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S-2

Probability and Statistics-2

The Poisson Distribution
Notes and Revision

SP-20	W-21	S-21	M-21
	W-20	S-20	M-20

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The Poisson Distribution

§ The Poisson Distribution:

It is a discrete probability distribution, in which the events occur, singly, at random and independently, in a given interval of space or time.

The mean and variance of a Poisson distribution are equal, hence a Poisson distribution has only one parameter 'λ' [$\mu = \sigma^2 = \lambda$ (let)] : $\lambda > 0$

We write $P(X=r) = \dfrac{e^{-\lambda} \cdot \lambda^r}{r!}$; $r = 0, 1, 2, \ldots$

We write $X \sim Po(\lambda)$. Here $E(X) = \lambda$, $Var(X) = \lambda$

Example 1: A clinic deals only with flu vaccinations. The number of patients arriving every 15 minutes is modelled by random variable X with distribution $Po(4.2)$

(i) State two assumptions required for the Poisson model to be valid. --- [2]

(ii) Find the probability that at least 1 patent will arrive in a 15-minute period. -- [2]

[S-10/73/Q10]

Solution (i) Patients arrive at constant mean rate.
Patients arrive at random.
Patients arrive independently.
Patients arrive singly.

(ii)
$$P(X \geq 1) = 1 - P(0)$$
$$= 1 - \frac{e^{-\lambda} \cdot \lambda^0}{0!}$$
$$= 1 - \frac{e^{-4.2} \times 1}{1}$$
$$= 1 - e^{-4.2}$$
$$= 1 - 0.0145$$
$$= 0.985$$

$\left\{ \begin{array}{l} P(X=r) = \dfrac{e^{-\lambda} \cdot \lambda^r}{r!} \\ Po(4.2) \Rightarrow \lambda = 4.2 \end{array} \right.$

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Example 2: The number of goals scored per match by Everly Rovers is represented by the random variable X, which has mean 1.8.

(i) State two conditions for X to be modelled by a Poisson distribution. ---[2]

Assume now that $X \sim Po(1.8)$

(ii) Find $P(2 < X < 6)$ ---[2]

(iii) The manager promises the team a bonus if they score at least 1 goal in each of the next 10 matches. Find the probability that they win the bonus. |S-11| 72|Q3| ---[3]

Solution (i) Constant average rate of goals scored,
Goals at random
Goals independent.

(ii) $X \sim Po(1.8) \Rightarrow \lambda = 1.8$ $\quad [P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}]$

$$P[2 < X < 6] = P(3) + P(4) + P(5)$$

$$= e^{-1.8} \times \frac{(1.8)^3}{3!} + e^{-1.8} \cdot \frac{(1.8)^4}{4!} + e^{-1.8} \cdot \frac{(1.8)^5}{5!}$$

$$= e^{-1.8}\left[\frac{(1.8)^3}{3!} + \frac{(1.8)^4}{4!} + \frac{(1.8)^5}{5}\right]$$

$$= \underline{0.259}$$

(iii) In each match $P(\text{at least 1 goal}) = P(X \geq 1)$

$$= 1 - P(X=0)$$

$$= 1 - \frac{e^{-\lambda} \cdot \lambda^0}{r!}$$

$$= 1 - e^{-1.18}$$

$\therefore$ Prob. that in next 10 matches they score at least one goal

$$P(\text{win bonus}) = (1 - e^{-1.8})^{10}$$

$$= \underline{0.164}$$

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§ Poisson distribution for different intervals:

In Poisson distribution, events occur at a constant rate; The mean average number of events in a given interval is proportional to that interval.

Example 3: Customers arrive at an enquiry desk at a constant rate of 1 every 5 minutes.

(i) State one condition for the number of customers arriving in a given period to be modelled by a Poisson distribution. --[1]

(ii) Find the probability that exactly 5 customers will arrive during a randomly chosen 30-minute period. --[2]

(iii) Find the probability that fewer than 3 customers will arrive during a randomly chosen 12-minute period. --[3]

W-11/71/Q6

Solution: (i) Customers arrive independently/singly and arrive at random (times)

(ii) for a 5 minute period $\lambda = 1$

$\therefore$ for a 30-minute period $\lambda = \frac{1}{5} \times 30 = 6$ ✓

$\therefore P(X=5) = e^{-6} \cdot \frac{6^5}{5!} = 0.161$ ✓

$\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

(iii) No. for a 12-minute period, $\lambda = \frac{1}{5} \times 12 = 2.4$

$$P(X<3) = P(X=0) + P(X=1) + P(X=2)$$

$$= e^{-2.4}\left(\frac{2.4^0}{0!} + \frac{2.4^1}{1!} + \frac{2.4^2}{2!}\right)$$

$$= 0.570 ✓$$

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Example 4: The number of planes arriving at an airport every hour day time is modelled by the random variable X with distribution $Po(5.2)$.

(i) State two assumptions required for the Poisson model to be valid in this context. ---[2]

(ii) (a) Find the probability that the number of planes arriving in a 15-minute period is greater than 1 and less than 4. ---[3]

(b) Find the probability that more than 3 planes will arrive in a 40-minute period. $\boxed{M-17/72/Q7}$ --[2]

Solution (i) Planes arrive at constant mean rate.

Planes arrive at random.

(ii)(a) $P(5.2)$ and planes arrive every hour (60 minutes) $\Rightarrow \lambda = 5.2$

for 15 minutes period $\rightarrow \lambda = \frac{15}{60} \times 5.2 = 1.3$

$P(1 < X < 4) = P(2) + P(3)$ $\quad \left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^{r}}{r!} \right.$

$= e^{-1.3}\left[\frac{1.3^2}{2!} + \frac{1.3^3}{3!}\right]$

$= 0.330$

(b) In 40 minute period, $\lambda = \frac{40}{60} \times 5.2 = 3.467$

$P(X > 3) = 1 - P(0, 1, 2, 3)$

$= e^{-3.467}\left[1 + 3.467 + \frac{3.467^2}{2!} + \frac{3.467^3}{3!}\right]$

$= 0.456$ (3 s.f)

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§ Poisson distribution as an approximation to binomial distribution:

$X \sim B(n,p)$, when the value of 'n' is large and 'p' is small, such that np is moderate ($n>50$ and $np<5$) the Poisson distribution with mean np can be used as an approximation for the binomial distribution.

$$X \sim B(n,p) \longrightarrow P_o(\lambda) \text{ where } \lambda = np.$$

Example 5: On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of X, giving the values of any parameters --[1]

(b) State an approximating distribution for X, giving the values of parameters -[2

(c) Use this approximating distribution to find each of the following.

(i) $P(X=4)$ (ii) $P(2 \leq X \leq 4)$ ---[2]+[2]

(d) Use a suitable approximating distribution to find the probability that, in a random sample of 700 microchips, there will be at least 1 faulty one

M-21/62/Q4 ---[3]

Solution (a) $p = \frac{1}{400}$, $n = 1000 \Rightarrow$ Binomial distribution: $X \sim B\left(1000, \frac{1}{400}\right)$.

(b) n is very large and p is small hence Binomial distribution approximates to Poisson: $\lambda = np = 1000 \times \frac{1}{400} = 2.5$

or $P_o(2.5)$

(c) (i) $P(X=4) = e^{-2.5} \times \frac{2.5^4}{4!} = \underline{0.134}$ (3 s.f.) $\left\{ P(X=r) = e^{-\lambda} \frac{\lambda^r}{r!} \right.$

(ii) $P(2 \leq X \leq 4) = P(X=2) + P(X=3) + P(X=4)$

$$= e^{-2.5}\left[\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!}\right] = \underline{0.604} \text{ (3 s.f.)}$$

(d) Now $\lambda = \frac{1}{400} \times 700 = 1.75$ $\{\lambda = np\}$

$P(X \geq 1) = 1 - P(X=0)$ $P(0) = e^{-\lambda}$

$$= 1 - e^{-1.75} = \underline{0.826}$$

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Example 6: The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30000 letters and these errors occur randomly. A randomly chosen booklet from this publisher contains 12500 pages.
Use a suitable approximating distribution to find the probability that this booklet contains atleast 2 errors. --[3]

[M-20/62/Q1]

Solution: $\lambda = np = 12500 \times \frac{1}{30,000} = \frac{5}{12} = 0.417$ $\left\{ \begin{array}{l} p = 1/30000 \\ n = 12,500 \end{array} \right.$ $B(n,p) \sim Po(\lambda)$

$P(X \geq 2) = 1 - P(X=0 \text{ or } X=1)$

$= 1 - e^{-0.417}(1 + 0.417) = 0.0661$ ✓ $\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

Example 7: In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution X, including the values of any parameters. --[1]

(ii) State an appropriate approximating distribution for X, including the values of parameters. Justify your choice of approx. distribution. --[3]

(iii) Use your approximating distribution to find $P(X>2)$ --[2]

(b) Another large random sample of 'n' data items is chosen. The prob. that the sample contains no data items that are entered incorrectly is more than 0.1. Find the largest value of n. [S-20/62/Q3] --[3]

Solution: (a) (i) $B(3600, 0.0012)$ $\{p = 0.12\%$

(ii) $Po(4.32)$; $\left. \begin{array}{l} n = 3600 \\ p = .0012 \end{array} \right\} \Rightarrow \lambda = np = 4.32$

(iii) $P(X>2) = 1 - P(0,1,2)$

$= 1 - e^{-4.32}\left[1 + 4.32 + \frac{4.32^2}{2}\right]$

$= \underline{0.805}$ ✓ (3 s.f.)

(b) $P(X=0) = e^{-\lambda} > 0.1$ (Given)

$\Rightarrow -\lambda > \ln 0.1$

$\Rightarrow \lambda < -\ln 0.1$

$\Rightarrow \lambda < \ln 10$

$\Rightarrow 0.0012n < \ln 10$

$\Rightarrow n < \frac{\ln 10}{0.0012}$

$n < 1918.8$

$\therefore$ Largest value of $n = \underline{1918}$ ✓

§ Normal distribution as an approximation to Poisson distribution:

For $\lambda > 15$, the Poisson distribution with λ can be approximated by the normal distribution with mean $\mu = \lambda$ and variance $\sigma^2 = \lambda$, with a continuity correction applied.

The accuracy of this approximation improves as λ increases.

$$X \sim Po(\lambda) \to N(\mu, \sigma^2) \to N(\lambda, \lambda) \text{ for } \lambda > 15$$

Example 8: The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. --[4]

[W-21/61/Q2]

Solution: $Po(\lambda) = Po(45.2) \Rightarrow \lambda = 45.2$ ($\lambda > 15$, leads to Normal distribution)

Poisson distribution approaches → Normal distribution $N(\mu, \sigma^2)$

$$\to N(\lambda, \lambda) = N(45.2, 45.2) \{\mu = \sigma^2 = \lambda\}$$

$$P(X > 60) = P\left(z > \frac{X-\mu}{\sigma}\right) = P\left(z > \frac{X-\lambda}{\sqrt{\lambda}}\right) = P\left(z > \frac{60-45.2}{\sqrt{45.2}}\right)$$

$$= P(z > 2.276)$$

$$= 1 - P(z < 2.276)$$

$$= 1 - \phi(2.276)$$

$$= 1 - 0.9886$$

$$= 0.0114$$

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Example 9: In a certain document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. ---[2]

(b) Use an approximating distribution to find the probability that there are more than 50 typing errors in 200 randomly chosen pages. ---[4]

In same document, formatting errors occurs at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. --[3]

W-21/62/Q5

Solution (a) $n = 10$, $p = 0.2 \Rightarrow \lambda = np = 10 \times 0.2 = 2$, use $Po(2)$

$$P(X<3) = P(X=0, X=1, X=2) = e^{-\lambda}\left(1+\lambda+\frac{\lambda^2}{2!}\right) \quad \left\{ P(X=r) = \frac{e^{-\lambda}\cdot\lambda^r}{r!} \right.$$

$$= e^{-2}\left(1+2+\frac{2^2}{2!}\right) = \underline{0.677} \checkmark (3s.f)$$

(b) $n = 200$, $p = 0.2 \Rightarrow \lambda = np = 200 \times 0.2 = 40 > 15$

$\therefore$ Poisson distribution approximating to Normal distribution $N(\mu, \sigma^2)$

$\mu = \sigma^2 = \lambda = 40 \quad \therefore X \sim N(40, 40) \checkmark$

$$P(X > 50) = P\left(z > \frac{50.5-40}{\sqrt{40}}\right) \quad \left\{ \begin{array}{l} \text{Continuity correction} \\ P(X>50) = P(X \geq 51) \sim N \to X > 50.5 \end{array} \right.$$

$$= P(z > 1.660) = 1 - \phi(1.660)$$

$$= 1 - 0.9515$$

$$= \underline{0.0485} \checkmark$$

(C) Now $n = 20$, p = typing and formatting error per page $= 0.2 + 0.3$, $p = 0.5 \checkmark$

$\therefore \lambda = np = 20 \times 0.5 = 10 \checkmark$

$$P(8 \leq X \leq 11) = P(X = 8, 9, 10, 11) = e^{-\lambda}\left(\frac{\lambda^8}{8!}+\frac{\lambda^9}{9!}+\frac{\lambda^{10}}{10!}+\frac{\lambda^{11}}{11!}\right)$$

$$= e^{-10}\left(\frac{10^8}{8!}+\frac{10^9}{9!}+\frac{10^{10}}{10!}+\frac{10^{11}}{11!}\right)$$

$$= \underline{0.477} \checkmark (3sf)$$

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Example 10: On average, 1 in 75 000 adults has a certain genetic disorder.

(a) Use a suitable approximating distribution to find the probability that, in a random sample of 10 000 people, at least 1 has a genetic disorder. ---[3]

(b) In a random sample of n people, where n is large, the probability that no-one has the genetic disorder is more than 0·9.
Find the largest possible value of n. ---[4]

[S-21/61/Q5]

Solution (a) $n = 10\,000,\ p = \frac{1}{75\,000}$ $\quad$ n is large and p is small

$np = 10\,000 \times \frac{1}{75\,000} = \frac{2}{15}$

hence it is Poisson distribution. $\lambda = \frac{2}{15}$

$Po\left(\frac{2}{15}\right)$

$$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-2/15} \quad \left\{ P(0) = e^{-\lambda} \right.$$

$$= \underline{0.125} \ (3\,sf)$$

(b) $\lambda = \frac{n}{75\,000}$ $\quad \left\{ \lambda = np = n \times \frac{1}{75{,}000} \right.$

$$P(X=0) = e^{-\frac{n}{75000}} > 0.9 \ (\text{Given})$$

$$\Rightarrow -\frac{n}{75000} > \ln 0.9$$

$$\Rightarrow -n > -0.10536 \times 75000$$

$$\Rightarrow n < 7902.04$$

$\therefore$ Largest value of $\underline{n = 7902}$

Example 11: Customers arrive at a particular shop at random times. It has been found that the mean number of customers who arrive during a 5-minute interval is 2.1.

(a) Find the probability that exactly 4 customers arrive during a 10-minute interval. ---[2]

(b) Find the probility that at least 4 customers arrive during 20-minute interval. ---[2]

(c) Use a suitable approximating distribution to find the probability that fewer then 40 customers arrive during a 2-hour interval. [S-21/62/Q7] ---[4]

Solution (a) Random Variabe, X denotes the number of customers.

During a 5-minute interval- mean number of Customers $\lambda = 2.1$; $Po(2.1)$

$\therefore$ During 10-minute interval- mean number of Customers $\lambda = \frac{10}{5} \times 2.1 = 4.2$

$$P(X=4) = e^{-4.2} \times \frac{4.2^4}{4!} \quad \left[P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right]$$

$$= \underline{0.194 \ (3\,sf)}$$

(b) During 20-minute interval: $\lambda = \frac{20}{5} \times 2.1 = 8.4$ $\quad Po(8.4)$

$$P(X \geq 4) = 1 - P(X=0,1,2,3) = 1 - e^{-8.4}\left\{1 + 8.4 + \frac{8.4^2}{2!} + \frac{8.4^3}{3!}\right\}$$

$$= \underline{0.968 \ (3sf)}$$

(c) During 2 Hr (120 minutes) interval $\lambda = \frac{5}{120} \times 2.1 = 50.4 > 15$

$\therefore$ approximating distribution is normal distribution. $X \sim N(\mu, \sigma^2) = N(\lambda, \lambda) = N(50.4, 50.4)$

$$P(X < 40) = P\left(z < \frac{39.5 - 50.4}{\sqrt{50.4}}\right) \quad \left[\text{Continuity Correction. } X<40 \sim X < 39.5\right]$$

$$= P(z < -1.535)$$

$$= 1 - \phi(1.535)$$

$$= 1 - 0.9376$$

$$= \underline{0.0624}$$

12 The number of calls received at a small call centre has a Poisson distribution with mean 2.4 calls per 5-minute period.

(a) Find the probability of exactly 4 calls in an 8-minute period --[2]

(b) Find the probability of atleast 3 calls in 3-minute period. ---[3]

The number of calls received at a large call centre has a Poisson distribution with mean 41 calls per 5-minute period.

(c) Use an approximating distribution to find the probability that the number of calls received in a 5-minute period is between 41 and 59 inclusive. --[5]

SP 2006/Q3

Solution (a) $\lambda = 2.4$ per 5-minute period.

$\Rightarrow$ for $\rightarrow$ 8-minute period $\lambda = \frac{2.4}{5} \times 8 = 3.84$ ✓

$$P(r=4) = e^{-3.84} \times \frac{3.84^4}{4!} = \underline{0.195} \text{ (3sf)} \quad \left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$$

(b) For 3 minute-period: $\lambda = \frac{2.4}{5} \times 3 = 1.44$

$$P(X \geq 3) = 1 - P(X=0,1,2)$$
$$= 1 - e^{-1.44} \times \left(1 + 1.44 + \frac{1.44^2}{2}\right) = \underline{0.176} ✓$$

(c) $X \sim Po(\lambda) \sim N(\mu, \sigma^2) = N(\lambda, \lambda)$ for $\lambda = 41 > 15$

$X \sim N(41, 41)$, $\mu = 41$, $\sigma^2 = 41$

$$P(41 \leq X \leq 59) = P(X < 59.5) - P(X < 40.5) \otimes$$
$$= P\left(Z < \frac{59.5 - 41}{\sqrt{41}}\right) - P\left(Z < \frac{40.5 - 41}{\sqrt{41}}\right)$$
$$= P(Z < 2.889) - P(Z < -0.078)$$
$$= \phi(2.889) - \phi(-0.078)$$
$$= \phi(2.889) - [1 - \phi(0.078)]$$
$$= 0.9981 - (1 - 0.5311)$$
$$= \underline{0.529} \text{ (3 sf)} ✓$$

$\otimes$ Continuity Correction:
$X \geq 41 \sim X > 40.5$
$X \leq 59 \sim X < 59.5$

13. Most plants of a certain type have three leaves, However, it is known that, on average, 1 in 10000 of these plants have four leaves and plants with four leaves are called "lucky". The number of lucky plants in a random sample of 25000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. --[2]

Use your approximating distribution to answer parts (b) and (c)

(b) Find $P(X \leq 3)$ ---[2]

(c) Given that $P(X=k) = 2P(X=k+1)$, find k. --[2]

The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geq 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. --[3]

[S-21/63/Q5]

Solution:

$n = 25000, \; p = \frac{1}{10,000}$

(a) $\lambda = np = 25000 \times \frac{1}{10,000} = 2.5 < 5$

$n > 50$ and $p = 0.0001 < 0.1$

$\therefore$ Poission distorbution $Po(\lambda)$

$= \underline{Po(2.5)}$ ✓

(b) $P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$

$P(X \leq 3) = P(X=0,1,2,3)$

$= e^{-2.5} \times \left(1 + 2.5 + \frac{2.5^2}{2!} + \frac{2.5^3}{3!}\right)$

$= \underline{0.758}$ (3sf)

(c) $P(X=k) = 2P(X=k+1)$

$\Rightarrow e^{-2.5} \times \frac{2.5^k}{k!} = 2 \times e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$

$\Rightarrow k+1 = 5$

$\Rightarrow \underline{k=4}$ ✓

(d) $P(Y \geq 1) = 0.963$

$\Rightarrow 1 - P(Y=0) = 0.963$

$\Rightarrow 1 - e^{-\lambda} = 0.963$

$\Rightarrow e^{-\lambda} = 0.037$

$\Rightarrow -\lambda = \ln 0.037$

$\Rightarrow \lambda = -(-3.296837\ldots)$

$np = 3.296837$

$n \times 0.0001 = 3.296837$

$n = 32968.37$

$\underline{n = 33000}$ ✓ (3sf)

14. The random variable X has the distribution $Po(\lambda)$

(a) (i) State the values that X can take --- [1]

It is given that $P(X=1) = 3 \cdot P(X=0)$

(ii) Find λ. --- [1]

(iii) Find $P(4 \leq X \leq 6)$ --- [2]

(b) The random variable Y has distribution $Po(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $P(Y < 46) = 0.0668$, correct to 4 decimal places. --- [5]

[S-20 / 62 / Q5]

Solution (a) (i) $0, 1, 2, 3, ----$

(ii) $P(X=1) = 3 \cdot P(0)$ $\left\{ P_0(X=\lambda) = e^{-\lambda} \cdot \frac{\lambda^{\lambda}}{\lambda!} \right.$

$\Rightarrow e^{-\lambda} \times \frac{\lambda^1}{1!} = 3e^{-\lambda} \Rightarrow \underline{\lambda = 3}$ ✓

(iii) $P(4 \leq X \leq 6) = e^{-3}\left(\frac{3^4}{4!} + \frac{3^5}{5!} + \frac{3^6}{6!}\right) = \underline{0.319}$ (3 sf)

(b) Given μ is large $\Rightarrow Po(\mu) \sim N(\mu, \sigma^2) = N(\mu, \mu)$

Given $P(Y < 46) = 0.0668$ $\left\{ \begin{array}{l} \text{continuity correction} \\ Y < 46 \sim Y < 45.5 \end{array} \right.$

$P(Y<46) = P\left(Z < \frac{45.5-\mu}{\sqrt{\mu}}\right) = 0.0668$ $\left\{ \begin{array}{l} < 0.5 \\ \phi(-z) = 0.066 \\ \Rightarrow \phi(z) = 1 - 0.0668 \\ \quad = 0.9332 \\ z = \phi^{-1}(0.9332) \\ \quad = 1.50 \\ \therefore -z = -1.50 \end{array} \right.$

$\Rightarrow \frac{45.5-\mu}{\sqrt{\mu}} = \phi^{-1}(0.0668)$

$\Rightarrow \frac{45.5-\mu}{\sqrt{\mu}} = -1.5$

$\Rightarrow 45.5 - \mu = -1.5\sqrt{\mu}$

$\Rightarrow \mu - 1.5\sqrt{\mu} - 45.5 = 0$

$\Rightarrow \sqrt{\mu} = \frac{1.5 \pm \sqrt{184.25}}{2} = \frac{1.5 \pm 13.5738}{2}$

$\Rightarrow \sqrt{\mu} = \frac{15.0738}{2}$ or $\cancel{\frac{-12.0738}{2}}$

$\sqrt{\mu} = 7.5369 \Rightarrow \underline{\mu = 56.8}$ ✓

15 It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an approximating distribution to find the probability that more than 1 flower in the sample is white. ---[3]

(b) Justify the approximating distribution used in part (a) ---[1]

The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in two samples is less than 4. ---[3]

W-20/61/Q1

Solution: $P(\text{white}) = p = \frac{1}{300}$, $n = 200 \Rightarrow \lambda = np = \frac{1}{300} \times 200 = \frac{2}{3}$

(a) $P_0(2/3)$

$$P(X>1) = 1 - P(X=0,1) = 1 - e^{-2/3}\left(1+\frac{2}{3}\right) \quad \left\{P(X=r) = e^{-\lambda}\cdot\frac{\lambda^r}{r!}\right.$$
$$= 1 - 0.856\ldots$$
$$= \underline{0.144} \ (3sf)$$

(b) $n > 50$ and $np = \frac{2}{3} < 5$ (or $p = \frac{1}{300} < 0.1$ and $n < 50$)

(c) for the first kind $\lambda_1 = 2/3$

Now for another kind: $\lambda_2 = np = 150 \times 0.02 = 3$

$\therefore$ for combined $\lambda = \lambda_1 + \lambda_2 = \frac{2}{3} + 3 = \frac{11}{3}$

$$\therefore P(X<4) = P(X=0,1,2,3)$$
$$= e^{-\frac{11}{3}}\left(1 + \frac{11}{3} + \frac{\left(\frac{11}{3}\right)^2}{2!} + \frac{\left(\frac{11}{3}\right)^3}{3!}\right) \quad \left(\because \lambda = \frac{11}{3}\right)$$
$$= \underline{0.501} \ (3sf)$$

16. On average, 1 in 50000 people have a certain gene. Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150000 have the gene. ---[3]

[W-20/62/Q1]

Solution: $p = \frac{1}{50000}$, $n = 150000 \Rightarrow \lambda = np = \frac{150000}{50000} = 3$; ∴ $Po(3)$

$P(X>2) = 1 - P(X=0,1,2) = 1 - e^{-3}\cdot\left(1+3+\frac{3^2}{2!}\right) = \underline{0.577}$ (3sf)

17. Customers arrive at a shop at a constant average rate of 2.3 per minute.

(a) State another condition for the number of customers arriving per minute to have a Poisson distribution. ---[1]

It is now given that the number of customers arriving per minute has the distribution $Po(2.3)$

(b) Find the probability that exactly 3 customers arrive during 1-minute period. ---[2]

(c) Find the prob. that more than 3 customers arrive during a 2-minute period. ---[3]

(d) Five 1-minute periods are chosen at random. Find the prob. that no customers arrive during exactly 2 of these 5 periods. ---[3]

[W-20/62/Q5]

Solution (a) Customers arrive independently or singly or at random.

(b) $Po(2.3) \Rightarrow \lambda = 2.3$

$P(X=3) = e^{-3} \times \frac{2.3^3}{3!} = 0.203$ (3sf) $\left\{ P(X=r) = e^{-\lambda}\cdot\frac{\lambda^r}{r!} \text{ for } Po(\lambda) \right.$

(c) for 2-minute interval $\lambda = 2\times2.3 = 4.6$

$P(X>3) = e^{-4.6}\left(1+4.6+\frac{4.6^2}{2!}+\frac{4.6^3}{3!}\right)$

$= \underline{0.674}$ (3sf)

(d) For 1-minute period, $\lambda = 2.3$

$P(\text{none arrive}) = P(X=0) = e^{-2.3}$

$p = 0.10026$

Now for n trial, using Binomial dis

$n=5$, $r=2$, $p=0.10026$, $q = 0.89974$

Using $P(X=r) = {}^nC_r \cdot p^r \cdot q^{n-r} \Rightarrow P(r=2) = {}^5C_2 \cdot (0.10026)^2 \cdot (0.89974)^3$

$= \underline{0.0732}$ (3sf)

18. On average, 1 in 150 components made by certain machine are faulty. The variable X denotes the number of faulty components in a random sample of 500 components.

(i) Describe fully the distribution of X. ---[2]

(ii) State a suitable approximating distribution for X, giving a justification for your choice. ---[2]

(iii) Use your approximating distribution to find the probability that the sample will include at least 3 faulty components. ---[3]

[W-19/72/Q1]

Solution (i) Binomial distribution $B(n, p)$.

$n = 500$ and $p = \frac{1}{150}$

(ii) Poisson distribution. $\left.\begin{matrix} n > 50 \\ \lambda < 5 \end{matrix}\right\}$

$n = 500$ is large and mean $\lambda = np = 500 \times \frac{1}{150} = \frac{10}{3} = 3.3 < 5$

$\therefore Po\left(\frac{10}{3}\right)$

(iii) $P(X \geq 3) = 1 - P(X = 0, 1, 2)$ $\left\{ P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right.$

$= 1 - e^{-\frac{10}{3}}\left(1 + \frac{10}{3} + \frac{\left(\frac{10}{3}\right)^2}{2!}\right) = 1 - 0.353 = \underline{0.647}$ (3sf)

19. Cars arrive at filling station randomly and at a constant rate of 2.4 cars per minutes. ---[2]

(i) Calculate the prob. that the fewer than 4 cars arrive in a 2-minute period.

(ii) Use a suitable approximating distribution to calculate the prob. that at least 140 cars arrive in a 1-hour period. ---[4]

[W-19/73/Q2]

Solution (i) rate of 1-minute period $\lambda = 2.4$

$\therefore$ at a rate of 2-min. period $\lambda = 2.4 \times 2$

$\lambda = 4.8$

$P(X < 4) = e^{-4.8} \cdot \left(1 + 4 + \frac{4.8^2}{2!} + \frac{4.8^3}{3!}\right)$

$= \underline{0.294}$ (3sf)

$\left\{ P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!} \right.$

(ii) for 1-hour period. $\lambda = 60 \times 2.4 = 144$

$Po(\lambda) \sim N(\lambda, \lambda)$, $\mu = 144$, $\sigma^2 = 144$

$P(X \geq 140) = P\left(z > \frac{139.5 - 144}{\sqrt{144}}\right)$ $\left\{ \begin{matrix} \text{Continuity} \\ \text{correction.} \\ X \geq 140 \rightarrow 139.5 \end{matrix} \right.$

$= P(z > -0.375)$

$= P(z < 0.375)$

$= \phi(0.375) = \underline{0.643}$ (3sf)

20 (i) The random variable X has a distribution $B(300, 0.01)$. Use a Poisson approximation to find $P(2<X<6)$ ---[3]

(ii) The random variable Y has the distribution $Po(\lambda)$, and $P(Y=0) = P(Y=2)$. Find λ. ---[2]

(iii) The random variable Z has the distribution $Po(5.2)$ and it is given that $P(Z=n) < P(Z=n+1)$.

(a) Write down an inequality in n. ---[1]

(b) Hence or otherwise find the largest possible value of n. ---[2]

[W-19|73|Q5]

Solution (i) $B(300, 0.01) \sim Po(\lambda)$; $\lambda = np = 300 \times 0.01 = 3$ ✓

$P(2<X<6) = P(X=3,4,5)$ $\left\{ P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right.$

$= e^{-3}\left(\frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}\right) = \underline{0.493}$ (3sf) ✓

(ii) $P(0) = P(2) \Rightarrow e^{-\lambda} = e^{-\lambda} \cdot \frac{\lambda^2}{2} \Rightarrow \lambda^2 = 2 \Rightarrow \lambda = \sqrt{2}$ or $\underline{1.41}$ (3sf) ✓

(iii) (a) $Po(5.2)$, $\lambda = 5.2$

$P(Z=n) < P(Z=n+1)$

$\Rightarrow e^{-5.2} \cdot \frac{5.2^n}{n!} < e^{-5.2} \cdot \frac{5.2^{n+1}}{(n+1)!}$

(b) $\Rightarrow 1 < \frac{5.2}{n+1} \Rightarrow n+1 < 5.2 \Rightarrow n < 4.2$

$\therefore$ largest value of $\underline{n = 4}$ ✓

21. A random variable X has the distribution $B(75, 0.03)$

(i) Use Poisson approximation to the binomial distribution to calculate $P(X<3)$ ---[3]

(ii) Justify the use of the Poisson approximation. ---[1]

[S-18|73|Q1]

Solution (i) $B(75, 0.03) \rightarrow n = 75,\ p = 0.03 \Rightarrow \lambda = np = 75 \times 0.03 = 2.25$

$\sim Po(2.25) \rightarrow P(X<3) = e^{-2.25}\left(1 + 2.25 + \frac{2.25^2}{2!}\right) = \underline{0.609}$ (3sf) ✓

(ii) $n = 75 > 50$ and $\lambda = 2.25 < 5$ ✓

n is large. ✓

22. The number of eagles seen per hour in a certain location has the distribution $Po(1.8)$. The number of vultures seen per hour in the same location has the independent distribution $Po(2.6)$

(i) Find the probability that, in a randomly chosen hour, at least 2 eagles are seen. ---[2]

(ii) Find the prob. that, in a randomly chosen half-hour period, the total number of eagles and vultures seen is less than 5. ---[3]

Alex wants to be at least 99% certain of seeing atleast 1 eagle.

(iii) Find the minimum time for which she should watch for eagles. ---[3]

[M-19 | 72 | Q5]

Solution (i) For eagles $Po(1.8) \rightarrow \lambda = 1.8$

$$P(X \geq 2) = 1 - P(X = 0, 1)$$
$$= 1 - e^{-1.8}(1 + 1.8) = 1 - 0.463 = \underline{0.537} \ (3 \text{ sf})$$

(ii) Per hour $Po(1.8)$ and $Po(2.6)$

for half-hour period combine for both eagles and vultures:

$$\lambda = \frac{1}{2}(1.8 + 2.6) = 2.2$$

$Po(2.2)$

$$P(X + Y < 5) = P(X+Y = 0, 1, 2, 3, 4)$$
$$= e^{-2.2}\left[1 + 2.2 + \frac{2.2^2}{2!} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!}\right] = \underline{0.928} \ (3 \text{ sf})$$

(iii) To watch eagles: Let t hour period $\rightarrow \lambda = 1.8t$ $Po(1.8t)$

$$P(X > 0) = 1 - e^{-1.8t} \geq 0.99 \quad (99\%)$$

$$\Rightarrow e^{-1.8t} \leq 0.01$$
$$\Rightarrow -1.8t \leq \ln 0.01$$
$$\Rightarrow -1.8t \leq -4.60517$$
$$t \geq \frac{-4.60517}{-1.8} = 2.5584$$

$$t \geq 2.56 \ (3 \text{ sf})$$

She must watch for atleast $\underline{2.56}$ (hours)

23(a) The random variable X has the distribution Po(2·3)

(i) Find $P(2 \leq X \leq 4)$ ---[2]

(ii) Find the probability that the sum of two independent values of X is greater than 2. ---[3]

(iii) The random variable S is the sum of 50 independent values of X. Use a suitable approximating distribution to find $P(S \leq 110)$. ---[4]

(b) The random variable Y has the distribution Po(λ). Given that $P(Y=3) = P(Y=5)$, find λ. ---[3]

S-19/71/Q5

Solution (a) Po(2·3) → $\lambda = 2.3$ and $P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!}$

(i) $P(2 \leq X \leq 4) = P(X=2,3,4) = e^{-2.3}\left[\frac{2.3^2}{2!} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!}\right] = \underline{0.585}$ ✓

(ii) for two independent values of X, $\lambda = 2.3 \times 2 = 4.6$

$P(X>2) = 1 - P(X=0,1,2) = 1 - e^{-4.6}\left[1 + 4.6 + \frac{4.6^2}{2!}\right] = \underline{0.837}$ ✓

(iii) for 50 independent value of X; $\lambda = 2.3 \times 50 = 115$ ✓

$S \sim N(115, 115)$; $\mu = \sigma^2 = 115$

Continuity Correct. $S \leq 110 \sim 110.5$

$$P(S \leq 110) = P\left(z < \frac{110.5 - 115}{\sqrt{115}}\right) = P(z < -0.420)$$

$$= 1 - \phi(0.420) = 1 - 0.663 = \underline{0.337}$$ ✓

(b) $Y \sim Po(\lambda)$

$P(Y=3) = P(Y=5)$

$\Rightarrow e^{-\lambda} \cdot \frac{\lambda^3}{3!} = e^{-\lambda} \cdot \frac{\lambda^5}{5!}$

$\Rightarrow \lambda^2 = 20$

$\lambda = \underline{2\sqrt{5}}$ (or 4.47 (3sf)) ✓

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24 The random variable X has the distribution Po(5)

(i) Find X=2 --[1]

(ii) It is given that $P(X=n)=P(X=n+1)$; Find the value of n. --[2]

[S-19/72/Q1]

Solution (i) $Po(5) \rightarrow \lambda=5$

$$P(X=2) = e^{-5} \times \frac{5^2}{2!} = \underline{0.0842} \text{ (3sf)}$$

$$\left\{ P(X=r) = e^{-\lambda} \cdot \frac{\lambda^r}{r!} \right.$$

(ii) $P(X=n) = P(X=n+1) \Rightarrow e^{-\lambda} \cdot \frac{\lambda^n}{n!} = e^{-\lambda} \cdot \frac{\lambda^{n+1}}{(n+1)!} \Rightarrow 1 = \frac{\lambda}{n+1}$

$$\Rightarrow n+1=5 \Rightarrow n=4$$

$[\therefore n+1=\lambda=5]$

25. Each day at a certain doctor's surgery there are 70 appointments available in the morning and 60 in the afternoon. All the appointments are filled every day. The prob. that any patient misses a particular morning appointment is 0.04, and the prob. that any patient misses a particular afternoon appointment is 0.05. All missed appointments are independent of each other.

Use suitebe approximating distribution to answer the following.

(i) Find the prob. that on a randomly chosen morning there are at least 3 missed appointments. --[3]

(ii) Find the prob. that on a randomly chosen day there are a total of exacly 6 missed appointments. --[3]

(iii) Find the prob. that in a randomly chosen 10-day period there are more than 50 missed appointments. [S-19/73/Q7] --[4]

Solution (i) $n=70$, $p(\overset{X}{\text{morning}})=0.04 \Rightarrow \lambda=70\times.04 = 2.8$

$Po(2.8)$, as n is large,

$$P(X \geq 3) = 1-P(X=0,1,2)$$

$$= 1-e^{-2.8}\left(1+2.8+\frac{2.8^2}{2!}\right)$$

$$= \underline{0.531}$$

(ii) In afternoon $p=\overset{(Y)}{0.05} \Rightarrow \lambda=np=60\times0.05 = 3.0$, $n=60$

$\therefore$ for both combined $\lambda = 2.8+3 = 5.8$

$$P(X+Y=6) = e^{-5.8} \times \frac{5.8^6}{6!} = \underline{0.16}$$

(iii) for 10 day-period $\lambda=5.8\times10=58$

$Po(58) \sim N(58, 58)$, $\mu=58=\sigma^2$

$$P(X+Y>50) = P\left(z > \frac{50.5^{\otimes}-58}{\sqrt{58}}\right)$$

$$= P(z > -0.985)$$

$$= \phi(0.985)$$

$$= \underline{0.838 \text{ (3 Sf)}}$$

$\otimes$ Continuity Correction

$X+Y>50$

$\sim X+Y>50.5$

26 The number of phone calls arriving in a 10-minute period at a Switchboard is modelled by the random variable X which has the distribution Po(4.1). Use an approximating distribution to find the probability that more than 90 calls arrive in a 4-hour period. --[5]

[M-18|72|Q2]

Solution: For 10-minute period $\lambda = 4.1 \Rightarrow$ in 4-hour period, $\lambda = \frac{4.1}{10} \times 4\times60 = 98.4$

now $Po(98.4) \sim N(98.4, 98.4) \rightarrow \mu = \sigma^2 = 98.4$

$$P(X > 90) = P\left(Z > \frac{90.5 - 98.4}{\sqrt{98.4}}\right) = P(Z > -0.796) = P(Z < 0.796)$$

$$= \Phi(0.796) = 0.787 \checkmark \ (3\,sf)$$

27. The number of e-readers sold in 10-day period in a Shop is modelled by the distribution Po(5.1). Use an approximating distribution to find the probability that fewer than 140 e-readers are sold in a 300-day period. --[4]

[S-18|71|Q3]

Solution: In 10 day-period $\lambda = 5.1 \Rightarrow$ in 300 day-period, $\lambda = \frac{5.1}{10} \times 300 = 153 \checkmark$

$\therefore Po(153) \sim N(153, 153) \rightarrow \mu = 153 = \sigma^2$

$$P(X < 140) = P\left(Z < \frac{139.5 - 153}{\sqrt{153}}\right)$$

continuity correction
$X < 140 \Rightarrow X \leq 139$
for Normal ~ 139.5

$$= P(Z < -1.091)$$
$$= 1 - \Phi(1.091)$$
$$= 1 - 0.862$$
$$= \underline{0.138} \checkmark \ (3\,Sf)$$

28. The number of alpha, beta and gamma particles emitted per minute by a certain piece of rock have independent distributions $Po(0.2)$, $Po(0.3)$ and $Po(0.6)$ respectively. Find the probability that the total number of particals emmited during a 4-minute period is less than 4. [S-18/72/Q1] --[3]

Solution: $\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 0.2 + 0.3 + 0.6 = 1.1$ for per minute period.

$\therefore$ for 4-minute period $\lambda = 1.1 \times 4 = 4.4$ ✓

$$P(X<4) = P(X=0,1,2,3) = e^{-4.4}\left(1 + 4.4 + \frac{4.4^2}{2!} + \frac{4.4^3}{3!}\right) = 0.359 ✓ \text{ (3sf)}$$

29. Accidents on a particular road occur at a constant average rate of 1 in every 4.8 weeks.

(i) State, in context, one condition for the number of accidents in a given period to be modelled by a Poisson distribution. --[1]

Assume that a Poisson distribution is a suitable model.

(ii) Find the prob. that exactly 4 accidents will occur during a randomly chosen 12-week period. --[2]

(iii) Find the probability that more than 3 accidents will occur during a randomly chosen 10-week period. --[3]

(iv) Use a suitable approximating distribution to find the probability that fewer than 30 accidents will occur during a randomly chosen 2-year period ($104\frac{2}{7}$ weeks) [S-18/72/Q6] --[4]

Solution (i) Accidents occur independenly or randomly. ✓

(ii) for 12-week period $\lambda = \frac{1}{4.8} \times 12 = 2.5$ $P(X=4) = e^{-2.5} \times \frac{2.5^4}{4!} = 0.134$ ✓ (iii) for 10-week Period $\lambda = \frac{1}{4.8} \times 10 = \frac{25}{12}$ $P(X>3) = 1 - P(X=0,1,2,3)$ $= 1 - e^{-\frac{25}{12}}\left[1 + \frac{25}{12} + \frac{(\frac{25}{12})^2}{2!} + \frac{(\frac{25}{12})^3}{3!} + \frac{(\frac{25}{12})^4}{4!}\right]$ $= 0.158$ ✓ (3sf)	(iv) for 2year ($104\frac{2}{7}$ week)-period. $\lambda = \frac{1}{4.8} \times \frac{730}{7} = 1825/4$ $Po(\lambda) \sim N\left(\frac{1825}{4}, \frac{1825}{4}\right)$ $P(X<30) = P\left(Z < \frac{29.5 - \frac{1825}{4}}{\sqrt{1825/4}}\right)$ $= P(Z < 1.668)$ $= \phi(1.668)$ $= 0.952$ ✓ (3sf)

30. The numbers, M and F, of male and female students who leave a particular school each year to study engineering have means 3.1 and 0.8 respectively.

(i) State, in context, one condition required for M to have a Poisson distribution. [1]

Assume that M and F can be modelled by independent Poisson distribution.

(ii) Find the probability that the total number of students who leave to study engineering in a particular year is more than 3.

(iii) Given that the total number of students who leave to study engineering in a particular year is more that 3, find the probability that no female students leave to study engineering in that year. ...[3]

[S-18/73/Q4]

Solution (i) Males leave at random. (to do engineering)
or Males leave indepent of other males leaving (to do eng)
or Number of males leaving each year at a constant rate (to do eng.)

(ii) For both males and females combined $\lambda = 3.1 + 0.8 = 3.9$ (each year)
$P_o(3.9)$.

$$P(M+F > 3) = 1 - P((M+F) = 0, 1, 2, 3)$$
$$= e^{-3.9}\left(1 + 3.9 + \frac{3.9^2}{2!} + \frac{3.9^3}{3!}\right) = 0.54675$$
$$= 0.547 \checkmark \text{ (3 sf)}$$

(iii) $P(F = 0 \text{ and } M > 3)$ for $F \to \lambda_1 = 0.8$
for $M \to \lambda_2 = 3.1$

$$= P(F=0) \times P(M > 3)$$
$$= e^{-0.8} \times [1 - P(M = 0, 1, 2, 3)]$$
$$= e^{-0.8} \cdot \left[1 - e^{-3.1}\left(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!}\right)\right]$$
$$= 0.16857$$

Now $P(F=0 \text{ and } M=3 / M+F=3) = \dfrac{P(F=0 \text{ and } M=3)}{P(M+F=3)}$
(Conditional Prob.)

$$= \frac{0.16857}{0.54675}$$
$$= 0.308 \checkmark \text{ (3 sf)}$$

31. Small drops of two liquids, A and B, are randomly and indepently distributed in the air. The average numbers of drops of A and B per cubic centimetre of air are 0.25 and 0.36 respectively.

(i) A sample of $10\,cm^3$ of air is taken at random. Find the prob. that the total number of drops of A and B in this sample is at least 4. [3]

(ii) A sample of $100\,cm^3$ of the air is taken at random. Use an approximating distribution to find the probability that the total number of A and B in this sample is less than 60. --[5]

[W-18/71/Q4]

Solution: Per cubic cm $\rightarrow \lambda_1 = 0.25, \lambda_2 = 0.36$

(i) for $10\,cm^3 \rightarrow \lambda = 10\times0.25 + 10\times0.36 = 6.1$

$$P(X\geq4) = 1 - e^{-6.1}\left(1 + 6.1 + \frac{6.1^2}{2!} + \frac{6.1^3}{3!}\right) = \underline{0.857}$$

(ii) for $100\,cm^3 \rightarrow \lambda = 100\times0.25 + 100\times0.36 = 61$

$Po(61) \sim N(61, 61)$; $\lambda = \mu = \sigma^2 = 61$

$$P(X<60) = P\left(Z < \frac{59.5 - 61}{\sqrt{61}}\right)$$

{⊗ continuity correction
$X < 60 \rightarrow X \leq 59 \sim 59.5$

$$= P(Z < -0.192) = 1 - \phi(0.192) = \underline{0.424}$$

---

32. The random variable X has the distribution Po(2.3). Find $P(2\leq X<5)$. --[3]

[W-18/72/Q1]

Solution: $Po(2.3) \rightarrow \lambda = 2.3$

$\left\{ P(X=r) = e^{-\lambda}\cdot\frac{\lambda^r}{r!} \right.$

$$P(2\leq X<5) = P(X=2,3,4)$$

$$= e^{-2.3}\left(\frac{2.3^2}{2!} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!}\right)$$

$$= \underline{0.585} \; (3\,sf)$$