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(S-2)

Probabily and Statistics-2

Sampling and Estimation
Exercise-2 Solution (Revision)

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Contents:

1. Population and sample; Random sample of size n.
2. The distribution of sample means. $E(\bar{X}(n)) = \mu = E(X)$; $Var(\bar{X}(n)) = \frac{\sigma^2}{n}$
3. Test statistic: $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$ for normal distribution
4. Estimation:
   Unbiased estimate of population variance σ^2:
   $$s^2 = \frac{n}{(n-1)}\sigma^2 = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right) = \frac{1}{n-1}\left(\Sigma x^2 - \frac{(\Sigma x)^2}{n}\right)$$
5. Confidence Interval for the population mean $\bar{x}$. (for α% confidence interval)
   $$\bar{x} - z\cdot\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z\cdot\frac{\sigma}{\sqrt{n}}; \quad z = \phi(\beta) \Leftrightarrow \beta = \phi^{-1}(z)$$

Width of the C.I: $= 2z\cdot\frac{\sigma}{\sqrt{n}}$

$$\begin{cases} \beta = \alpha + \frac{(1-\alpha)}{2} \\ \text{and } \alpha = \beta - (1-\beta) \end{cases}$$

6. Confidence interval (C.I) for population proportion p:
   (i) $$\left(p - z\cdot\sqrt{\frac{p(1-p)}{n}},\ p + z\cdot\sqrt{\frac{p(1-p)}{n}}\right)$$
   (ii) Width of C.I $= 2z\sqrt{\frac{p(1-p)}{n}}$

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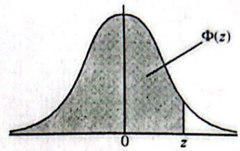
THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z, the table gives the value of $\Phi(z)$, where

$$\Phi(z) = \mathrm{P}(Z \le z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that

$$\mathrm{P}(Z \le z) = p.$$

$z = \phi^{-1}(p)$

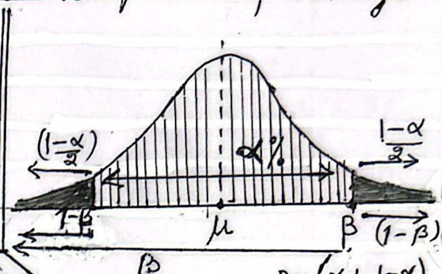
p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Confidence interval (C.I = $\alpha\%$) for population mean.

§ **$\alpha\%$ of confidence interval:**

If we work out sample means for a large number of samples.

$\alpha\%$ of the time we would expect the sample mean $\bar{x}$ to lie within shaded area.



β

$\beta = \left(\alpha + \frac{1-\alpha}{2}\right)$

$z = \phi^{-1}(\beta)$

$\boxed{\alpha = \beta - (1-\beta)}$

§ For $\alpha\%$ confidence interval (C.I) for a population mean from a normal population with given variance

$$\bar{x} - z \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + z \cdot \frac{\sigma}{\sqrt{n}} \quad \text{--- (1)}$$

Example: For 95% confidence interval.

$$\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}}$$

$$\begin{cases} \alpha = 95\% = 0.95 \\ 1-\alpha = 1 - 0.95 = 0.05 \\ \beta = \alpha + \frac{(1-\alpha)}{2} = 0.95 + \frac{0.05}{2} \\ \quad = 0.95 + 0.025 \\ \beta = 0.975 \\ z = \phi^{-1}(\beta) = \phi^{-1}(0.975) = 1.96 \end{cases}$$

§ For large sample:

C.I is $\bar{x} \pm z \cdot \frac{s}{\sqrt{n}}$

$$\begin{cases} \text{using unbaised estimation Variance } s^2 \\ s^2 = \frac{1}{n-1}\left(\sum x^2 - n \cdot \bar{x}^2\right) \end{cases}$$

C.I = $\alpha\%$	90%	95%	98%	99%
β = p	0.95	0.975	0.99	0.995
z	1.645	1.960	2.326	2.576

$P(Z \leq z) = p \Rightarrow z = \phi^{-1}(p)$

$z = \phi^{-1}(\beta)$

$$\begin{cases} \beta = \left(\alpha + \frac{1-\alpha}{2}\right) \\ \alpha = \beta - (1-\beta) \end{cases}$$

(*) "Given in the normal distribution table."

§ Total width of the confidence interval $= 2z \cdot \frac{\sigma}{\sqrt{n}}$ from (1)

Date ___/___/___
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1. Leaves from a certain type of tree have lengths that are distributed with standard deviation 3.2 cm. A random sample of 250 of these leaves is taken and the mean length of this sample is found to be 12.5 cm.

(a) Calculate a 99% confidence interval for the population mean length. [3]

(b) Write down the probability that whole of a 99% confidence interval will lie below the population mean. [1]

[SP-20/06/Q1]

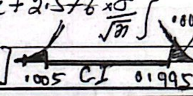
Solution Standard deviation $\sigma = 3.2$; $\bar{x} = 12.5$, $n = 250$,

(a) for 99% confidence interval; ($z = 2.576$ for $\beta = 0.99 + 0.005 = 0.995$)

CI: $12.5 - 2.576 \times \frac{3.2}{\sqrt{250}} < \mu < 12.5 + 2.576 \times \frac{3.2}{\sqrt{250}}$ $\left\{ \begin{aligned} \beta &= \alpha + \frac{(1-\alpha)}{2} \\ &= 0.99 + \frac{(1-0.99)}{2} \end{aligned} \right.$

$\Rightarrow 12.0 < \mu < 13.0$ (3 sf) ✓ $\left\{ \bar{x} - 2.576 \times \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.576 \times \frac{\sigma}{\sqrt{n}} \right\}$

(b) 0.005 (or 0.5%) $\left[\frac{(1-0.99)}{2} = \frac{0.1}{2} = 0.005 \right]$



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2. Describe briefly how to use random numbers to choose a sample of n students from a year-group of 276 students.

[SP-20/06/Q2] ---[3]

Solution: Generate 3 digit numbers. (Number students from 1 to 276).

Ignore number > 276

Ignore repeats.

3. The lengths of a certain species of lizard are known to be normally distributed with standard deviation 3.2 cm. A naturalist measures the lengths of a random sample of 100 lizards of this species and obtains an $\alpha\%$ confidence interval for the population mean. He finds the total width of this interval is 1.25 cm. Find α. ---[5]

[M-20/62/Q2]

Solution: $\sigma = 3.2,\ n = 100,$

Total width of confidence interval $2z.\frac{\sigma}{\sqrt{n}} = 1.25$ (Given)

$\Rightarrow 2z \times \frac{3.2}{\sqrt{100}} = 1.25 \Rightarrow z = \frac{1.25 \times 10}{2 \times 3.2} = 1.953$

$\beta = \phi(z) = \phi(1.953) = 0.9746$ $\quad\left[z = \phi^{-1}(\beta) : \beta = \alpha + \left(\frac{1-\alpha}{2}\right) \right.$

$\Rightarrow \alpha = 2\beta - 1 = 2 \times 0.9746 - 1 = 0.9492$ $\quad \Rightarrow \alpha = 2\beta - 1$

$\therefore \alpha\% = \underline{94.9\%}$ (or $\underline{95\%}$)

4. A construction company notes the time, t days, that it takes to build each house of certain design. The results of a random sample of 60 such houses are summarised as follows; $\Sigma t = 4820, \Sigma t^2 = 392050$

(a) Calculate a 98% confidence interval for the population mean time. ---[6]

(b) Explain why it was necessary to use Central Limit Theo. in part (a) ---[1]

[M-21/62/Q1]

Solution: (a) $\bar{x} = $ Est$(\mu) = \frac{\Sigma t}{n} = \frac{4820}{60} = \frac{241}{3}$ (or 80.3); Est $\sigma^2 = \frac{60}{59}\left(\frac{392050}{60} - \left(\frac{241}{3}\right)^2\right)$

$s^2 = $ Est $\sigma^2 = 82.0904$ ✓

for 98% C.I $z = \phi^{-1}(0.98 + 0.01)$

$z = \phi^{-1}(0.99) = 2.326$ ✓

C.I $= \bar{x} \pm z\sqrt{\frac{s^2}{n}} = \frac{241}{3} \pm 2.326 \times \sqrt{\frac{82.0904}{60}} = \underline{77.6 \text{ to } 83.1}$

C.I $= \underline{77.6 < \mu < 83.1}$ ✓

(b) Population distribution of times is unknown. ($n = 60 > 50$)

Sampling and Estimation-Revision

5. The lengths, in millimetres, of a random sample of 12 rods made by a certain machine are as follows:
200, 201, 198, 202, 200, 199, 199, 201, 197, 202, 200, 199.

(a) Find unbiased estimates of the population mean and variance. ---[3]
(b) Give a statistical reason why these estimates may not be reliable. ---[1]

M-22/62/Q1

Solution: (a) $Est(\mu) =$ mean of random sample $\bar{x} = \frac{\Sigma x_i}{n} = \frac{2398}{12} = 199.833$
$= \underline{200}$ (3s.f)

$$Est(\sigma^2) = s^2 = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2\right) = \frac{12}{11}\left[\frac{479226}{12} - \left(\frac{2398}{12}\right)^2\right]$$
$$= \underline{2.33} \text{ (3 s.f) } (mm^2)$$

(b) Small sample.

6. A random sample of 500 households in certain town was chosen. Using this sample, a confidence interval for proportion, p, of all households in this town that owned two or more cars was found to be $0.355 < p < 0.445$.
Find the confidence level of this confidence interval. Give your answer correct to the nearest integer. ---[5]

M-22/62/Q3

Solution:

$0.355 < p < 0.445 \Rightarrow 2p = 0.445 + 0.355 = 0.8$
$\Rightarrow p = 0.8/2 = 0.4$
$\Rightarrow est(p) = 0.4$

$\Rightarrow p + z\sqrt{\frac{p(1-p)}{n}} = 0.445$

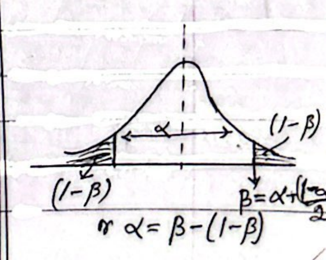
$\left[C.I: \left(p - z\sqrt{\frac{p(1-p)}{n}},\ p + z\sqrt{\frac{p(1-p)}{n}}\right)\right]$

$\Rightarrow 0.4 + z\sqrt{\frac{0.4(1-0.4)}{500}} = 0.445 \Rightarrow z = 0.045 \div \sqrt{\frac{0.4(1-0.4)}{500}}$
$\Rightarrow z = 2.054$

$\beta = \phi(z) = \phi(2.054)$
$= 0.98 = \beta$ (let)

det $C.I = \alpha = \beta - (1-\beta)$

$\alpha = 0.98 - (1 - 0.98)$
$C.I = 0.96$ or 96% confidence



7. Anita carried out a survey of 140 randomly selected students at her college. She found that 49 of these students watched a TV programme called Bunch.
(a) Calculate an approximate 98% confidence interval for the proportion, p, of students at Anita's college who watch Bunch. ---[3]
Carlos says that the confidence interval found in (a) is not useful because it is too wide.
(b) Without calculation, explain briefly how Carlos can use the results of Anita's survey to find a narrower confidence interval for p. ---[1]
[M-23/62/Q1]

Solution:
(a) Proportion $p = \frac{49}{140} = 0.35$

$\alpha\% = 98\% \rightarrow \alpha = 0.98$
$\beta = \alpha + \frac{(1-\alpha)}{2} = 0.98 + \frac{(1-.98)}{2}$
$= 0.98 + 0.01$
$\beta = .99$
$\therefore\ z = \phi^{-1}(\beta) = \phi^{-1}(.99) = 2.326$

Confidence interval for proportion:
$p \pm z\sqrt{\frac{p(1-p)}{n}}$, $p = 0.35$
$= 0.35 \pm 2.326 \times \sqrt{\frac{0.35(1-0.35)}{140}}$
$\Rightarrow$ Confidence interval is: 0.256 to 0.444 (3sf)

(b) Find a smaller percentage confidence interval/lower level of confidence.

8. The lengths, X centimeters, of a random sample of 7 leaves from a certain variety of tree are as follows: 5.2, 4.8, 5.5, 6.1, 4.8, 3.9, 4.4.
(a) Calculate unbiased estimates of the population mean and variance of X. ---[3]
It is now given that the true value of the population variance of X is 0.55, and that X has normal distribution.
(b) Find a 95% confidence interval for the population mean of X ---[3]
[S-20/61/Q1]

Solution
(a) Est mean $\bar{x} = \frac{\Sigma x}{n} = \frac{34.7}{7} = 4.9571$ or 4.96 (3sf) ; $\Sigma x^2 = 175.15$

$s^2 = \text{Est Var. } \sigma^2 = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right) = \frac{7}{6}\left(\frac{175.15}{7} - (4.9571)^2\right) = 0.523$ (3sf)

(b) 95% C.I $\rightarrow z = 1.96$

$z = \phi^{-1}\left(0.95 + \frac{0.05}{2}\right)$
$= \phi^{-1}(0.975)$
$z = 1.96$

$C.I = \bar{x} \pm z\sqrt{\frac{s^2}{n}} = 4.96 \pm 1.96 \times \sqrt{\frac{0.523}{7}}$

C.I: 4.42 to 5.49 (3sf)

$\beta = \alpha + \frac{(1-\alpha)}{2}$
$z = \phi^{-1}(\beta)$

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9. The score on one spin of a 5-sided spinner is denoted by the random variable X with prob. distribution as shown in the table:

x	0	1	2	3	4
$P(X=x)$	0.1	0.2	0.4	0.2	0.1

(a) Show that $Var(X) = 1.2$ --[2]

The spinner is spun 200 times. The score on each spin is noted and mean, $\bar{x}$, of the 200 scores is found.

(b) Given that $P(\bar{X} > a) = 0.1$, find the value of a. ---[4]

(c) Explain whether it was necessary to use Central Limit theorem in your answer to part(b). [S-20/62/Q4] ---[1]

Solution: $E(X) = \Sigma p_i x_i = 0\times.1 + 1\times.2 + 2\times.4 + 3\times.2 + 4\times0.1 = 2$ ✓

(a) $E(X^2) = \Sigma px^2 = 0^2\times.1 + 1^2\times.2 + 2^2\times.4 + 3^2\times.2 + 4^2\times.1 = 5.2$

$Var(X) = E(X^2) - (E(X))^2 = 5.2 - 2^2 = 1.2$ ✓

(b) Now $n = 200$ (large), $E(\bar{X}) = 2$, $Var(\bar{X}) = \frac{\sigma^2}{n} = \frac{1.2}{200}$; using Normal distribution

$P(\bar{X} > a) = P\left(z > \frac{a-2}{\sqrt{1.2 \div 200}}\right) = .1 \Rightarrow 1 - \phi\left(\frac{a-2}{\sqrt{1.2 \div 200}}\right) = 0.1$

$\Rightarrow \frac{a-2}{\sqrt{1.2 \div 200}} = \phi^{-1}(0.9) = 1.282 \Rightarrow a = 2 + \sqrt{1.2 \div 200} \times 1.282$

$\Rightarrow a = 2.10$ (3 sf)

(c) Yes, use CLT, as X is not normally distributed. ($n = 200 > 50$)

10. A random sample of 100 values of X is taken. These value are summarised as: $n = 100$, $\Sigma x = 1556$, $\Sigma x^2 = 29004$

Calculate unbiased estimates of the population mean and variance of X [S-20/63/Q1] ---[3]

Solution: Est $\mu = \frac{\Sigma x}{n} = \frac{1556}{100} = 15.56$ ✓

$$\text{Est } \sigma^2 = \frac{n}{n-1}\left[\frac{\Sigma x^2}{n} - \mu^2\right]$$

$$= \frac{100}{99}\left[\frac{29004}{100} - (15.56)^2\right] = 48.4105$$

Est $\sigma^2 = s^2 = 48.4$ ✓ (3 sf)

11. Sunita has a six-sided die with faces marked 1, 2, 3, 4, 5, 6. The probability that the die shows a six on any throw is p. Sunita throws the die 500 times and finds that it shows a six 70 times.

(a) Calculate an approximate 99% confidence interval for p. --- [4]

(b) Sunita believes that the die is fair. Use your answer to part (a) to comment on her belief. --- [1]

(c) Sunita uses the result of her 500 throws to calculate an $\alpha\%$ confidence interval for p. This interval has width 0.04. Find the value of α. --- [5]

S-20/63/Q5

Solution: $n = 500$, $p = \frac{70}{500} = 0.14$, Confidence interval $= 99\%$, $(1 - .99) = .01$

(a)

$\phi(z) = k = .99 + \frac{0.01}{2} = 0.995$

$z = \phi^{-1}(0.995) = 2.576$

$$CI = p \pm z \times \sqrt{\frac{p(1-p)}{n}}$$

$$CI = 0.14 + 2.576 \times \sqrt{\frac{0.14(1-0.14)}{500}} = 0.14 \pm 0.04 \Rightarrow 0.100 \leq p \leq 0.180$$

(b) $P(X=6) = 1/6 = 0.1666$--- which lies within the confidence interval, hence the belief is justified.

(c) width of $CI = 2z \cdot \sqrt{\frac{p(1-p)}{n}} = 0.04$

$$\Rightarrow 2z\sqrt{\frac{0.14 \times (1-0.14)}{500}} = 0.04 \Rightarrow z = 1.289$$

$$\phi(1.289) = 0.9013$$

$$C.I \ \alpha\% = 0.9013 - (1 - 0.9013) = 0.803$$

$$\therefore CI = 80.3\%$$

or $\alpha = 80.3$

§ For a sample size 'n' drawn from a normal distribution with known variance σ^2, and sample mean $\bar{x}$, the test statistic is: $z = \left(\frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}\right)$

12. The time, in minutes, taken by students to complete a test has the distribution $N(125, 36)$

(a) Find the probability that the mean time taken to complete the test by a random sample of 40 students is less than 123 minutes. ---[3]

(b) Explain whether it was necessary to the Central Limit theorem in the solution to part (a). ---[1]

S-21/61/Q2

Solution: $N(125, 36) \rightarrow \mu = 125, \sigma^2 = 36, \; E(\bar{X}) = 125$, random variable mean time $= \bar{x}$

(a) for $n = 40$, Variance $(\bar{X}(n)) = \frac{\sigma^2}{n} = \frac{36}{40}$ ✓

$$P(\bar{x} < 123) = P\left(z < \frac{123-125}{\sqrt{\frac{36}{40}}}\right) = P(z < -2.108)$$

$$= 1 - \phi(2.108) = 1 - 0.9825 = 0.0175$$ ✓

(b) No, population is normal.

13. 100 randomly chosen adults each throw a ball once. The length, l, metres, of each throw is recorded. The results are summarised as: $n = 100$, $\Sigma l = 3820$, $\Sigma l^2 = 182200$
Calculate a 94% confidence interval for the population mean length of throws by adults. ---[6]

S-21/ 61/ Q4

Solution: $n = 100$, $\Sigma l = 3820$, $\Sigma l^2 = 182200$

mean. $\bar{l} = \frac{\Sigma l}{n} = \frac{3820}{100} = \underline{38.2}$ ✓

Unbiased estimated variance $s^2 = \frac{1}{(n-1)}\left(\Sigma x^2 - n\bar{x}^2\right)$

$= \frac{1}{(100-1)}\left(182200 - 100 \times 38.2^2\right) = \frac{12032}{33}$

or $s^2 = \underline{366.424}$ ✓

for 94% CI:

$\left(CI: \bar{x} - z\cdot\frac{s}{\sqrt{n}} < \mu < \bar{x} + z\cdot\frac{s}{\sqrt{n}}\right)$

for 94% C.I $\Rightarrow \alpha = 0.94$
$\beta = \alpha + (1-\alpha)/2$
$= .94 + \frac{0.06}{2} = .97$
0.3
$\beta = 0.97$ ... 0.3

CI: $38.2 - 1.881 \times \sqrt{\frac{366.424}{100}} < 38.2 + 1.881 \times \sqrt{\frac{366.424}{100}}$

Check;
$\phi^{-1}(0.97)$
$z = 1.881$ ✓
$[z = \phi^{-1}(\beta)]$

$\Rightarrow$

CI: $\underline{34.6 < \mu < 41.8}$ (3 sf)

14. The height, h centimetres, of a random sample of 100 fully grown animals of a certain species were measured. The results are summarised as: $n = 100$, $\Sigma h = 7570$, $\Sigma h^2 = 588050$

(a) Find unbiased estimates of population mean and variance. --[3]

(b) Calculate a 99% confidence interval for the mean height of animals of this species. --[3]

Four random samples were taken and a 99% confidence interval for the population mean, μ, was found from each sample.

(c) Find the probability that all four of these confidence intervals contain the true value of μ. --[2]

S-21/62/Q6

Solution: $n = 100$, $\Sigma h = 7570$, $\Sigma h^2 = 588050$

(a) $est(\mu) = \frac{\Sigma h}{n} = \frac{7570}{100} = \underline{75.70}$ ✓

$$est(\sigma^2) = s^2 = \frac{1}{(n-1)}\left(\Sigma h^2 - \frac{(\Sigma h)^2}{n}\right) = \frac{1}{(100-1)}\left(588050 - \frac{(7570)^2}{100}\right)$$

$$\Rightarrow s^2 = 152\ (3sf)\ (= 151.525)$$

(b) CI: $\bar{h} - z\cdot\frac{s}{\sqrt{n}} < \mu < \bar{h} + z\cdot\frac{s}{\sqrt{n}}$ $\left[z = 2.576 \left(\text{for } 99\%\ C.I \atop \beta = 0.995\right)\right]$

$$\Rightarrow 75.7 - 2.576 \times \sqrt{\frac{151.525}{100}} < \mu < 75.7 + 2.576 \times \sqrt{\frac{151.525}{100}}$$

C.I: $\underline{72.5 < \mu < 78.9}$ ✓

(c) $P(\text{four } C.I) = (0.99)^4$
$= \underline{0.961\ (3sf)}$

$$\left\{\begin{aligned} \beta &= \alpha + \left(\frac{1-\alpha}{2}\right) \\ &= .99 + \frac{(1-0.99)}{2} \\ &= 0.99 + 0.005 \\ &= 0.995 \\ z &= \phi^{-1}(\beta) = \phi^{-1}(0.995) \\ &= 2.576 \end{aligned}\right.$$

15. The masses, m kilograms, of flour in a random sample of 90 sacks of flour are summarised as follows:

$$n = 90, \quad \Sigma m = 4509, \quad \Sigma m^2 = 225950$$

(a) Find unbaised estimates of the population mean and variance. --[3]

(b) Calculate a 98% confidence interval for the population mean. --[3]

(c) Explain why it was necessary to use Central Limit theorem in answering part (b). --[1]

(d) Find the probability that the confidence interval found in part (b) is wholly above the true value of the population mean. --[2]

S-21 | 63 | Q4

Solution: (a) est mean $\bar{x} = \frac{\Sigma m}{n} = \frac{4509}{90} = 50.1$ ✓

$s^2 = \text{est Var}(m) = \frac{n}{n-1}\left(\frac{\Sigma m^2}{n} - \mu^2\right) = \frac{90}{89}\left(\frac{225950}{90} - (50.1)^2\right) = \frac{481}{890}$

$= \underline{0.552}$ ✓ (3sf)

(b) C.I $\quad \bar{x} \pm z\sqrt{\frac{\sigma^2}{n}}$

$\left\{\begin{array}{l} 98\%\ C.I \\ \Rightarrow z = \phi^{-1}(98 + 1\%) \\ z = \phi^{-1}(0.99) = 2.326 \end{array}\right.$

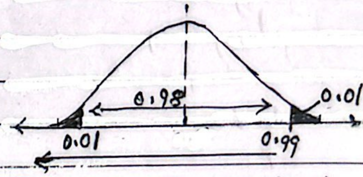
$= 50.1 \pm 2.326 \times \sqrt{\frac{481/890}{90}}$

$= 49.9$ to 50.3 (3 sf) $\Rightarrow$ CI: $\underline{49.9 < \mu < 50.3}$ ✓

(c) Population of masses is not given to be Normal distribution.

(d) $1 - 0.98 = 0.02$

Req Prob $= 0.02 \div 2 = 0.01$ ✓



$\beta = \alpha + \frac{(1-\alpha)}{2}$

$= 0.98 + \frac{(1-0.98)}{2}$

$= 0.98 + 0.01$

$\beta = 0.99$

16 The diameters, x millimetres, of a random sample of 200 discs made by a certain machine were recorded. The results are summerised as:

$$n = 200, \quad \Sigma x = 2520, \quad \Sigma x^2 = 31852.$$

(a) Calculate a 95% confidence interval for the population mean diameter. --- [6]

(b) Joan chose random samples and used each sample to calculate a 95% confidence interval for the population mean diameter.

How many of these 40 confidence intervels would be expected to include the true value of the population mean diameter? --- [1]

S-22 | 61 | Q1

Solution: $n = 200, \ \Sigma x = 2520, \ \Sigma x^2 = 31852$

(a) $Est(\mu) = \frac{\Sigma x}{n} = \frac{2520}{200} = 12.6$ ✓

$Est(\sigma^2) = s^2 = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\mu)^2\right)$

$= \frac{200}{199}\left(\frac{31852}{200} - (12.6)^2\right)$

$= 0.5025$ (or 0.503)

$z = 1.96$, for 95% confidence interval

$C.I = \mu \pm z \cdot \sqrt{\frac{Est(\sigma^2)}{n}}$

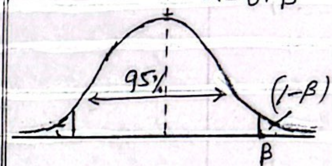
$= 12.6 \pm 1.96 \times \sqrt{\frac{0.5025}{200}}$

$= 12.6 \pm 0.0982$

or $= 12.6 \pm 0.1$

$C.I =$ '12.5 to 12.7' ✓

$\alpha = 0.95$



$\beta = \alpha + \frac{(1-\alpha)}{2}$

$= 0.95 + \frac{(1-0.95)}{2}$

$= 0.95 + 0.025$

$\beta = 0.975$ (or p)

$\Rightarrow z = \phi^{-1}(0.975)$

$= 1.96$

(b) $0.95 \times 40 = \underline{38}$ ✓

17 X is a random variable with distribution $Po(2.90)$. A random sample of 100 values of X is taken. Find the prob. that the sample mean is less than 2.88. ---[5]

[S-22/62/Q7]

Solution: $\bar{x} \sim N\left(2.9, \frac{2.9}{100}\right)$ $[Po(2.90) \Rightarrow \lambda = 2.90 \sim N(\lambda, \lambda)$

$P(\bar{x} < 2.88) = P\left(z < \frac{2.88 - 2.90}{\sqrt{\frac{2.9}{100}}}\right) = P(z < -0.1174)$

$= 1 - \phi(0.1174) = \underline{0.453}$ ✓

18. The number of characters in emails sent by a particular company is modelled by the distribution $N(1250, 480^2)$

Find the probability that the mean number of characters in a random sample of 100 emails sent by the company is more than 1300. ---[3]

[S-22/63/Q1]

Solution: $N(1250, 480^2) \sim N(\mu, \sigma^2)$.

Now $n = 100$, sample variance $Var(\bar{x}) = \frac{480^2}{100} \Rightarrow S.D(\bar{x}) = \frac{480}{10}$

Sample mean $\bar{x} = \mu = 1250$

$P(\bar{x} > 1300) = P\left(z > \frac{1300 + \frac{1}{200} - 1250}{\frac{480}{10}}\right) = P(z > 1.042)$

$= 1 - \phi(1.042)$

$= \underline{0.149\ (3\ S.f)}$

Continuity correction: $\bar{x} > 1300 \rightarrow$ $1300 + \frac{1}{2n} \rightarrow 1300 + \frac{1}{200}$

19. A random sample of 5 values of a variable X is given as: 2, 3, 3, 5, a

(a) Find an expression, in terms of a, for the mean of these values. ---[1]

It is given that an unbiased estimate of population variance of X, using these values, is 4. It is also given that a is positive.

(b) Find and simplify a quadratic equation in terms of a and hence find the value of a. [S-22/63/Q6] ---[3]

Solution (a) Mean $\bar{x} = \Sigma x/n = \frac{13+a}{5}$ ---(1)

unbiased Est(Var) $= \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right)$

$\Rightarrow \frac{5}{4}\left(\frac{47+a^2}{5} - \left(\frac{13+a}{5}\right)^2\right) = 4$ (Given)

$2a^2 - 13a - 7 = 0$

$\Rightarrow (a-7)(2a+1) = 0$

$\underline{a = 7}$ ✓ or $a = -\frac{1}{2}$ ✗ $(a > 0)$

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20 A certain train journey takes place everyday throughout the year. The time taken, in minutes, for the journey is normally distributed with variance 11.2.

(a) The mean time for a random sample of 'n' of these journeys was found. A 94% confidence interval for the population mean time was calculated and found to have a width of 1.4076 minutes, correct to 4dp.
Find the value of n. ---[3]

(b) A passengers noted the times for 50 chosen journeys in January, February and March. Give a reason why this sample is unsuitable for use in finding a confidence interval for the population mean time. [1]

(c) A reseacher took 4 random samples and a 94% confidence interval for the population mean was found from each sample.
Find the probability that exactly 3 of these confidence intervals contain the true value of the population mean. [2]

[S-23 | 61 | Q4]

Solution (a) width of confidence interval (Var. $\sigma^2 = 11.2$)

$$2z \cdot \frac{\sigma}{\sqrt{n}} = 1.4076 \text{ (given)} \Rightarrow z \times \frac{\sqrt{11.2}}{\sqrt{n}} = \frac{1.4076}{2}$$

$$\Rightarrow 1.881 \times \frac{\sqrt{11.2}}{\sqrt{n}} = 0.7038 \Rightarrow \sqrt{n} = \frac{1.881 \times \sqrt{11.2}}{0.7038} = 8.944$$

$$\Rightarrow n = (8.944)^2 = \underline{80} \checkmark$$

for 94% $\rightarrow \alpha = 0.94$

$$\beta = \alpha + \frac{1-\alpha}{2} = 0.94 + \frac{1-.94}{2}$$

$$\beta = 0.97$$

$$z = \phi^{-1}(0.97) = 1.881 \checkmark$$

(b) Jan, Feb and March does not represent the whole year.

(c) $\alpha = 94\% = 0.94$; $p = 0.94$, $q = 0.06$, $n = 4$

$$P(X=3) = {}^4C_3 (.94)^3 \times 0.06$$

$$= \underline{0.199} \text{ (3 sf)}$$

21. A sample of 5 randomly selected values of a variable X is as follows;
1, 2, 6, 1, a, where $a > 0$
Given that the unbaised estimate of the variance X calculated from this sample is $\frac{11}{2}$. Find the value of a. ---[3]

S-23 | 61 | Q6

Solution: Unbaised est var $X = \frac{n}{n-1}\left[\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2\right]$

$$= \frac{5}{(5-1)}\left[\frac{(1^2+2^2+6^2+1^2+a^2)}{5} - \frac{(1+2+6+1+a)^2}{5^2}\right] = \frac{11}{2} \text{ (Given)}$$

$$= \frac{5}{4}\left[\frac{(42+a^2)}{5} - \frac{(10+a)^2}{5^2}\right] = \frac{11}{2} \Rightarrow \frac{1}{4}\left[(42+a^2) - \left(\frac{100+a^2+20a}{5}\right)\right] = \frac{11}{2}$$

$$\Rightarrow 4a^2 - 20a + 0 = 0 \Rightarrow 4a(a-5) = 0 \Rightarrow \underline{a = 5} \; (a > 0)$$

22 In a survey of 200 randomly chosen students from a certain college, 23% of the students said that they own a car.
Calculate an approximate 93% confidence interval for the proportion of students from the college who own a car. ---[3]

S-23 | 62 | Q1

Solution: C.I for the population proportion p:

$$p \pm z \times \sqrt{\frac{p(1-p)}{n}} \quad ; \quad p = 0.23, n = 200$$

$$= 0.23 \pm 1.811 \sqrt{\frac{0.23(1-0.23)}{200}}$$

for 93%
$\alpha = 0.93$, $1-\alpha = 0.07$
$\beta = \alpha + \frac{(1-\alpha)}{2} = 0.93 + 0.035$
$= 0.965$
$z = \phi^{-1}(\beta) = \phi^{-1}(0.965)$
$z = 1.811$

$$= 0.23 \pm 1.811 \times 0.02975$$

$$= 0.23 \pm 0.54$$

C.I is $\underline{0.176 \text{ to } 0.284}$ (3 sf)

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23. A club has 264 members, numbered from 1 to 264. Donash wants to choose a random sample of members for a survey. In order to choose the members for the sample he uses his calculator to generate random digits. His first 20 random digits are as follows:

10612 , 11801 , 21473, 22759

(a) The numbers of the first two members in the sample are 106 and 121. Write down the numbers of next two members in the sample. --[2]

(b) To obtain the numbers for members after 4_{th} member, Donash start with the second random digit, 0, and obtain the numbers 061, 211. Explain why this method will not produce a random sample. --[1]

S-23/63/Q2

Solution (a) 180 and 227.

(b) These numbers are not independent of previous numbers.

or only a finite number of digits used.

24. In a random sample of 100 students at Luciana's college, x-students said that they liked exams. Lusiana used this result to find an approximate 90% confidence interval for the proportion, p, all students at her college who liked exams. Her confidence interval had width 0.15792.

(a) Find the possible values of x. --[4]

Suzma independently took another random sample and found another approximate 90% confidence interval for p.

(b) Find the probability that neither of the two confidence intervals contain the true value of p. --[1]

S-23/63/Q3

Solution: width of the C.I , $n=100$, $p = x/100$

(a) $\Rightarrow 2z\sqrt{\dfrac{p(1-p)}{n}} = 0.15792$

$\Rightarrow 1.645\sqrt{\dfrac{\frac{x}{100}\left(1-\frac{x}{100}\right)}{100}} = 0.07896$

$\Rightarrow x(100-x) = \dfrac{100^3 \times (0.7896)^2}{(1.645)^2}$

$\Rightarrow x^2 - 100x + 2304 = 0$

$\Rightarrow x = 36$ or 64 ✓

90% CI $\rightarrow \alpha = .9$, $\beta = \alpha + \frac{(1-\alpha)}{2}$

$= .9 + \frac{(1-.9)}{2}$

$\beta = 0.95$

$z = \phi^{-1}(0.95) = 1.645$

(b) $p = 90\% = .9$, $q = 0.1$

Now $n = 2$

$P(X=0) = {}^2C_0\, p^0 q^2 = 1 \times 1 \times (0.1)^2$

$= 0.01$ ✓

25. In a survey, a random sample of 250 adults in Fromleigh were asked to fill a questionnaire about their travel.

(a) It was found that 102 adults in the sample travel by bus. Find an approximate 90% confidence interval for the proportion of all the adults in Fromleigh who travel by bus. ---[3]

(b) The survey included a question about the amount, x dollars, spent on travel per year. The results are summarised as follows: $n = 250$, $\Sigma x = 50460$, $\Sigma x^2 = 19854200$

Find unbiased estimates of the population mean and variance of the amount spent per year on travel. ---[3]

A councillor wanted to select a random sample of houses in Fromleigh. He planned to select the first house on each of the 143 streets in Fromleigh.

(c) Explain why this would not provide a random sample. ---[1]

W-20/61/Q2

Solution: (a) $n = 250$, travel by bus $= 102$, 90% confidence interval

$p = \frac{102}{250}$

$$CI = p \pm z\sqrt{\frac{p(1-p)}{n}}$$

$\phi(z) = 0.90 + \frac{(0.1)}{2} = 0.95$

$z = \phi^{-1}(0.95) = 1.645$ ✓

$$= \frac{102}{250} \pm 1.645\sqrt{\frac{\frac{102}{250}\left(1-\frac{102}{250}\right)}{250}}$$

CI is 0.357 to 0.459 (3s.f) ✓

(b) $n = 250$, $\Sigma x = 50460$, $\Sigma x^2 = 19854200$

Estimate of mean $= \frac{\Sigma x}{n} = \frac{50460}{250} = \$201.84$ ✓

Estimate of Var. $s^2 = \frac{1}{(n-1)}\left(\Sigma x^2 - \frac{\Sigma x^2}{n}\right) = \frac{1}{249}\left(19854200 - \frac{50460^2}{250}\right)$

$= 38832.75$ dollars². ✓

(c) Every house does not have an equal chance of being selected, (or most houses have no chance of being selected). ✓

26. A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws.

(a) Calculate an approximate confidence interval for the probability that the die shows a six on one throw.

(b) Marcella claims the die is biased. Use your answer to part (a) to comment on this claim.

[W-20/62/Q2] -- [1]

Solution (a) $CI = \frac{56}{300} \pm 2.054\sqrt{\frac{\frac{56}{300}\times\frac{244}{300}}{300}}$; $p = \frac{56}{300}$ and $z = \phi^{-1}[0.96 + 0.02]$
$= \phi^{-1}(0.98) = 2.054$

$= 0.14$ to 0.233 ✓

(b) $\frac{1}{6} = 0.167$ [Prob. of getting six on a fair die]

This within confidence interval, so no reason to believe die is biased.

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27 It is known that the height H, in metres, of trees of certain kind has the distribution $N(12.5, 10.24)$. A scientist takes a random sample of 25 trees of this kind and finds the sample mean $\bar{H}$, of the heights.

(a) State the distribution of $\bar{H}$, giving the values of any parameters. ---[2]

(b) Find $P(12 < \bar{H} < 13)$ --[3]

W-21/61/Q1

Solution: Population distribution $X \sim N(12.5, 10.24)$

(a) Now for sample $\bar{H}$, $\bar{H} \sim$ distribution will be Normal. , $n = 25$

$E(\bar{H}) = 12.5$, $Var(\bar{H}) = \frac{\sigma^2}{n} = \frac{10.24}{25} = 0.4025$

$\Rightarrow \bar{H} \sim N(12.5, 0.4025)$ ✓

(b) $P(12 < \bar{H} < 13) = P\left[\left(z < \frac{13-12.5}{\sqrt{0.4025}}\right) - \left(z < \frac{12-12.5}{\sqrt{0.4025}}\right)\right]$

$= P[(z < 0.781) - (z < -0.781)]$

$= \phi(0.781) - [1 - \phi(0.781)] = 2\phi(0.781) - 1 = 0.565$ ✓

28. A random sample of 75 students at a large college was selected for a survey. 15 of these students said they owned a car. From this result an approximate $\alpha\%$ confidence interval from the proportion of all students at the college who own a car was calculated. The width of this interval was found to to be 0.162.
Calculate the value of α correct to 2 significant figures. --[5]

[W-21/61/Q3]

Solution: $\text{est}(p) = \frac{15}{75} = 0.2$ ✓ and $(1-p) = 1-0.2 = 0.8$ ✓, $n = 75$

Let z corresponds to $\alpha\%$ CI

Given width of C.I $= 2z\sqrt{\frac{p(1-p)}{n}} = 0.162$ (Given)

$$\Rightarrow 2z\sqrt{\frac{0.2\times0.8}{75}} = 0.162 \Rightarrow z = 1.754$$

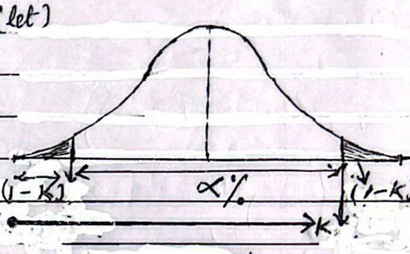
Now $\phi(z) = \phi(1.754) = 0.96 = k$ (let)

$\therefore$

$\alpha = 0.96 - (1-0.96)$

$\alpha = 2\times0.96 - 1 = 0.92$

CI: $\alpha = 92\%$



$\alpha = k - (1-k)$
$= .96 - (1-.96) = .92$

(or $k\% = \frac{1+\alpha\%}{2}$)

29 The random variable T denotes the time, in seconds, for 100m races run by Tania. T is normally distributed with mean μ and variance σ^2. A random sample of 40 races run by Tania gave the results. $n = 40$, $\Sigma t = 560$, $\Sigma t^2 = 7850$

(a) Calculate unbaised estimate of μ and σ^2. ---[3]

The random variable S denotes the time, in seconds, for 100m races run by Suki. S has the independent distribution $N(14.2, 0.3)$

(b) Using your answers for part (a), find the probability that, in a randomly chosen 100m race, Suki's time will be at least 0.1 s more than Tania's time. [5]

[W-21/61/Q6]

Solution (a) $n = 40$, $\Sigma t = 560$, $\Sigma t^2 = 7850$

$\text{est } \mu = \frac{\Sigma t}{n} = \frac{560}{40} = 14$; and $\text{est } \sigma^2 = \frac{n}{n-1}\left(\frac{\Sigma t^2}{n} - \mu^2\right)$

$$\text{est } \sigma^2 = \frac{40}{39}\left(\frac{7850}{40} - 14^2\right) = 0.25641$$

(b) $S \sim N(14.2, 0.3)$

Now $E(S-T) = E(S) - E(T) = 14.2 - 14 = 0.2$

and $Var(S-T) = Var(S) + Var(T) = 0.3 + 0.256 = 0.55641$

$$P(S - T \geq 0.1) = P\left(Z > \frac{0.1 - 0.2}{\sqrt{0.55641}}\right) = P(Z \geq -0.134)$$

$$= P(Z < 0.134)$$

$$= \phi(0.134)$$

$$= 0.553 \text{ (3sf)}$$

30. The mass, in kilograms, of a block of cheese sold in a supermarket is denoted by the random variable M. The masses of random sample of 40 blocks are summarised as: $n=40, \Sigma m=20.50, \Sigma m^2=10.7280$

(a) Calculate unbiased estimate of population mean and variance of M. ---[3]

(b) The price \$P, of a block of cheese of mass M kg is found using the formula $P=11M+0.50$.

Find the estimates of population mean and variance of P. --[3]

W-21/62/Q1

Solution: (a) $n=40, \ \Sigma m=20.50, \ \Sigma m^2=10.7280$

Est mean $= \frac{\Sigma m}{n} = \frac{20.50}{40} = 0.5125$ ✓

Est var $= \frac{n}{n-1}\left(\frac{\Sigma m^2}{n} - (E(m))^2\right) = \frac{40}{39}\left(\frac{10.7280}{40} - (0.5125)^2\right)$

$= \underline{0.00569}$ ✓ (3 sf) (0.0056859)

(b) $P = 11M + 0.50$

Est (Mean price) $= 11 \times 0.5125 + 0.50 = \underline{6.14}$ ✓ (3sf)

Est Var of price $= 11^2 \times 0.0056859$

$= \underline{0.688}$ (3sf)

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31. Andy and Jessica are doing a survey about musical preferences. They plan to choose a representative sample of six students from the 256 students at their college.

(a) Andy suggests that they go to the music building during the lunch hour and choose six students at random from the students who are there. Give a reason why this method is unsatisfactory. ---[1]

(b) Jessica decides to use another method. She numbers all the students in the college from 1 to 256. Then she uses her calculator and generates the following random numbers.

204393, 162007, 204028, 587119, 207395.

From these numbers, she obtains six student numbers. The first three of her student numbers are 204, 162 and 7.

Continue Jessica's method to obtain the next three student numbers. --[2]

[W-21/62/Q2]

Solution: (a) Bias towards students who play instruments or only music students or the six will possibly be friends/have similar music preferences.

(b) 28, 119, 207. .. (Choose first three digits/or last three digits < 257)

32 The probability that a certain spinner lands on red on any spin is p. The spinner is spun 140 times and it lands on red 35 times.

(a) Find an approximate 96% confidence interval for p. --- [3]

From further three experiments, Jack finds a 90% confidence interval, a 95% confidence interval and a 99% confidence interval for p.

(b) Find the probability that exactly two of these confidence intervals contain the true value of p. --- [3]

[W-21/62/Q3]

Solution (a) For 96% C.I ; $p = \frac{35}{140} = 0.25$

for 96% C.I
$z = \phi^{-1}(0.96 + 0.02)$
$= \phi^{-1}(0.98)$
$z = 2.054$ ✓

CI is: $p \pm z \sqrt{\frac{p(1-p)}{n}}$

$= 0.25 \pm 2.054 \sqrt{\frac{0.25 \times 0.75}{140}}$

$=$ 0.175 to 0.325 (3 sf)

(b) P(Exactly 2 CI out of 3 contain true p) $= 0.90 \times 0.95 \times 0.01 + 0.1 \times .95 \times .99$
$+ 0.90 \times 0.05 \times 0.99 = \underline{0.147}$ ✓

33\. The heights, in metres, of a random sample of 10 mature trees of a certain variety are given: 5·9, 6·5, 6·7, 5·9, 6·9, 6·0, 6·4, 6·2, 5·8, 5·8.
Find unbiased estimate of the population mean and variance of the of the heights of all mature trees of this variety. ---[3]

W-22/61/Q1

Solution: $\text{Est mean}(\mu) = \frac{\Sigma x}{n} = \frac{62\cdot 1}{10} = \underline{6\cdot 21}$ ✓ ; $\Sigma x^2 = 387\cdot 05$

$\text{Est Var}(\sigma^2) = s^2 = \frac{n}{n-1}\left(\frac{\Sigma x^2}{n} - (\bar{x})^2\right) = \frac{10}{9}\left(\frac{387\cdot 05}{10} - (6\cdot 21)^2\right)$

$= \underline{0\cdot 157}$ (3s.f) ✓

34\. A builder's merchant sells stone of different sizes:

(a) The masses of size A stones have standard deviation 6 grams. The mean mass of random sample of 200 size A stones 45 grams.
Find a 95% confidence interval for the population mean mass of size A stones. --[3]

(b) The masses of size B stones have standard deviation 11 grams. Using a random sample of size 200, an α% confidence interval for the population mean mass is found to have width 4 grams. Find α. ---[4]

W-22/61/Q5

Solution (a) $C\cdot I = 45 \pm 1\cdot 96 \times \frac{6}{\sqrt{200}}$

$= 45 \pm 0\cdot 831$

$C\cdot I$ is $\underline{44\cdot 2 \text{ to } 45\cdot 8}$ (3sf)

$\left\{\begin{array}{l}\bar{x} = 45 \\ \sigma = 6 \\ n = 200\end{array}\right.$ {Confidence Interval: $\bar{x} \pm z\cdot\frac{\sigma}{\sqrt{n}}$ }

$\left\{\begin{array}{l} C\cdot I = 95\% = \alpha \quad (1-0\cdot 95) = 0\cdot 05 \\ \beta = \alpha + \frac{(1-\alpha)}{2} = 0\cdot 95 + 0\cdot 0275 \\ \quad = 0\cdot 975 \\ z = \phi^{-1}(\beta) = \phi(0\cdot 975) \\ \quad z = 1\cdot 96 \end{array}\right.$

(b) $\bar{x} \pm z\cdot\frac{\sigma}{\sqrt{n}} = 4$ given.

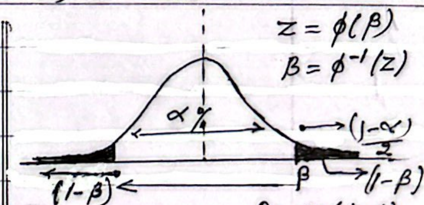
$\{\sigma = 11,\ n = 200$

$\therefore z\cdot\frac{\sigma}{\sqrt{11}} = 2$

$\Rightarrow z \times \frac{11}{\sqrt{200}} = 2$

$\Rightarrow z = \frac{2\times\sqrt{200}}{11} = 2\cdot 571$

$\phi(z) = \phi(2\cdot 571) = 0\cdot 9945 = \beta$



$\alpha = \beta - (1-\beta) = 0\cdot 9945 - (1 - 0\cdot 9945) = 0\cdot 9898 \Rightarrow \alpha = 99\%$

35. Each of a random sample of 80 adults gave an estimate, h metres, of the height of a particular building. The results are as:

$$n = 80, \quad \Sigma h = 2048, \quad \Sigma h^2 = 52760$$

(a) Calculate unbiased estimate of the population mean and variance. --[3]
(b) Using this sample, the upper boundary of an $\alpha\%$ confidence interval for the population mean is 26.0. Find the value of α. --[4]

[W-22/62/Q1]

Solution:
(a) $Est(\mu) = \dfrac{\Sigma h}{n} = \dfrac{2048}{80} = \underline{25.6}$ ✓

$Est(\sigma^2) = \dfrac{n}{(n-1)}\left(\dfrac{\Sigma x^2}{n} - (\bar{x})^2\right) = \dfrac{80}{79}\left(\dfrac{52760}{80} - (25.6)^2\right) = \underline{4.19}$ ✓ (3 s.f)

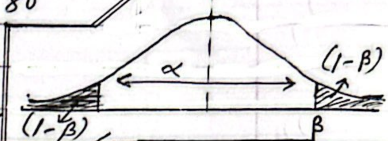
(b) $\bar{x} + Z\sqrt{\dfrac{Est\,\sigma^2}{n}}$ = upper boundary of C.I

$\Rightarrow \quad 25.6 + Z\sqrt{\dfrac{4.19}{80}} = 26$

$\Rightarrow Z = 1.748$

$\beta = \phi(Z) = \phi(1.748) = 0.960$

$\alpha = \beta - (1-\beta) = 0.960 - (1 - 0.960)$
$= 0.92 = \underline{92\%}$ ✓



36. X is a random variable with distribution $B(10, 0.2)$. A random sample of 160 values of X is taken.
(a) Find the approximate distribution of the sample mean, including the values of the parameters. --[3]
(b) Hence find the probability that the sample mean is less than 1.8. --[3]

[W-22/62/Q5]

Solution: $B(10, 0.2) \rightarrow n = 10, p = 0.2 \Rightarrow \mu = np = 10 \times 0.2 = 2$ ✓; $\sigma^2 = npq = 10 \times 0.2 \times 0.8 = 1.6$ ✓

For sample with $N = 160$,

Sample mean $\bar{x} = \mu = \underline{2}$ ✓ and Sample variance $= \dfrac{\sigma^2}{n} = \dfrac{1.6}{160} = \dfrac{1}{100} = 0.01$ ✓

Formal sample mean distribution is Normal $\underline{(2, 0.01)}$ ✓ $\longleftrightarrow N\left(\mu, \dfrac{\sigma^2}{n}\right)$

(b) $P(\bar{X} < 1.8) = P\left(Z < \dfrac{1.8 - \frac{1}{320} - 2}{\sqrt{0.01}}\right) = P(Z < -2.03)$
$= 1 - \phi(2.03)$
$= \underline{0.0212}$ ✓

Continuity correction. $\dfrac{1}{2n}$

$\bar{X} < 1.8 \rightarrow 1.8 - \dfrac{1}{320}$

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