\Rightarrow \underline{k = 4}$ ✓

(d) $P(Y \geq 1) = 0.963$

$\Rightarrow 1 - P(Y=0) = 0.963$

$\Rightarrow 1 - e^{-\lambda} = 0.963$

$\Rightarrow e^{-\lambda} = 0.037$

$\Rightarrow -\lambda = \ln 0.037$

$\Rightarrow \lambda = -(-3.296837\ldots)$

$np = 3.296837$

$n \times 0.0001 = 3.296837$

$n = 32968.37$

$\underline{n = 33000}$ ✓ (3sf)

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1.3 Cars arrive at a fuel station at random at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. --- [3]

(b) Use an approximating distribution to find the prob. that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. -- [4]

Independently of cars, trucks arrive at fuel station at random at a constant average rate of 3.6 per 15-minute period.

(c) Find the prob. that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. --[3]

[S-22/61/Q5]

Solution: $\lambda = 13.5$ per hour

(a) $\lambda = \frac{20}{60} \times 13.5 = 4.5$ per 20-minute period.

$P(r > 4) = 1 - P(r = 0,1,2,3,4)$ $\qquad P(r) = e^{-\lambda} \frac{\lambda^r}{r!}$

$= 1 - e^{-4.5}\left[1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^3}{4!}\right] = \underline{0.468}$ ✓ (3 s.f)

(b) for 12-hour period $\lambda = 12 \times 13.5 = 162 > 15$

$X \sim Po(162) \Rightarrow X \sim N(162, 162)$; $\mu = 162$, $\sigma^2 = 162$ $[N(\mu, \sigma^2)]$

$P(150 \leq X \leq 160) = P(149.5 \leq Z \leq 160.5)$ $\left\{ \begin{array}{l} \text{Continuity correction} \\ Z \geq 150 \to Z \sim 149.5 \\ Z \leq 160 \to Z \sim 160.5 \end{array} \right.$

$= P\left(Z \leq \frac{160.5 - 162}{\sqrt{162}}\right) - P\left(Z < \frac{149.5 - 162}{\sqrt{162}}\right)$

$= P(Z \leq -0.118) - P(Z < -0.982)$

$= 1 - \phi(0.118) - (1 - \phi(0.982)) = \phi(0.982) - \phi(0.118)$

$= \underline{0.290}$ (3 s.f) ✓

(c) $\lambda = \frac{10}{60} \times 13.5 + \frac{10}{15} \times 3.6 = 4.65$

$P(3 < r < 7) = P(r = 4, 5, 6)$

$= e^{-4.65}\left[\frac{4.65^4}{4!} + \frac{4.65^5}{5!} + \frac{4.65^6}{6!}\right]$

$= \underline{0.494}$ (3 s.f)

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14. It is known that 1.8% of children in a certain country have not been vaccinated against measles. A random sample of 200 children in this country is chosen.

(a) Use a suitable approximating distribution to find the probability that there are fewer than 3 children in the sample who have not been vaccinated against measles. ---[4]

(b) Justify approximating distribution. ---[2]

[S-22/62/Q3]

Solution (a) $p = 0.018$, $n = 200$, $\lambda = np = 200 \times 0.018 = 3.6$ ✓ $\left\{ P(x) = \frac{e^{-\lambda} \cdot \lambda^x}{x!} \right.$

$P(x < 3) = P(x = 0, 1, 2)$ using $P_o(3.6)$

$= e^{-3.6}\left(1 + 3.6 + \frac{3.6^2}{2}\right) = \underline{0.303}$ ✓ (3 s.f)

(b) $B(200, 0.018)$, $n > 50$,

$np = 200 \times 0.018 = 3.6 < 5$ and p is small $p = 0.018 < 0.1$

$\therefore$ Poissons distribution ✓

15. The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5 minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. ---[3]

(b) If more than 4 clients arrive during a 5-minute period, they cannot be served. Find the prob. that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. --[2]

(c) Use a suitable approximating distribution to find the prob. that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at desk

[S-22/63/Q5] --[4]

Solution: $\lambda = 2.2$ per 5-minute period

(a) $\Rightarrow \lambda = \frac{15}{5} \times 2.2 = 6.6$ per 15-min.

$P(x=6) = e^{-6.6} \times \frac{6.6^6}{6!} = \underline{0.156}$ ✓ (3 s.f).

(b) $\lambda = 2.2$ per 5-minute

$P(x > 4) = 1 - P(x = 0, 1, 2, 3, 4)$

$= 1 - e^{-2.2}\left(1 + 2.2 + \frac{2.2^2}{2} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!}\right)$

$= \underline{0.0725}$ ✓ (3 s.f)

(c) for 1 hr period $\lambda = \frac{60}{5} \times 2.2 = 26.4 > 15$

$P_o(\lambda) \rightarrow N(\lambda, \lambda)$

$\rightarrow N(\mu, \sigma^2) \sim N(26.4, 26.4)$

$\mu = 26.4$, $\sigma = \sqrt{26.4}$

$P(x < 20) = P\left(z < \frac{19.5 - 26.4}{\sqrt{26.4}}\right)$ Continuity correction $x < 20 \rightarrow x \le 19 \sim 19.5$

$= P(z < -1.343)$

$= 1 - \phi(1.343)$

$= \underline{0.0897}$ (3 s.f)

(or 0.896)

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16. In a certain country, 20540, adults out of population of 6012300 have a degree in medicine.

(a) Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. --[4]

(b) Justify the approximating distribution used in part (a). --[2]

S-23/61/Q1

Solution: Given population prob. for medicine degree $= \frac{2054}{6012300} = 0.0034163$

(a) for 1000 adults sample,

$$\lambda = 1000 \times 0.0034163 = 3.4163$$

$\therefore$ approximation distribution is $Po(\lambda)$; $\lambda = 3.4163$

$$P(X<4) = e^{-3.4163}\left[1 + 3.4163 + \frac{3.4163^2}{2} + \frac{3.4163^3}{3!}\right]$$

$$= 0.555$$

$\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

(b) $n = 1000 > 50$, hence the approximating distribution is $P_0(\lambda)$

17 (a) The random variable W has a poisson distribution. State the relationship between $E(W)$ and $Var(W)$ ---[1]

(b) The random variable X has the distribution $B(n, p)$. Jyothi wishes to use poisson distribution as an approximating distribution for X.

Use the formula for $E(X)$ and $Var(X)$ to explain why it is necessary for p to be closed to 0 for this to be reasonable approximation. ---[1]

(c) Given that Y has a distribution $B(20000, 0.00007)$, use a poisson distribution to calculate an estimate of $P(Y>2)$ --[3]

S-23/62/Q2

Solution: (a) $E(W) = Var(W)$

(b) $np \simeq np(1-p)$,

Hence $(1-p)$ must be close to 1

or p is close to 0.

(c) $\lambda = np = 20000 \times 0.00007 = 1.4$

$$P(Y>2) = 1 - \{P(0) + P(1) + P(2)\}$$

$$= 1 - e^{-1.4}\left\{1 + 1.4 + \frac{1.4^2}{2}\right\}$$

$$= 0.167 \text{ (3 sf)}$$

18. The number, X, of books received at a charity shop has a constant mean of 5·1 per day.

(a) State, in context, one condition for X to be modelled by a Poisson distribution. --[1]

Assume now that X can be modelled by a Poisson distribution.

(b) Find the probability that exactly 10 books are received in a 3-day period. --[2]

(c) Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. --[4]

The number of DVD's received at the same shop is modelled by an independent Poisson distribution with mean 2·5 per day.

(d) Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. --[3]

[S-23/62/Q4]

Solution: Mean = 5·1, per day (Given)

(a) Books received independently or singly or randomly.

(b) for 3 day period; $\lambda = 3\times5\cdot1 = 15\cdot3$

$P(X=10) = e^{-15\cdot3}\times\frac{15\cdot3^{10}}{10!} = \underline{0\cdot0439}$ ✓

(c) Now for 180 books, > 50

for 30 day period $\lambda = 5\cdot1\times30 = 153$

$X \sim N(\lambda,\sigma^2) = N(153, 153)$

$P(X>180) = P\left(z > \frac{180\cdot5-153}{\sqrt{153}}\right)$

$= P(z>2\cdot223)$ Continuity Correction. $X>180 \rightarrow X\geq181$, $X \sim 180\cdot5$

$= 1-\phi(2\cdot223)$

$= \underline{0\cdot0131}$ (3 s.f)

Poisson distribution: $P(X=r) = \frac{e^{-\lambda}\cdot\lambda^r}{r!}$

(d) $\lambda = 5\cdot1$ for books, $\lambda = 2\cdot5$ for DVDs, per day

Hence for both books and DVDs $\lambda = 5\cdot1+2\cdot5 = 7\cdot6$ per day.

$P(X>3) = 1-\{P(0)+P(1)+P(2)+P(3)\}$

$= 1-e^{-7\cdot6}\left\{1+7\cdot6+\frac{7\cdot6^2}{2}+\frac{7\cdot6^3}{3!}\right\}$

$= \underline{0\cdot945}$ ✓ (3 sf)

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19. It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X.

(a) Use an approximating distribution to find $P(X \leq 3)$ ---[3]

(b) Find the value of E(X) and Var(X), and explain how your answer suggest that the approximating distribution used in part (a) is likely to be appropriate. ---[2]

S-23/63/Q6

Solution: $p = \frac{1}{5000}$; $n = 12500 \Rightarrow \lambda = np = 12500 \times \frac{1}{5000} = 2.5 < 5$

(a) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$ $\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

$= e^{-2.5}\left\{1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!}\right\}$

$= \underline{0.758}$ (3 sf)

(b) $E(X) = np = 2.5$ ✓ and $Var(X) = npq = 12500 \times \frac{1}{5000} \times \frac{4999}{5000} = 2.4995$ ✓

E(X) & Var(X) are almost equal. Hence Poisson distribution.

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20. It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an approximating distribution to find the probability that more than 1 flower in the sample is white. ---[3]

(b) Justify the approximating distribution used in part (a) ---[1]

The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in two samples is less than 4. ---[3]

W-20/61/Q1

Solution: $P(\text{white}) = p = \frac{1}{300}$, $n = 200 \Rightarrow \lambda = np = \frac{1}{300} \times 200 = \frac{2}{3} < 5$

(a) $Po(2/3)$

$$P(X>1) = 1 - P(X=0,1) = 1 - e^{-2/3}\left(1+\frac{2}{3}\right) \quad \left\{P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!}\right.$$

$$= 1 - 0.856\ldots$$

Using Poisson distribution as an approximation to Binomial dist. $= \underline{0.144}$ (3 s.f.)

(b) $n > 50$ and $np = \frac{2}{3} < 5$ (or $p = \frac{1}{300} < 0.1$ and $n > 50$)

(c) for the first kind $\lambda_1 = 2/3$

Now for another kind: $\lambda_2 = np = 150 \times 0.02 = 3 < 5$

$\therefore$ for combined $\lambda = \lambda_1 + \lambda_2 = \frac{2}{3} + 3 = \frac{11}{3} < 5$

$$\therefore P(X<4) = P(X=0,1,2,3)$$

$$= e^{-\frac{11}{3}}\left(1 + \frac{11}{3} + \frac{\left(\frac{11}{3}\right)^2}{2!} + \frac{\left(\frac{11}{3}\right)^3}{3!}\right) \quad \left(\because \lambda = \frac{11}{3}\right)$$

$$= \underline{0.501} \text{ (3 s.f.)}$$

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21. The number of absences per week by workers at a factory has the distribution $Po(2.1)$

(a) Find the standard deviation of the number of absences per week. ---[1]

(b) Find the probability that the number of absences in a 2-week period is at least 2. ---[3]

(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. ---[2]

W-20/61/Q5

Solution: $Po(2.1)$ (Given, per week)

(a) Standard deviation $\sigma = \sqrt{2.1}$ $\left(\text{as } \sigma^2 = \lambda = 2.1 \quad \text{variance} = \text{mean}\right)$

or $= \underline{1.45}$ (3 sf)

(b) per week $\lambda = 2.1$

$\Rightarrow$ for 2-week period $= 2 \times 2.1 = 4.2$

$$P(X \geq 2) = 1 - \{P(0) + P(1)\} \qquad \left\{P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}\right.$$

$$= 1 - e^{-4.2}(1 + 4.2)$$

$$= \underline{0.922} \text{ (3 sf)}$$

(c) For a 3-week period $\lambda = 3 \times 2.1 = 6.3$

$$P(4 < X < 8) = P(5) + P(6) + P(7)$$

$$= e^{-6.3}\left\{\frac{6.3^5}{5!} + \frac{6.3^6}{6!} + \frac{6.3^7}{7!}\right\}$$

$$= \underline{0.455} \text{ (3 sf)}$$

22. On average, 1 in 50000 people have a certain gene. Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150000 have the gene. ---[3]

[W-20/62/Q1]

Solution: $p = \frac{1}{50000}$, $n = 150000 \Rightarrow \lambda = np = \frac{150000}{50000} = 3 < 5$; $P_o(3)$

$P(X>2) = 1 - P(X=0,1,2) = 1 - e^{-3}\cdot\left(1+3+\frac{3^2}{2!}\right) = \underline{0.577}$ (3sf)

$\boxed{B(n,p) \rightarrow P_o(\lambda)}$

23. Customers arrive at a shop at a constant average rate of 2.3 per minute.

(a) State another condition for the number of customers arriving per minute to have a Poisson distribution. ---[1]

It is now given that the number of customers arriving per minute has the distribution $P_o(2.3)$

(b) Find the probability that exactly 3 customers arrive during 1-minute period. ---[2]

(c) Find the prob. that more than 3 customers arrive during a 2-minute period. ---[3]

(d) Five, 1-minute periods are chosen at random. Find the prob. that "no customers arrive" during exactly 2 of these 5 periods. ---[3]

[W-20/62/Q5]

Solution (a) Customers arrive independently or singly or at random.

(b) $P_o(2.3) \Rightarrow \lambda = 2.3$

$P(X=3) = e^{-3} \times \frac{2.3^3}{3!} = 0.203$ (3sf)

$\left\{ P(X=r) = e^{-\lambda}\cdot\frac{\lambda^r}{r!} \right.$ for $P_o(\lambda)$

(c) for 2-minute interval $\lambda = 2\times2.3 = 4.6$

$P(X>3) = e^{-4.6}\left(1+4.6+\frac{4.6^2}{2!}+\frac{4.6^3}{3!}\right)$

$= \underline{0.674}$ (3sf)

(d) For 1-minute period, $\lambda = 2.3$

P(none arrive) $= P(X=0) = e^{-2.3}$

$p = 0.10026$

Now for n trial, using Binomial dis

$n=5$, $r=2$, $p=0.10026$,

$q = 0.89974$

Using $P(X=r) = {}^{n}C_r\cdot p^r\cdot q^{n-r} \Rightarrow P(r=2) = {}^5C_2\cdot(0.10026)^2\cdot(0.89974)^3$

$= \underline{0.0732}$ (3sf)

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§ **Normal distribution as an approximation to Poisson distribution:**

For $\lambda > 15$, the Poisson distribution with λ can be approximated by the normal distribution with mean $\mu = \lambda$ and variance $\sigma^2 = \lambda$, with a continuity correction applied.

The accuracy of this approximation improves as λ increases.

$$X \sim Po(\lambda) \rightarrow N(\mu, \sigma^2) \rightarrow N(\lambda, \lambda) \text{ for } \lambda > 15.$$

Example 24. The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. --[4]

[W-21/61/Q2]

Solution: $Po(\lambda) = Po(45.2) \Rightarrow \lambda = 45.2$ ($\lambda > 15$, leads to Normal distribution)

Poisson distribution approaches $\rightarrow$ Normal distribution $N(\mu, \sigma^2)$

$$\rightarrow N(\lambda, \lambda) = N(45.2, 45.2) \{\mu = \sigma^2 = \lambda\}$$

$$P(X > 60) = P\left(z > \frac{x-\mu}{\sigma}\right) = P\left(z > \frac{x-\lambda}{\sqrt{\lambda}}\right) = P\left(z > \frac{60.5 - 45.2}{\sqrt{45.2}}\right)$$

$$= P(z > 2.276)$$

$$= 1 - P(z < 2.276)$$

$$= 1 - \phi(2.276)$$

$$= 1 - 0.9886$$

$$= 0.0114$$

Continuity correction.
$x > 60 \rightarrow x \geq 61$
$x \rightarrow 60.5$

25.(a) The proportion of people having a particular medical condition is 1 in 100000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution X, giving the values of any parameters. --[2]

(ii) Use the approximating distribution to calculate $P(X>0)$ --[2]

W-21/61/Q5a

Solution: Medical condition is: 1 in 100000 and $n = 2500$

(a) $n = 2500$; $p = \frac{1}{100000}$ $\Rightarrow$ $mean = n\lambda = \frac{1}{100000} \times 2500 = 0.025 = \lambda$

(i) Poission distribution $Po(0.025)$, $\lambda = 0.025$, $n = 2500 > 50$, $np = 0.025 < 5$
The medical condition occur singly, at random, independently.

(ii) $P(X>0) = 1 - P(0)$ $\left\{ P(X=r) = \frac{e^{-\lambda}\lambda^r}{r!} \right.$

$= 1 - e^{-0.025}$

$= \underline{0.0247}$ (3 sf)

Example 25 In a certain document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. ---[2]

(b) Use an approximating distribution to find the probability that there are more than 50 typing errors in 200 randomly chosen pages. ---[4]

In same document, formatting errors occurs at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. --[3]

W-21/62/Q5

Solution: (a) $n=10;\ p=0.2 \Rightarrow \lambda = np = 10\times 0.2 = 2 < 5$, Use $Po(2)$

$$P(X<3) = P(X=0, X=1, X=2) = e^{-\lambda}\left[1+\lambda+\frac{\lambda^2}{2!}\right) \quad \left\{ P(X=r) = \frac{e^{-\lambda}\cdot\lambda^r}{r!} \right.$$

$$= e^{-2}\left[1+2+\frac{2^2}{2!}\right) = \underline{0.677}\ (3s.f)$$

(b) $n=200,\ p=0.2 \Rightarrow \lambda = np = 200\times 0.2 = 40 > 15$

$\therefore$ Poisson distribution approximating to Normal distribution $N(\mu,\sigma^2)$

$\mu = \sigma^2 = \lambda = 40 \quad \therefore X \sim N(40, 40)$

$$P(X>50) = P\left(Z > \frac{50.5-40}{\sqrt{40}}\right) \quad \left\{ \begin{array}{l}\text{Continuity correction} \\ P(X>50) = P(X\geq 51) \sim N \rightarrow X > 50.5\end{array}\right.$$

$$= P(Z > 1.660) = 1-\phi(1.660)$$

$$= 1-0.9515$$

$$= \underline{0.0485}$$

(c) Now $n=20$, p = typing and formatting error per page $= 0.2+0.3$

$p = 0.5$

$\therefore \lambda = np = 20\times 0.5 = 10$

$$P(8\leq X\leq 11) = P(X=8,9,10,11) = e^{-\lambda}\left[\frac{\lambda^8}{8!}+\frac{\lambda^9}{9!}+\frac{\lambda^{10}}{10!}+\frac{\lambda^{11}}{11!}\right)$$

$$= e^{-10}\left(\frac{10^8}{8!}+\frac{10^9}{9!}+\frac{10^{10}}{10!}+\frac{10^{11}}{11!}\right)$$

$$= \underline{0.477}\ (3\,sf)$$

27. Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. ---[3]

(b) Use a suitable approximating distribution to find the prob. that at least 650 drops fall during a randomly chosen 2-hour period. ---[4]

[W-22/61/Q3]

Solution (a) $\lambda = 5.2$ for one minute.

$\lambda = \frac{30}{60} \times 5.2 = 2.6$ for 30 seconds

$P(r \geq 3) = 1 - P(r = 0, 1, 2)$

$= 1 - e^{-2.6}\left(1 + 2.6 + \frac{2.6^2}{2}\right)$

$= 0.482$ ✓ (3 sf)

(b) $\lambda = 5.2 \times 120 = 624$ for 2 hr period

$P(\lambda) \to N(\lambda, \lambda) \to N(624, 624) = N(\mu, \sigma^2)$

$P(r \geq 650) = P\left(Z \geq \frac{649.5 - 624}{\sqrt{624}}\right)$ $\left\{ \begin{array}{l} \text{Continuity Correction} \\ r \geq 650 \to r = 649.5 \end{array} \right.$

$= P(Z \geq 1.021)$

$= 1 - \phi(1.021) = 0.154$ ✓ (3 sf)

28. 1.6% of adults in a certain town ride a bicycle. A random sample of 200 adults from this town is selected.

(a) Use a suitable approximating distribution to find the probability that more than 3 of these adults ride a bicycle. ---[4]

(b) Justify your approximating distribution. ---[2]

[W-22/62/Q3]

Solution: $\lambda = np = 200 \times 1.6 = 3.2$, using $Po(3.2)$

(a) $P(X > 3) = 1 - P(X = 0, 1, 2, 3) = 1 - e^{-3.2}\left(1 + 3.2 + \frac{3.2^2}{2} + \frac{3.2^3}{3!}\right)$

$= 1 - (0.04076 + 0.1304 + 0.2087 + 0.2226)$

$= 0.397$ ✓ (3 sf)

(b) $B(n, p)$, $n = 200 > 50$

$np = 200 \times 0.016 = 3.2 < 5$ and $p = 0.016 < 0.1$

$\therefore$ Poisson distribution.

29. The number of calls received at a small call centre has a Poisson distribution with mean 2.4 calls per 5-minute period.

(a) Find the probability of exactly 4 calls in an 8-minute period --[2]

(b) Find the probability of atleast 3 calls in 3-minute period. ---[3]

The number of calls received at a large call centre has a Poisson distribution with mean 41 calls per 5-minute period.

(c) Use an approximating distribution to find the probability that the number of calls received in a 5-minute period is between 41 and 59 inclusive. --[5]

[SP 2006/Q3]

Solutions (a) $\lambda = 2.4$ per 5-minute period.

$\Rightarrow$ for $\rightarrow$ 8-minute period $\lambda = \frac{2.4 \times 8}{5} = 3.84$ ✓

$$P(r=4) = e^{-3.84} \times \frac{3.84^4}{4!} = \underline{0.195} \text{ (3sf)} \quad \left\{ P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right.$$

(b) For 3 minute-period: $\lambda = \frac{2.4}{5} \times 3 = 1.44$

$$P(X \geq 3) = 1 - P(X=0,1,2)$$

$$= 1 - e^{-1.44} \times \left(1 + 1.44 + \frac{1.44^2}{2}\right) = \underline{0.176} \checkmark$$

(c) $X \sim Po(\lambda) \sim N(\mu, \sigma^2) = N(\lambda, \lambda)$ for $\lambda = 41 > 15$

$X \sim N(41, 41)$, $\mu = 41$, $\sigma^2 = 41$

$$P(41 \leq X \leq 59) = P(X < 59.5) - P(X < 40.5)$$

$$= P\left(Z < \frac{59.5 - 41}{\sqrt{41}}\right) - P\left(Z < \frac{40.5 - 41}{\sqrt{41}}\right)$$

$= P(Z < 2.889) - P(Z < -0.078)$

$= \phi(2.889) - \phi(-0.078)$

$= \phi(2.889) - [1 - \phi(0.078)]$

$= 0.9981 - (1 - 0.5311)$

$= \underline{0.529} \text{ (3 sf)}$ ✓

{Continuity Correction}
$X \geq 41 \sim X > 40.5$
$X \leq 59 \sim X < 59.5$