PROBABILITY AND STATISTICS-2

9709

(March, June and November series 2020 – 2023 With marking scheme)

POISSON DISTRIBUTION

EXERCISE -2

Manjula Balaji

1) **March 2020 9709_62 Q1**

The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30 000 letters, and these errors occur randomly. A randomly chosen booklet from this publisher contains 12 500 letters.

Use a suitable approximating distribution to find the probability that this booklet contains at least 2 errors. [3]

2) **March 2020 9709_62 Q4**

The number of accidents on a certain road has a Poisson distribution with mean 0.4 per 50-day period.

(a) Find the probability that there will be fewer than 3 accidents during a year (365 days). [3]

(b) The probability that there will be no accidents during a period of n days is greater than 0.95.

Find the largest possible value of n. [4]

3) **March 2021 9709_62 Q4**

On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of X, giving the values of any parameters. [1]

(b) State an approximating distribution for X, giving the values of any parameters. [2]

(c) Use this approximating distribution to find each of the following.

(i) $P(X = 4)$. [2]

(ii) $P(2 \leqslant X \leqslant 4)$. [2]

(d) Use a suitable approximating distribution to find the probability that, in a random sample of 700 microchips, there will be at least 1 faulty one. [3]

4) **March 2022 9709_62 Q7**

(a) Two ponds, A and B, each contain a large number of fish. It is known that 2.4% of fish in pond A are carp and 1.8% of fish in pond B are carp. Random samples of 50 fish from pond A and 60 fish from pond B are selected.

Use appropriate Poisson approximations to find the following probabilities.

(i) The samples contain at least 2 carp from pond A and at least 2 carp from pond B. [3]

(ii) The samples contain at least 4 carp altogether. [3]

(b) The random variables X and Y have the distributions $\text{Po}(\lambda)$ and $\text{Po}(\mu)$ respectively. It is given that

- $P(X = 0) = [P(Y = 0)]^2$,
- $P(X = 2) = k[P(Y = 1)]^2$, where k is a non-zero constant.

Find the value of k. [4]

5) **March 2023 9709_62 Q2**

The number of orders arriving at a shop during an 8-hour working day is modelled by the random variable X with distribution Po(25.2).

(a) State **two** assumptions that are required for the Poisson model to be valid in this context. [2]

(b) **(i)** Find the probability that the number of orders that arrive in a randomly chosen 3-hour period is between 3 and 5 inclusive. [3]

(ii) Find the probability that, in two randomly chosen 1-hour periods, exactly 1 order will arrive in one of the 1-hour periods, and at least 2 orders will arrive in the other 1-hour period. [4]

(c) The shop can only deal with a maximum of 120 orders during any 36-hour period.

Use a suitable approximating distribution to find the probability that, in a randomly chosen 36-hour period, there will be too many orders for the shop to deal with. [4]

6) **June 2020 9709_61 Q5**

Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of one another.

(a) A week is chosen at random.

(i) Find the probability that the team scores a total of 4 goals in their two matches. [2]

(ii) Find the probability that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. [3]

(b) Use a suitable approximating distribution to find the probability that the team scores fewer than 25 goals in 10 randomly chosen weeks. [4]

(c) Justify the use of the approximating distribution used in part **(b)**. [1]

7) **June 2020 9709_62 Q3**

In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution of X, including the values of any parameters. [1]

(ii) State an appropriate approximating distribution for X, including the values of any parameters.

Justify your choice of approximating distribution. [3]

(iii) Use your approximating distribution to find $P(X > 2)$. [2]

(b) Another large random sample of n data items is chosen. The probability that the sample contains no data items that are entered incorrectly is more than 0.1.

Use an approximating distribution to find the largest possible value of n. [3]

8) **June 2020 9709_62 Q5**

(a) The random variable X has the distribution $\mathrm{Po}(\lambda)$.

(i) State the values that X can take. [1]

It is given that $P(X = 1) = 3 \times P(X = 0)$.

(ii) Find λ. [1]

(iii) Find $P(4 \leqslant X \leqslant 6)$. [2]

(b) The random variable Y has the distribution $\mathrm{Po}(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $P(Y < 46) = 0.0668$, correct to 4 decimal places.

Find μ. [5]

9) **June 2021 9709_61 Q1**

Accidents at two factories occur randomly and independently. On average, the numbers of accidents per month are 3.1 at factory A and 1.7 at factory B.

Find the probability that the total number of accidents in the two factories during a 2-month period is more than 3. [4]

10) June 2021 9709_61 Q5

On average, 1 in 75 000 adults has a certain genetic disorder.

(a) Use a suitable approximating distribution to find the probability that, in a random sample of 10 000 people, at least 1 has the genetic disorder. [3]

(b) In a random sample of n people, where n is large, the probability that no-one has the genetic disorder is more than 0.9.

Find the largest possible value of n. [4]

11) June 2021 9706_62 Q7

Customers arrive at a particular shop at random times. It has been found that the mean number of customers who arrive during a 5-minute interval is 2.1.

(a) Find the probability that exactly 4 customers arrive during a 10-minute interval. [2]

(b) Find the probability that at least 4 customers arrive during a 20-minute interval. [2]

(c) Use a suitable approximating distribution to find the probability that fewer than 40 customers arrive during a 2-hour interval. [4]

12) June 2021 9706_63 Q5

Most plants of a certain type have three leaves. However, it is known that, on average, 1 in 10 000 of these plants have four leaves, and plants with four leaves are called 'lucky'. The number of lucky plants in a random sample of 25 000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. [2]

Use your approximating distribution to answer parts **(b)** and **(c)**.

(b) Find $P(X \leqslant 3)$. [2]

(c) Given that $P(X = k) = 2P(X = k + 1)$, find k. [2]

The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geqslant 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. [3]

13) June 2022 9709_61 Q5

Cars arrive at a fuel station at random and at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. [3]

(b) Use an approximating distribution to find the probability that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. [4]

Independently of cars, trucks arrive at the fuel station at random and at a constant average rate of 3.6 per 15-minute period.

(c) Find the probability that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. [3]

14) June 2022 9709_62 Q3

It is known that 1.8% of children in a certain country have not been vaccinated against measles. A random sample of 200 children in this country is chosen.

(a) Use a suitable approximating distribution to find the probability that there are fewer than 3 children in the sample who have not been vaccinated against measles. [4]

(b) Justify your approximating distribution. [2]

15) June 2022 9709_63 Q5

The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5-minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. [3]

(b) If more than 4 clients arrive during a 5-minute period, they cannot all be served.

Find the probability that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. [2]

(c) Use a suitable approximating distribution to find the probability that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at the desk. [4]

16) June 2023 9709_61 Q1

In a certain country, 20 540 adults out of a population of 6 012 300 have a degree in medicine.

(a) Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. [4]

(b) Justify the approximating distribution used in part **(a)**. [2]

17) June 2023 9709_62 Q2

(a) The random variable W has a Poisson distribution.

State the relationship between $E(W)$ and $Var(W)$. [1]

(b) The random variable X has the distribution $B(n, p)$. Jyothi wishes to use a Poisson distribution as an approximate distribution for X.

Use the formulae for $E(X)$ and $Var(X)$ to explain why it is necessary for p to be close to 0 for this to be a reasonable approximation. [1]

(c) Given that Y has the distribution $B(20\,000, 0.000\,07)$, use a Poisson distribution to calculate an estimate of $P(Y > 2)$. [3]

18) June 2023 9709_62 Q4

The number, X, of books received at a charity shop has a constant mean of 5.1 per day.

(a) State, in context, one condition for X to be modelled by a Poisson distribution. [1]

Assume now that X can be modelled by a Poisson distribution.

(b) Find the probability that exactly 10 books are received in a 3-day period. [2]

(c) Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. [4]

The number of DVDs received at the same shop is modelled by an independent Poisson distribution with mean 2.5 per day.

(d) Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. [3]

19) June 2023 9709_63 Q6

It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12 500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X.

(a) Use an appropriate approximating distribution to find $P(X \leqslant 3)$. [3]

(b) Find the values of $E(X)$ and $Var(X)$, and explain how your answers suggest that the approximating distribution used in **(a)** is likely to be appropriate. [2]

20) October 2020 9709_61 Q1

It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an appropriate approximating distribution to find the probability that more than 1 flower in the sample is white. [3]

(b) Justify the approximating distribution used in part **(a)**. [1]

The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in the two samples is less than 4. [3]

21) October 2020 9709_61 Q5

The number of absences per week by workers at a factory has the distribution Po(2.1).

(a) Find the standard deviation of the number of absences per week. [1]

(b) Find the probability that the number of absences in a 2-week period is at least 2. [3]

(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. [2]

22) October 2020 9709_62 Q1

On average, 1 in 50 000 people have a certain gene.

Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3]

23) October 2020 9709_62 Q5

Customers arrive at a shop at a constant average rate of 2.3 per minute.

(a) State another condition for the number of customers arriving per minute to have a Poisson distribution. [1]

It is now given that the number of customers arriving per minute has the distribution Po(2.3).

(b) Find the probability that exactly 3 customers arrive during a 1-minute period. [2]

(c) Find the probability that more than 3 customers arrive during a 2-minute period. [3]

(d) Five 1-minute periods are chosen at random. Find the probability that no customers arrive during exactly 2 of these 5 periods. [3]

24) October 2021 9709_61 Q2

The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

25) October 2021 9709_61 Q5a

(a) The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

26) October 2021 9709_62 Q5a & Q5c

In a certain large document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. [2]

In the same document, formatting errors occur at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. [3]

27) October 2022 9709_61 Q3

Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. [3]

(b) Use a suitable approximating distribution to find the probability that at least 650 drops fall during a randomly chosen 2-hour period. [4]

28) October 2022 9709_62 Q3

1.6% of adults in a certain town ride a bicycle. A random sample of 200 adults from this town is selected.

(a) Use a suitable approximating distribution to find the probability that more than 3 of these adults ride a bicycle. [4]

(b) Justify your approximating distribution. [2]

29) SP 2020 9709_6 Q3

The number of calls received at a small call centre has a Poisson distribution with mean 2.4 calls per 5-minute period.

(a) Find the probability of exactly 4 calls in an 8-minute period. [2]

(b) Find the probability of at least 3 calls in a 3-minute period. [3]

The number of calls received at a large call centre has a Poisson distribution with mean 41 calls per 5-minute period.

(c) Use an approximating distribution to find the probability that the number of calls received in a 5-minute period is between 41 and 59 inclusive. [5]

Marking Scheme

1)

$(\lambda =) \frac{5}{12} = 0.417$ or better	**B1**
$1 - e^{-\frac{5}{12}}(1 + \frac{5}{12})$	**M1**
$= 0.0661$ or 0.0662 (3 sf)	**A1**
	3

2) .

(a)	$\lambda (= 0.4 \times 365 \div 50) = 2.92$	**B1**
	$e^{-2.92}(1 + 2.92 + \frac{2.92^2}{2})$	**M1**
	$= 0.441$ (3 sf)	**A1**
		3
(b)	$e^{-\lambda} > 0.95$	**M1**
	$-\lambda > \ln 0.95$ or $\lambda < 0.051293$ OE	**M1**
	'0.051293' $\times 50 \div 0.4$ $(= 6.411)$	**M1**
	Largest n is 6 (3 sf) Allow $n = 6$ or $n \leqslant 6$ (NOT $n < 6$ or $n \geqslant 6$ as final answer)	**A1**

3)

(a)	$B(1000, \frac{1}{400})$	**B1**
		1
(b)	Po(2.5)	**B2**
		2
c)(i)	$e^{-2.5} \times \frac{2.5^4}{4!}$	**M1**
	0.134 (3 sf)	**A1**
		2

(c)(ii)	$e^{-2.5}(\frac{2.5^2}{2!}+\frac{2.5^3}{3!}+\frac{2.5^4}{4!})$	**M1**
	0.604 (3 sf)	**A1**
		2
(d)	$\lambda = 2.5 \times 0.7$ or $\lambda = 700\times\frac{1}{400}$ [= 1.75]	**M1**
	$1 - e^{-1.75}$	**M1**
	0.826	**A1**
		3

4) .

(a)(i)	0.024×50 [= 1.2] and 0.018×60 [= 1.08]	**B1**
	$(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.08}(1 + 1.08))$	**M1**
	= 0.0991 (3 sf)	**A1**
		3
(a)(ii)	$\lambda = 0.024 \times 50 + 0.018 \times 60$	**M1**
	$1-e^{-2.28}\times\left(1+2.28+\frac{2.28^2}{2!}+\frac{2.28^3}{3!}\right)$	**M1**
	= 0.197 (3 sf)	**A1**
		3
7(b)	$e^{-\lambda}=\left[e^{-\mu}\right]^2=e^{-2\mu}$	**M1**
	$e^{-\lambda}\times\frac{\lambda^2}{2}=k\left[e^{-\mu}\times\mu\right]^2$	**M1**
	$e^{-2\mu} \times 2\mu^2 = k \times e^{-2\mu} \times \mu^2$	**M1**
	$k = 2$	**A1**
		4

5) .

(a)	Orders arrive at constant mean rate (must say mean or rate) Orders arrive at random Orders arrive independently Orders arrive singly	B1 B1
		2
(b)(i)	$\lambda = \frac{3}{8} \times 25.2$ [= 9.45]	B1
	$e^{-'9.45'}(\frac{9.45^3}{3!} + \frac{9.45^4}{4!} + \frac{9.45^5}{5!})$ or $e^{-'9.45'}(140.65 + 332.29 + 628.03)$ or 0.01107 + 0.02615 + 0.04942	M1
	= 0.0866 (3 sf)	A1
		3
(b)(ii)	$e^{-3.15} \times 3.15$ **or** $(1 - e^{-3.15}(1 + 3.15))$ **or** 0.135 **or** 0.822 (3 sf)	B1
	$e^{-3.15} \times 3.15 \times (1 - e^{-3.15}(1 + 3.15))$	M1
	$\times 2$ or 0.111×2	M1
	0.222 (3 sf)	A1
		4
(c)	N(113.4, 113.4)	B1
	$\frac{120.5-113.4}{\sqrt{113.4}}$ [= 0.667]	M1
	$1 - \phi$(*their* '0.667')	M1
	= 0.252 (3 sf)	A1
		4

6) .

(a)(i)	$e^{-2.9} \times \frac{2.9^4}{4!}$	M1
	0.162 (3 sf)	A1
		2
(a)(ii)	$e^{-2.1} \times \frac{2.1^4}{4!} \times e^{-0.8} + e^{-2.1} \times \frac{2.1^3}{3!} \times e^{-0.8} \times 0.8$	B1
	(**B1** for either expression correct, **M1** for P(4, 0) + P(3, 1))	M1
	0.113 (3 sf)	A1
		3
(b)	N(29, 29)	M1
	$\frac{24.5-29}{\sqrt{29}} (= -0.83563)$	M1
	$1 - \Phi(\text{"0.836"})$	M1
	0.202 (3sf)	A1
		4
(c)	29 is large or $29 > 15$	B1
		1

7) .

(a)(i)	B(3600, 0.0012)	B1
		1
(a)(ii)	Po(4.32) (**B1** for Po. **B1** for $\lambda = 4.32$)	B2
	$n = 3600$ which is large, $p = 0.12$ which is small and $np = 4.32$ which is < 5	B1
		3
(a)(iii)	$1 - e^{-4.32}\left(1 + 4.32 + \frac{4.32^2}{2}\right)$	M1
	0.805 (3 sf)	A1
		2
(b)	$e^{-\lambda} > 0.1$	M1
	$(-\lambda > \ln 0.1)$ $(\lambda < \ln 10)$ $0.0012n < \ln 10$	A1
	$(n < 1918.8)$ largest n is 1918	A1
		3

8) .

(a)(i)	0, 1, 2, 3, ...	B1
		1
a)(ii)	3	B1
		1
a)(iii)	$e^{-3}\left(\frac{3^4}{4!}+\frac{3^5}{5!}+\frac{3^6}{6!}\right)$	M1
	0.319 (3 sf)	A1
		2
(b)	$\Phi^{-1}(0.0668)$ $(=-1.500)$	M1
	$N(\mu, \mu)$	M1
	$\frac{45.5-\mu}{\sqrt{\mu}} = \text{“}-1.500\text{”}$	M1
	$\mu - \text{“}1.500\text{”}\sqrt{\mu} - 45.5 = 0$ $\sqrt{\mu} = \frac{\text{"}1.5\text{"} \pm \sqrt{(\text{"}-1.5\text{"})^2 + 4\times 45.5}}{2} (= 7.5369)$	M1
	$\mu = 56.8$ (3 sf)	A1
		5

9) .

$\lambda = (3.1 + 1.7) \times 2$	M1
$= 9.6$	A1
$1 - e^{-9.6}\left(1 + 9.6 + \frac{9.6^2}{2} + \frac{9.6^3}{3!}\right)$	M1
$= 0.986$ (3 sf)	A1
	4

10) .

(a)	$\text{Po}\left(\frac{2}{15}\right)$	M1
	$P(X \geqslant 1) = 1 - e^{-\frac{2}{15}}$	M1
	$= 0.125$ (3 sf)	A1
		3

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(b)		
	$\lambda = \dfrac{n}{75000}$	B1
	$e^{-\frac{n}{75000}} > 0.9$	M1
	$-\dfrac{n}{75000} > \ln 0.9 \quad [n < 7902.04]$	M1
	Largest value of n is 7902	A1
	Alternative method for Question 5(b)	
	$e^{-\mu} > 0.9$	M1
	$-\mu > \ln 0.9 \quad [\mu < 0.10536]$	M1
	$n = \mu \times 75000$	B1
	Largest value of n is 7902	A1

Alternative method for Question 5(b)	
$\dfrac{74999}{75000}$	B1
$\left(\dfrac{74999}{75000}\right)^n > 0.9$	M1
$n\ln\dfrac{74999}{75000} > \ln 0.9$	M1
Largest value of n is 7901	A1
	4

11) .

a)		
	$e^{-4.2} \times \dfrac{4.2^4}{4!}$	M1
	0.194 (3 sf)	A1
		2

(b)	$1-e^{-8.4}\left(1+8.4+\frac{8.4^2}{2}+\frac{8.4^3}{3!}\right)$	M1
	0.968 (3 sf)	A1
		2
(c)	N(50.4, 50.4)	M1
	$\frac{39.5-50.4}{\sqrt{50.4}}$ [= –1.535]	M1
	Φ('–1.535') = 1 – Φ('1.535')	M1
	0.0624 (3 sf) or 0.0623	A1
		4

12) .

a)	Po(2.5)	**B1**
	n = 25 000 > 50 and np (or λ) = 2.5 which is < 5 or n = 25 000 > 50 and p = 0.0001 < 0.1	**B1**
		2
(b)	$e^{-2.5}(1+2.5+\frac{2.5^2}{2}+\frac{2.5^3}{3!})$	**M1**
	0.758 (3 sf)	**A1**
		2
(c)	$e^{-2.5}\times\frac{2.5^k}{(k)!}=2e^{-2.5}\times\frac{2.5^{k+1}}{(k+1)!}$	**M1**
	$k=4$	**A1**

(d)	$1 - e^{-\lambda} = 0.963$	M1
	$\lambda = -\ln 0.037$ (= 3.2968 or 3.30 or 3.3)	M1
	$n = 33\,000$ (3 sf)	A1
		3

13) .

(a)	$\lambda = 4.5$	B1
	$1 - e^{-4.5}\left(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!}\right)$	M1
	= 0.468 (3 sf)	A1
		3
(b)	$\lambda = 162$ ($X \sim \text{Po}(162) \Rightarrow X \sim \text{N}(162, 162)$)	B1
	$\frac{149.5 - \text{'162'}}{\sqrt{\text{'162'}}}$ and $\frac{160.5 - \text{'162'}}{\sqrt{\text{'162'}}}$ (= –0.982 and –0.118)	M1
	$\Phi(\text{'0.982'}) - \phi(\text{'0.118'})$ oe	M1
	= 0.290 (3 sf)	A1
		4
(c)	$\lambda = \frac{13.5}{6} + 3.6 \times \frac{2}{3}$ OE or 4.65	M1
	$e^{-4.65}\left(\frac{4.65^4}{4!} + \frac{4.65^5}{5!} + \frac{4.65^6}{6!}\right)$	M1
	0.494 (3 sf)	A1
		3

14) .

(a)	Poisson	B1
	Mean = 3.6	B1
	$e^{-3.6}(1 + 3.6 + \frac{3.6^2}{2})$	M1
	0.303 (3 s.f.)	A1
		4
(b)	[Binomial with] 200 > 50	B1
	[200 × 0.018 =] 3.6 < 5 or [p =] 0.018 < 0.1	B1
		2

15)

(a)	$\lambda = 6.6$	B1
	$e^{-6.6} \times \frac{6.6^6}{6!}$	M1
	0.156 (3 s.f.)	A1
		3
(b)	$1 - e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$	M1
	0.0725 (3 s.f.)	A1
		2
(c)	N(26.4, 26.4)	B1
	$\frac{19.5 - '26.4'}{\sqrt{'26.4'}}$ [= –1.343]	M1
	$\phi('-1.343') = 1 - \phi('1.343')$	M1
	0.0897 or 0.0896 (3 s.f.)	A1

16) .

(a)	20540/6012300 = 0.0034163	**B1**
	[1000 × 0.0034163 = 3.4163]	
	Po(3.4163)	**B1**
	$e^{-their\ '3.4163'}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR $e^{-their\ '3.4163'}(1 + 3.4163 + 5.8356+6.6453)$ or 0.03283 + 0.1122 +0.1916 + 0.21819)	**M1**
	= 0.555 (3sf)	**A1**
(b)	$n = 1000 > 50$	**B1**
	$np = 3.4163 < 5$	**B1**
		2

17) .

(a)	E(W) = Var(W).	**B1**
		1
(b)	$np \approx np(1-p)$, hence $1-p$ must be close to 1	**B1**
(c)	$\lambda = 1.4$	**B1**
	$1 - e^{-1.4}(1 + 1.4 + \frac{1.4^2}{2})$ or $1 - e^{-1.4}(1 + 1.4 + 0.98)$ or 1-(0.2466+0.3452+0.2417)	**M1**
	= 0.167 (3 sf) or 0.166	**A1**

18) .

(a)	Books received independently or singly or randomly.	**B1**
(b)	$e^{-15.3} \times \frac{15.3^{10}}{10!}$	**M1**
	= 0.0439 (3sf)	**A1**

(c)	N(153, 153)	B1
	$\frac{180.5-153}{\sqrt{153}}$ [= 2.223]	M1
	$1-\phi$('2.223')	M1
	= 0.0131 (3sf)	A1
		4
(d)	(λ =) 5.1 + 2.5 [= 7.6]	B1
	$1-e^{-7.6}(1+7.6+\frac{7.6^2}{2}+\frac{7.6^3}{3!})= 1-e^{-7.6}(1+7.6+28.88+73.16)$ = 1 – (0.0005005+0.003803+0.01445+0.03661)	M1
	= 0.945 (3sf)	A1

19) .

(a)	$X \sim \text{Po}(2.5)$	B1
	$e^{-2.5}(1+2.5+\frac{2.5^2}{2}+\frac{2.5^3}{3!})$	M1
	= 0.758 (3 sf)	A1
		3
(b)	$E(X) = \frac{5}{2}$ or 2.5 , $\text{Var}(X) = \frac{4999}{2000}$ or 2.4995	*B1
	These are almost equal	DB1
		2

20) .

(a)	$\text{Po}\left(\frac{2}{3}\right)$	B1
	$1-e^{-\frac{2}{3}}\left(1+\frac{2}{3}\right)$	M1
	= 0.144 (3 sf)	A1

(b)	$n > 50$ and $np = \dfrac{2}{3} < 5$ **or** $n > 50$ and $p = \dfrac{1}{300} < 0.1$	**B1**
		3
(c)	$\text{Po}\left(\dfrac{11}{3}\right)$	**B1**
	$e^{-\frac{11}{3}}\left(1+\dfrac{11}{3}+\dfrac{\left(\frac{11}{3}\right)^2}{2!}+\dfrac{\left(\frac{11}{3}\right)^3}{3!}\right)$	**M1**
	$= 0.501$ (3 sf)	**A1**

21) .

(a)	$\sqrt{2.1}$ or 1.45 (3 sf)	**B1**
		1
(b)	$\lambda = 4.2$	**B1**
	$1 - e^{-4.2}(1 + 4.2)$	**M1**
	$= 0.922$ (3 sf)	**A1**
		3
(c)	$\lambda = 6.3$ $e^{-6.3}\left(\dfrac{6.3^5}{5!}+\dfrac{6.3^6}{6!}+\dfrac{6.3^7}{7!}\right)$	**M1**
	$= 0.455$ (3 sf)	**A1**

22) .

Poisson, any λ	**M1**
$1 - e^{-3}\left(1+3+\dfrac{3^2}{2}\right)$	**M1**
$= 0.577$ (3sf)	**A1**

23) .

(a)	Customers arrive independently or singly or at random	**B1**
		1
(b)	$e^{-2.3} \times \frac{2.3^3}{3!}$	**M1**
	$= 0.203$ (3sf)	**A1**
		2
(c)	Po(4.6)	**B1**
	$1 - e^{-4.6}\left(1 + 4.6 + \frac{4.6^2}{2!} + \frac{4.6^3}{3!}\right)$	**M1**
	$= 0.674$ (3sf)	**A1**
		3
(d)	P(none arrive) = $e^{-2.3}$ (= 0.10026)	**M1**
	${}^5C_2(e^{-2.3})^2(1 - e^{-2.3})^3$	**M1**
	$= 0.0732$ or 0.0733 (3sf)	**A1**

24) .

N(45.2, 45.2)	**B1**
$\frac{60.5 - 45.2}{\sqrt{45.2}}$ [= 2.276]	**M1**
$1 - \phi$('2.276')	**M1**
0.0114	**A1**

25) .

(a)(i)	Po(0.025)	**B1**
	$n = 2500 > 50$, $np = 0.025 < 5$	**B1**
(a)(ii)	$1 - e^{-0.025}$	**M1**
	0.0247 (3sf)	**A1**

26) .

(a)	$e^{-2}(1 + 2 + \frac{2^2}{2!})$	M1
	0.677 (3sf)	A1
(c)	$\lambda = 10$	B1
	$e^{-10}\left(\frac{10^8}{8!}+\frac{10^9}{9!}+\frac{10^{10}}{10!}+\frac{10^{11}}{11!}\right)$	M1
	0.477 (3sf)	A1

27) .

(a)	$\lambda = 5.2 \div 2$ [= 2.6]	B1
	$1 - e^{-2.6}(1 + 2.6 + \frac{2.6^2}{2})$ or $1 - e^{-2.6}(1 + 2.6 + 3.38)$ **or** 1- (0.07427 + 0.1931 + 0.2510)	M1
	= 0.482 (3 sf)	B1
		3
(b)	N(120×5.2, 120×5.2)	B1
	$\frac{649.5 - \textit{their } '624'}{\sqrt{\textit{their } '624'}}$ [= 1.021]	M1
	$1 - \Phi$(*their* '1.021')	M1
	= 0.154 (3 sf)	A1

28) .

(a)	Use of Poisson. mean = 3.2	B1 B1
	$1 - e^{-3.2}\left(1 + 3.2 + \frac{3.2^2}{2} + \frac{3.2^3}{3!}\right)$ or $1 - e^{-3.2}(1 + 3.2 + 5.12 + 5.46133)$ or 1 – (0.04076 + 0.1304 + 0.2087 + 0.2226)	M1
	= 0.397 or 0.398	A1
(b)	[Binomial with] [n =] 200 > 50	B1
	[np =][200 × 0.016 =] 3.2 < 5 or [p =]0.016 < 0.1	B1

29) .

(a)	$e^{-3.84} \times \frac{3.84^4}{4!}$	1	**M1**
	= 0.195 (3 sf)	1	**A1**
		2	
(b)	$1 - P(X = 0, 1, 2)$	1	**M1**
	$1 - e^{-1.44}\left(1 + 1.44 + \frac{1.44^2}{2}\right)$	1	**M1**
	= 0.176	1	**A1**
		3	
(c)	$X \sim N(41, 41)$	1	**B1**
	$\frac{40.5 - 41}{\sqrt{41}}$ (= –0.078) $\frac{59.5 - 41}{\sqrt{41}}$ (= 2.889)	2	**M1M1**
	Φ('2.889') – Φ('–0.078') = Φ('2.889') – (1 – Φ('0.078')) = 0.9981 – (1 – 0.5311)	1	**M1**
	= 0.529 (3 sf)	1	**A1**