Date 15.10.21

FMe

Further Mechanics

Hook's law
Notes and Revision

SP-20 | S-20 | W-20 | S-21

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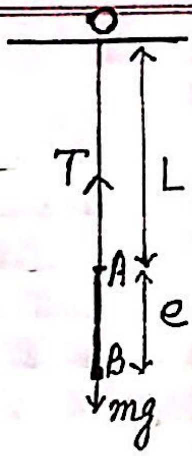
Notes



Hook's Law:

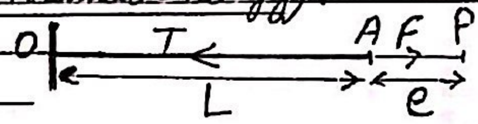
- The tension in an elastic spring (or string) is proportional to the extension 'e' produced.

$T \propto e$

or $T = \dfrac{\lambda e}{L}$

where:
'λ' is the modulus of elasticity of the string, its unit is Newton 'N'
'e' is the extension produced.
'L' is the natural length of the string.

- **Workdone (or EPE - Elastic Potential Energy):**



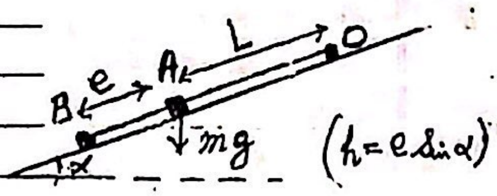
$EPE = \dfrac{\lambda \cdot e^2}{2L}$

- **GPE - Gravitational Potentional Energy:**

(i) When the string moves from A to B producing an extension 'e'

The loss in GPE = $\underline{mge}$.

(ii) From $A \rightarrow B$.
Loss in GPE = $\underline{mge \sin\alpha}$.



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Hook's Law_ Revision

1. One end of a light elastic spring, of natural length a and modulus of elasticity $5mg$, is attached to a fixed point A. The other end of the spring is attached to a particle P of mass m. The spring hangs with P vertically below A. The particle P is released from rest in the position where the extension of the spring is $\frac{1}{2}a$.

(a) Show that the initial acceleration of P is $\frac{3}{2}g$ upwards. ---[3]

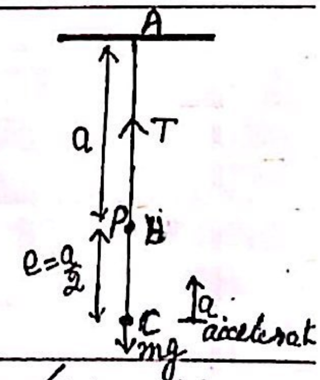
(b) Find the speed of P when the spring first returns to its natural length. ---[4]

[S-20/31/Q3]

Solution (a) $T - mg = ma$ ---(i) (Using Newton's law of Motion)

Using Hook's Law:

$$T = \frac{\lambda e^2}{L} = \frac{5mg \times \frac{1}{2}a}{a} = \frac{5}{2}mg \text{ ---(ii)}$$



from (i) and (ii)

$$\frac{5}{2}mg - mg = ma$$

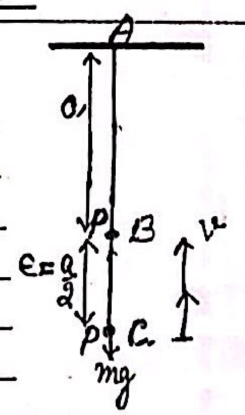
$$\Rightarrow \frac{3}{2}mg = ma$$

$$\Rightarrow \text{Acceleration } a = \frac{3}{2}g \checkmark \text{ (upwards)}$$

(b) Gain $K.E = \frac{1}{2}mv^2 \checkmark$; Gain in $G.P.E = mg.(\frac{1}{2}a) \checkmark$

Loss in $EPE = \frac{1}{2}\frac{\lambda e^2}{L} = \frac{\frac{1}{2} \times 5mg.(\frac{a}{2})^2}{a}$

$\Rightarrow$ Loss in $EPE = \frac{5}{8}mga \checkmark$



Now when P returns to C to B position:

Gain in K.E + Gain in GPE = Loss in EPE

$$\frac{1}{2}mv^2 + \frac{1}{2}mga = \frac{5}{8}mga$$

$$\Rightarrow v^2 = \frac{1}{4}ga$$

$$v = \frac{1}{2}\sqrt{ga} \checkmark$$

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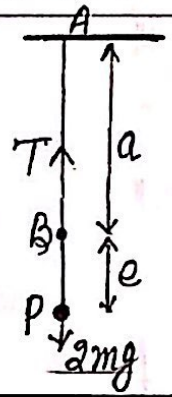
2. A light elastic string has natural length a and modulus of elasticity $24mg$. One end of the string is attached to a fixed point A. The other end of the string is attached to a particle of mass $2m$.

(a) Find, in terms of a, the extension of the string when the particle hangs freely in equilibrium below A. --[2]

(b) The particle is released from rest at A. Find, in terms of a, the distance of the particle A when it first comes instantaneous rest. --[6]

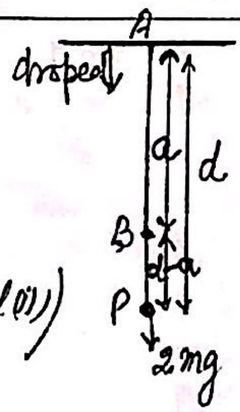
[SP-20/03/Q2]



Solution (a) $L = a$, $\lambda = 24mg$, $e = ?$

$T = \frac{\lambda}{L} e \Rightarrow 2mg = \frac{24mg \times e}{a}$

$\Rightarrow$ extension $e = a/12$



(b) Let the particle P comes to rest at a distance of 'd' below A; Extension $e = (d-a)$

G.P.E of the particle at $A = 2mg.d$ ---(i)

EPE at $A = \frac{\lambda e^2}{2L} = \frac{24mg.(d-a)^2}{2a}$ ---(ii)

No change in the K.E of the particle.

$\therefore$ at P, Loss in GPE = Gain in EPE

$\Rightarrow 2mgd = 24mg(d-a)^2/2a$ (from (i) and (ii))

$\Rightarrow 6(d-a)^2 = ad$

$\Rightarrow 6(d^2 - 2ad + a^2) = ad$

$\rightarrow 6d^2 - 13ad + 6a^2 = 0$

$\Rightarrow (2d-3a)(3d-2a) = 0$

$\Rightarrow d = \frac{3}{2}a$ or $d = \frac{2}{3}a$ ($\because d > a$)

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3. A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity 3mg, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.
Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. ---[3]

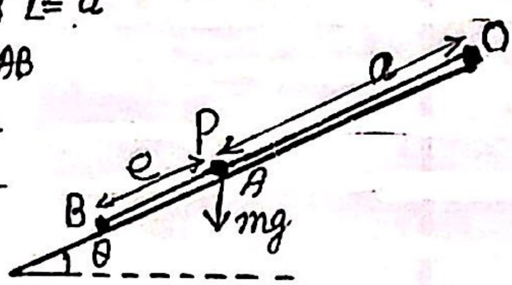
W-20/31/Q1

Solution: mass of particle $= m$; natural length $L = a$

$\lambda = 3mg$, Let extension $= e = AB$

Gain in EPE $= \frac{1}{2}\frac{\lambda e^2}{L}$

$= \frac{1}{2} \times \frac{3mg e^2}{a}$ ---(i)



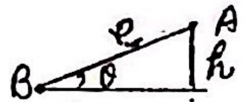
Loss in GPE $= mgh$

$= mg.e \sin\theta$ ---(ii)

Loss in GPE = gain in EPE

$\Rightarrow mg.e\sin\theta = \frac{1}{2} \cdot \frac{3mge^2}{a}$

$\Rightarrow e = \frac{2}{3} \cdot a \sin\theta$ ✓



$\frac{h}{AB} = \sin\theta$

$\Rightarrow \frac{h}{e} = \sin\theta$

$\Rightarrow h = e\sin\theta$

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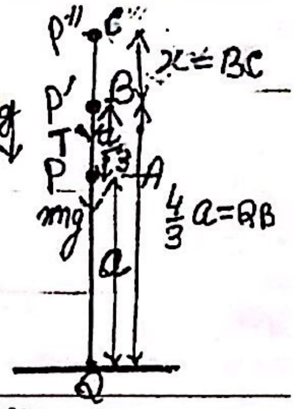
4. One end of a light elastic string, of natural length a and modulus of elasticity k, is attached to a particle P of mass m. The other end of the string is attached to a fixed point Q. The particle P is projected vertically upwards from Q. When P is moving upwards and at a distance $\frac{4}{3}a$ directly above Q, it has a speed $\sqrt{2ga}$. At this point, its acceleration is $\frac{7}{3}g$ downwards.
Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P. [W-20/32/Q6] --[8]

Solution: mass of the particle $= m$; natural length $L = a$
modulus of elasticity $\lambda = k$
Velocity at $B = \sqrt{2ga}$
Acceleration at $B = \frac{7}{3}g$ (downward)
Tension in the string $= T$ (downwards)



Using Newtons law of motion at B.
$$T + mg = m \cdot \frac{7}{3}g \quad \text{--- (i)}$$
Tension in the string. $T = \frac{\lambda e}{L} = \frac{k \cdot a/3}{a} = \frac{k}{3}$ --- (ii)
from (i) and (ii) $\frac{k}{3} + mg = \frac{7}{3}mg \Rightarrow \frac{k}{3} = \frac{4}{3}mg$
$\Rightarrow \underline{k = 4mg}$ ✓

Now let the greatest height above $Q = \left(\frac{4}{3}a + x\right) = QC$

from B to C:
Gain in GPE $= mgx$ --- (iii)
Loss in K.E $= \frac{1}{2}m(\sqrt{2ga})^2 = \frac{1}{2}m \times 2ag = mga$ --- (iv)
Gain in EPE $= \frac{1}{2}\frac{\lambda e^2}{L}$
$= \frac{1}{2} \times \frac{4mg}{a}\left\{\left(x+\frac{a}{3}\right)^2 - \left(\frac{a}{3}\right)^2\right\}$
$= \frac{2mg}{a}\left[x^2 + \frac{2ax}{3} + \cancel{\frac{a^2}{9}} - \cancel{\frac{a^2}{9}}\right]$ --- (v)

Now Gain in EPE + Gain in GPE = Loss in K.E

$\frac{2mg}{a}\left[x^2 + \frac{2ax}{3}\right] + mgx = mga$ (from (iii), (iv), (v))
$\Rightarrow 2\left(x^2 + \frac{2ax}{3}\right) + ax = a^2$
$\Rightarrow 2x^2 + \frac{4ax}{3} + ax = a^2$
$\Rightarrow 2x^2 + \frac{7ax}{3} - a^2 = 0$
$6x^2 + 7ax - 3a^2 = 0$
$\Rightarrow x = a/3$ or $x = -\cancel{3a/2}$

$\therefore$ Greatest height $QC = \frac{4}{3}a + x = \frac{4}{3}a + \frac{a}{3} = \frac{5a}{3}$ ✓

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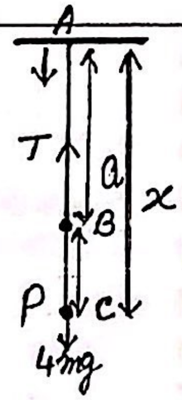
5. One end of a light elastic string, of natural length a and modulus of elasticity kmg, is attached to a fixed point A. The other end of the string is attached to a particle P of mass $4m$. The particle P hangs in equilibrium a distance x vertically below A.

(a) Show that $k = \frac{4a}{x-a}$ .... [1]

An additional particle, of mass $2m$, is now attached to P and the combined particle is released from rest at the original equilibrium position of P. When the combined particle has descended a distance $\frac{1}{3}a$, its speed is $\frac{1}{3}\sqrt{ga}$.

(b) Find x in terms of a. [S-21/31/Q3] ---[6]

Solution: AB = a (natural length), mass = $4m$, AC = x.



(a) Now using Hooke's Law $\{\lambda = kmg\}$

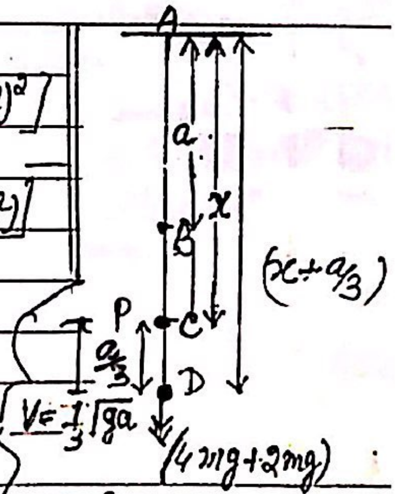
$T = \frac{\lambda e}{L} = \frac{kmg(x-a)}{a} = 4mg$ {Extension $= (x-a)$}

$\Rightarrow k(x-a) = 4a \Rightarrow k = \frac{4a}{(x-a)}$ ✓

(from C to D)

(b) Gain in K.E + gain in EPE = Loss in GPE

$\left(\frac{1}{2}mv^2 + \frac{1}{2}\frac{\lambda e^2}{L} = mgh\right)$



$\Rightarrow \frac{1}{2}\times 6m \times \frac{ga}{9} + \frac{1}{2}\times\frac{kmg}{a}\times\left[\left(x+\frac{a}{3}-a\right)^2 - (x-a)^2\right] = 6mg \times a/3$

$\left(k = \frac{4a}{(x-a)}\right)$

$\Rightarrow \frac{a}{3} + \frac{1}{2a}\times\frac{4a}{(x-a)}\times\left[\cancel{x^2} + \frac{4a^2}{9} - \frac{4ax}{3} - (\cancel{x^2} - 2ax + a^2)\right] = 2a$

$\Rightarrow \frac{a}{3} + \frac{2}{(x-a)}\left[\frac{4a^2}{9} - a^2 + 2ax - \frac{4ax}{3}\right] = 2a$

$\frac{2}{(x-a)}\left[\frac{6ax - 5a^2}{9}\right] = \frac{5a}{3}$

$\frac{2}{(x-a)}\times\frac{a}{3}\left[\frac{6x-5a}{3}\right] = 5a/3$

$2(6x-5a) = 15(x-a)$

$12x - 10a = 15x - 15a \Rightarrow 3x = 5a \Rightarrow x = \frac{5a}{3}$ ✓

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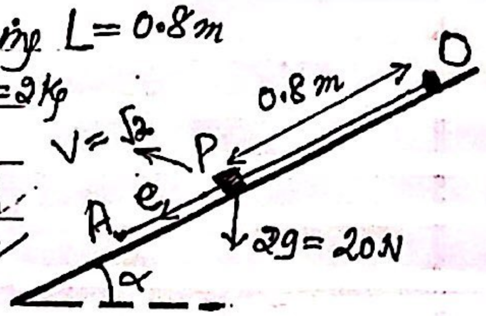
6. One end of a light elastic string of natural length 0.8 m and modulus of elasticity 36 N is attached to a fixed point O on a smooth plane. The plane is inclined at an angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$. A particle P of mass 2 kg is attached to the other end of the string. The string lies along a line of greatest slope of the plane with the particle below the level of O. The particle is projected with speed $\sqrt{2}\,ms^{-1}$ directly down the plane from the position where OP is equal to the natural length of the string.
Find the maximum extension of the string during the subsequent motion. [S-21/33/Q2] --- [5]

Solution: $\lambda = 36N$, natural length of string $L = 0.8m$
$\sin\alpha = \frac{3}{5}$, at P, $V = \sqrt{2}\ ms^{-1}$, $m = 2kg$
Let extension PA = e



at A, $EPE = \frac{1}{2}\frac{\lambda e^2}{L} = \frac{1}{2}\times\frac{36\times e^2}{.8} = 22.5e^2 = \frac{45}{2}e^2$ ✓

Loss in GPE $= mg \cdot e\sin\alpha$
$= 20\times\frac{3}{5}\times e = 12e$ ✓

Loss in K.E. $= \frac{1}{2}mV^2 = \frac{1}{2}\times 2\times(\sqrt{2})^2 = 2$ ✓

Loss in G.P.E + Loss in K.E = Gain in EPE

$$12e + 2 = \frac{45}{2}e^2 \Rightarrow 45e^2 - 24e - 4 = 0$$
$$(3e-2)(15e+2) = 0$$
$$e = \frac{2}{3} \quad ; \quad -\frac{2}{15}\ \times$$

$\therefore$ Max. Extension $e = \frac{2}{3}$ ✓

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7

One end of a light spring of natural length a and modulus of elasticity $4mg$ is attached to a fixed point O. The other end of the spring is attached to a particle A of mass km, where k is a constant. Initially the spring lies at rest on a smooth horizontal surface and has length a. A second particle B, of mass m, is moving towards A with speed $\sqrt{\frac{4}{3} ga}$ along the line of the spring from the opposite direction to O, (see diagram).

The particles A and B collide and coalesce. At a point C in the subsequent motion, the length of the spring is $\frac{3}{4}a$ and the speed of the combined particle is half of its initial speed.

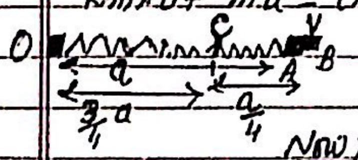
(a) Find the value of k. --- [6]

At the point C the horizontal surface becomes rough, with coefficient of friction μ between the combined particle and the surface. The deceleration of the combined particle at C is $\frac{9}{20}g$.

(b) Find the value of μ. $\boxed{S\text{-}20 \,|\, 33 \,|\, Q7}$ -- [4]

Solution: $m_A = km$, $m_B = m$
$u_A = 0$, $u_B = u$ (let) $= \sqrt{\frac{4}{3} ga}$
{Combined mass $= km + m = (k+1)m$
{After collision, Velocity $= V$ (let)

Using conservation of momentum law;
$m_A u_A + m_B u_B = (m_A + m_B) \times V$
$km \times 0 + m u = (k+1)V \Rightarrow V = \frac{u}{(k+1)}$ --- (1)



Velocity at C $= \frac{V}{2}$

Now for A to C;

Loss in K.E $= \frac{1}{2} MV^2$
$= \frac{1}{2}(k+1)m \left[V^2 - \left(\frac{V}{2}\right)^2\right]$

Loss in K.E $= \frac{1}{2} \times \frac{3}{4}(k+1)$
$= \frac{3}{8}(k+1)m \cdot \frac{u^2}{(k+1)^2}$ from (1)
$= \frac{3}{8} \frac{(k+1)m}{(k+1)^2} \times \frac{4}{3} g a$ [$\because u^2 = \frac{4}{3} ga$ given]
$= \frac{mga}{2(k+1)}$ —— (2)

Now Gain in EPE from A to C

$EPE = \frac{1}{2} \frac{\lambda}{l} \times e^2 = \frac{1}{2} \times \frac{4mg}{a} \cdot \left(\frac{a}{4}\right)^2$
$= \frac{mga}{8}$ —— (3)

Now Loss of K.E. = gain of EPE
$\Rightarrow \frac{mg.a}{2(k+1)} = \frac{mg.a}{8}$ [from (2) & (3)]
$\Rightarrow k+1 = 4 \Rightarrow \underline{k = 3}$ ✓

(Continued →)

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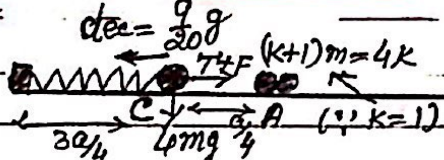
(continued)

7(b) At the point C, $R = 4mg$ --- ①
∴ Friction force $F = \mu R$



Using Hooke's law $T = \frac{\lambda e}{L} = \frac{4mg \times a/4}{a}$ ($\lambda = 4mg$) [$e = a/4 \leftarrow \left(a - \frac{3a}{4}\right)$]

$\Rightarrow T = mg$

Now using Newton's law

$(T + F = ma)$:

$\Rightarrow mg + F = 4m \times \frac{9}{20} g$

$\Rightarrow F = \frac{9}{5} mg - mg = \frac{4}{5} mg$ ✓ --- ②

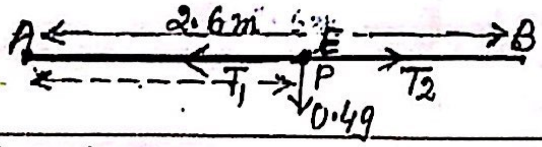
$F = \mu R = \frac{4}{5} mg$ (from 2)

($R = 4mg$ from ①)

$\mu \times 4mg = \frac{4}{5} mg$

$\Rightarrow \mu = \frac{1}{5}$

8. The fixed points A and B are on a smooth horizontal surface, with AB = 2.6 m, one end of a light elastic spring, of natural length 1.25 m and modulus of elasticity λ N is attached to A. The other end is attached to a particle P of mass 0.4 kg. One end of a second light elastic spring, of natural length 1.0 m and modulus of elasticity 0.6λ N, is attached to B; its other end is attached to P. The system is in equilibrium with P on the surface at point E. Show that AE = 1.4 m [W-18/21/Q5] -- [4]



Solution: $L_1 = 1.25\,m$; $L_2 = 1.0\,m$

$\lambda_1 = \lambda$; $\lambda_2 = 0.6\lambda$

extention $e_1 = (AE - 1.25)$ in first string and Extension in the second string $e_2 = (2.6 - AE - 1) = (1.6 - AE)$

Particle 'P' is equilibrium:

$T_1 = T_2$

$T_1 = \frac{\lambda_1 e_1}{L_1} = \frac{\lambda(AE - 1.25)}{1.25}$

$T_2 = \frac{\lambda_2 e_2}{L_2} = \frac{0.6\lambda(1.6 - AE)}{1.0}$

$\Rightarrow \frac{\lambda(AE - 1.25)}{1.25} = \frac{0.6\lambda(1.6 - AE)}{1}$

$\Rightarrow AE - 1.25 = 1.25 \times 0.6(1.6 - AE) \Rightarrow AE - 1.25 = 0.75(1.6 - AE)$

$1.75 AE = 2.45 \Rightarrow AE = 1.4$ ✓

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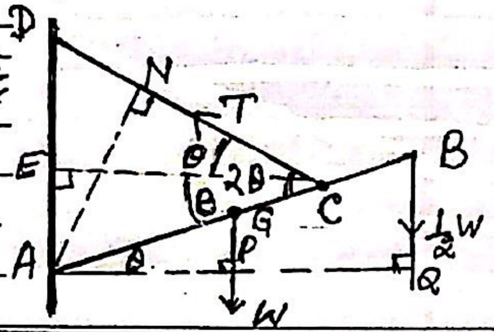
9. A uniform rod AB of length 4a and weight W is smoothly hinged to a vertical wall at the end A. The rod is held at an angle θ above the horizontal by a light elastic string. One end of the string is attached to the point C on the rod, where AC = 3a. The other end of the string is attached to a point D on the wall, with D vertically above A and such that angle ACD = 2θ. A particle of weight $\frac{1}{2}W$ is attached to the rod at B. It is given that $\tan\theta = 8/15$.

(i) Show that the tension in the string is $\frac{17}{12}W$. --[4]

(ii) Find the magnitude and direction of the reaction of the hinge. --[5]

(iii) Given that the natural length of the string is 2a, find its modulus of elasticity. [W-18/21/Q4] --[2]



Solution: $\tan\theta = \frac{8}{15}$, $\sin\theta = \frac{8}{17}$, $\cos\theta = \frac{15}{17}$

$AB = 4a$, $AC = 3a$, $AG = 2a$

Draw $AN \perp CD$

(i) $AN = AC\sin 2\theta = 3a\sin 2\theta$

$AP = AG\cdot\cos\theta = 2a\cos\theta$

$AQ = AB\cos\theta = 4a\cos\theta$

Taking moment about A,

$T\times AN = W\times AP + \frac{1}{2}W\times AQ$

$\Rightarrow T\times 3a\sin 2\theta = W\times 2a\cos\theta + \frac{1}{2}W\cdot 4a\cos\theta$

$\Rightarrow T\times 3a\times 2\sin\theta\cos\theta = 4Wa\cos\theta$

$\Rightarrow 6aT\sin\theta = 4aW \Rightarrow T = \frac{4aW}{6a\sin\theta}$

$\Rightarrow T = \frac{2}{3}W\times\frac{1}{8/17} = \frac{17}{12}W$ ✓

Draw $CE \perp AD \Rightarrow \angle ACE = \theta$

$\angle DCE = \theta$

$\therefore \angle A = \angle D = 90 - \theta$

$\therefore DC = AC = 3a$ →

(ii) horizontal component X of the force at A:

$X = T\cos\theta = \frac{17}{12}W\times\frac{15}{17} = \frac{5}{4}W$ ---(2)

and Vertically,

$Y = W + \frac{1}{2}W - T\sin\theta$

$= \frac{3}{2}W - \frac{17}{12}W\times\frac{8}{17} = \frac{5}{6}W$ —(3)

from (2) & (3) $F = \sqrt{X^2+Y^2} = \frac{5\sqrt{13}}{12}W$ ✓

upward at an angle to the horizontal

$\alpha = \tan^{-1}Y/X = \tan^{-1}2/3 = 33.7°$ ✓

(iii) $T = \frac{\lambda}{L}\times e \Rightarrow \frac{17W}{12} = \frac{\lambda(DC-2a)}{2a}$

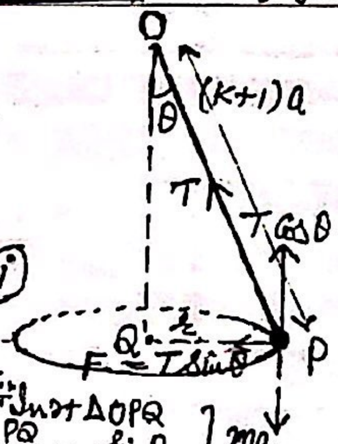
$\Rightarrow \frac{17}{12}W = \frac{\lambda\times(3a-2a)}{2a} = \lambda/2$

$\Rightarrow \lambda = \frac{17W}{6}$ ✓ $(DC = 3a)$

10. One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of string during this motion is $(k+1)a$.

(a) Find the value of k. ---[4]

(b) Find the value of $\cos\theta$ [W-20/31/Q3] --[2]



Solution: natural length $L = a$; length of the extended

(a) angular speed $w = \sqrt{\frac{g}{a}}$; string $OP = (k+1)a$

$\lambda = 4mg$. Extension $e = (k+1)a$

Using Hooke's law $\Rightarrow T = \frac{\lambda e}{L} = 4mg \cdot \frac{ka}{a} = 4kmg$ --- (i)

for circular motion:

Force toward centre; $F = mr\omega^2 = m a(k+1)\sin\theta \cdot \frac{g}{a}$

$\Rightarrow T \cancel{\sin\theta} = m(k+1)\cancel{a} \cdot \frac{g}{\cancel{a}} \cancel{\sin\theta}$

$\Rightarrow T = m(k+1) \cdot \frac{g}{a}$ --- (ii)

[In rt $\triangle OPQ$
$\frac{PQ}{OP} = \sin\theta$
$\Rightarrow r = PQ = OP \cdot \sin\theta$
$\Rightarrow r = (k+1)\sin\theta$]

from (i) and (ii)

$$m(k+1)g = 4k\,mg$$

$$\Rightarrow k+1 = 4k \Rightarrow 3k = 1 \Rightarrow k = \frac{1}{3} \checkmark$$

(b) $T\cos\theta = mg$

$\Rightarrow \frac{4}{3} mg \cos\theta = mg$ [from (i) $T = 4kmg = 4 \times \frac{1}{3} mg$ for $k = \frac{1}{3}$]

$$\Rightarrow \cos\theta = \frac{3}{4} \checkmark$$

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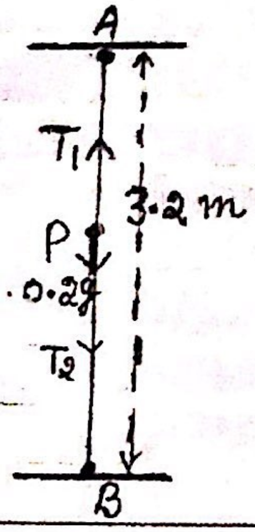
11 The fixed points A and B are such that AB = 3·2 m and A is vertically above B. One end of a light elastic string of natural length 0·8 m and modulus of elasticity 8N, is attached to a particle P of mass 0·2 kg. The other end of the string is attached to A. One end the second light elastic string of natural length 1·2 m and modulus of elasticity 4N, is attached to P and the other end is attached to B.
Find the length of AP when the particle is in equilibrium. ---[4]

|S-16|23|Q11|



Solution: For AP: $L_1 = 0.8\,m$; $\lambda_1 = 8N$, $e_1 = (AP - 0.8)\,m$

using Hooke's law:

$$T_1 = \frac{\lambda_1 e_1}{L_1} = \frac{8(AP-0.8)}{0.8} \quad ---(i)$$

for PB; $L_2 = 1.2\,m$, $\lambda_2 = 4N$, $e_2 = (3.2 - 1.2 - AP)$
or $e_2 = (2 - AP)$

Using Hooke's law:

$$T_2 = \frac{\lambda_2 e_2}{L_2} = \frac{4(2-AP)}{1.2} \quad -----(ii)$$

and a downward Force $= mg = 0.2g$ ... (iii)

$$\Rightarrow \quad T_1 = T_2 + mg$$

$$\Rightarrow \quad \frac{8(AP-0.8)}{0.8} = \frac{4(2-AP)}{1.2} + 0.2g$$

$$\Rightarrow \quad 10(AP-0.8) = \frac{10}{3}(2-AP) + 2$$

$$\Rightarrow \quad AP = \frac{5}{4} \text{ or } \underline{1.25\,m}$$

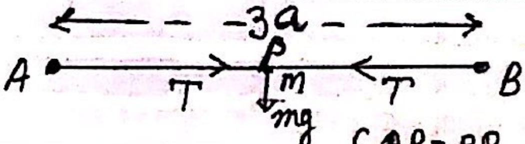
12 A and B are two fixed points on a smooth horizontal surface, with AB = 3a m. One end of the light elastic string of natural length a m and modulus of elasticity mg N, is attached to the point A. The other end of this string is attached to a particle P of mass m kg. One end of a second light elastic string of natural length ka m and modulus of elasticity 2mg N, is attached to B. The other end of the string is attached to P. Given that the system is in equilibrium when P is at M, the mid point of AB, find the value of k. --[3]

W-15/21/Q3

Solution: for AP: $L_1 = a$ m, $\lambda = mg$ N

$AB = 3a \rightarrow AP = \frac{3a}{2}$, $e_1 = \left(\frac{3a}{2} - a\right)$

$e_1 = a/2$



Using Hook's Law:

$$T = \frac{\lambda_1 e_1}{L_1} = \frac{mg \times a/2}{a} = \frac{1}{2} mg \quad \text{---(i)}$$

$\left\{ AP = BP = AM = \frac{3a}{2} \right.$

for BP: $L_2 = ka$ m, $\lambda_2 = 2mg$ N, $e_2 = \left(\frac{3a}{2} - ka\right) = \left(\frac{3}{2} - k\right)a$

$$\therefore T = \frac{\lambda_2 \cdot e_2}{L_2} = \frac{2mg \times \left(\frac{3}{2} - k\right)a}{ka} = \frac{\left(\frac{3}{2} - k\right)}{k} 2mg \quad \text{....(ii)}$$

from (i) and (ii)

$$\frac{2\left(\frac{3}{2} - k\right)mg}{k} = \frac{1}{2} mg$$

$$\Rightarrow \frac{3}{2} - k = \frac{k}{4}$$

$$\Rightarrow \frac{5}{4} k = 3/2$$

$$k = \frac{6}{5} = 1.2$$

$$\therefore \underline{k = 1.2}$$

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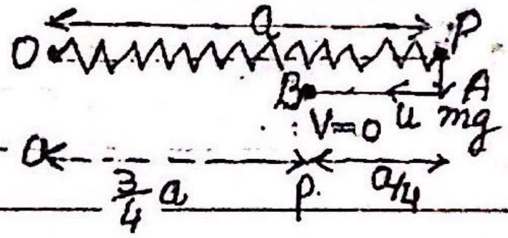
13. A light spring has natural length a and modulus of elasticity kmg. The spring lies on a smooth horizontal surface with one end attached to a fixed point O. A particle P of mass m is attached to the other end of the spring. The system is in equilibrium with $OP = a$. The particle is projected towards O with speed u and comes to instantaneous rest when $OP = \frac{3}{4}a$.

Use an energy method to show that $k = 16u^2/ag$ --- [2]

S-19/23/Q11



Solution: K.E at $A = \frac{1}{2}mu^2$ ---(i)

for spring $\lambda = kmg$, $l = a$

$$e = (a - \tfrac{3}{4}a) = \frac{a}{4}$$

$\therefore$ E.P.E at $B = \frac{\lambda e^2}{2l}$

$$= \frac{kmg.\left(\frac{a}{4}\right)^2}{2a} = \frac{kmg.a}{32} \text{ ---(ii)}$$

from (i) and (ii) gain in E.P.E = Loss in K.E

$$\Rightarrow \frac{kmga}{32} = \frac{1}{2}mu^2$$

$$\Rightarrow k = \frac{16u^2}{ag}$$

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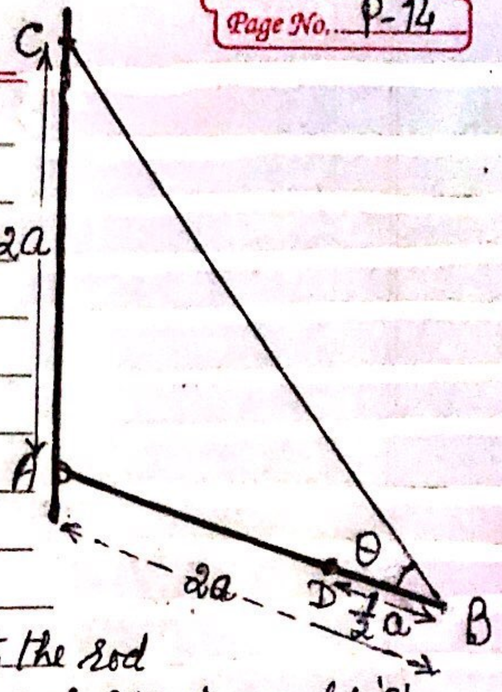
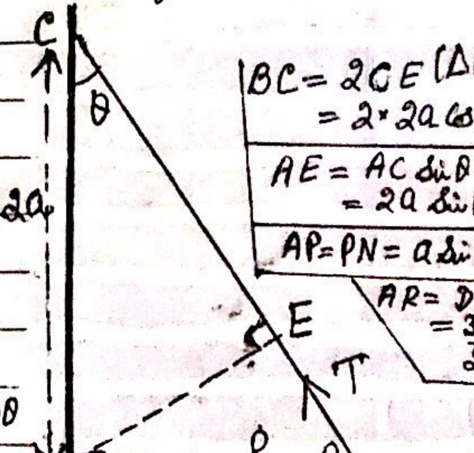
14. The end A of uniform rod AB, of length '2a' and weight 'W', is freely hinged to vertical wall. The end B of the rod is attached to light elastic string of natural length '$\frac{3}{2}a$', and modulus of elasticity '3W'. The other end of the string is attached to the point C on the wall, where C is vertically above A and AC = '2a'.



A particle of weight '2W' is attached to the rod at the point D, where DB = $\frac{1}{2}a$. The angle ABC is equal to 'θ'. Show that $\cos\theta = 3/4$ and find the tension in the string in terms of W. ---[10]

Find the magnitude of the reaction force at the hinge. [S-16/21/Q11] --[4]



$BC = 2CE$ (ΔACE)
$= 2 \times 2a\cos\theta$

$AE = AC\sin\theta$
$= 2a\sin\theta$

$AP = PN = a\sin\phi$

$AR = DQ$
$= \frac{3a}{2}\sin\phi$

<u>Solution:</u> $AB = AC = 2a \Rightarrow \angle ABC = \angle ACB = \theta$

Let $\angle QAB = \phi = 2\theta$ --- (1); Let $AE \perp BC$

<u>Taking moment about A,</u>

$T \times AE = W \times AP + 2W \times AR$

$\Rightarrow T \times 2a\sin\theta = W \times a\sin\phi + 2W \times \frac{3a}{2}\sin\phi$

$2T\sin\theta = W \times \sin 2\theta + 3W\sin 2\theta$ $\left\{\sin 2\theta = 2\sin\theta\cos\theta\right.$

$T = 2W\frac{\sin 2\theta}{\sin\theta} = 4W\cos\theta$

$\Longrightarrow \quad T = 4W\cos\theta$ --- (2)

<u>Now using Hook's law;</u> $\lambda = 3W$, $L = \frac{3a}{2}$,

$e = BC - \frac{3a}{2} = \left(2 \times 2a\cos\theta - \frac{3a}{2}\right)$

$T = \frac{\lambda e}{L} = \frac{3W}{3a/2}\left(4a\cos\theta - \frac{3a}{2}\right) = W(8\cos\theta - 3)$ --- (3)

from (2) & (3)

$4W\cos\theta = W(8\cos\theta - 3)$

$\Rightarrow \cos\theta = \frac{3}{4}$ ✓ --- (4)

$\Rightarrow \sin\theta = \frac{\sqrt{7}}{4}$ --- (5)

from (3) $T = 3W$ --- (6)

Horizontal component of reaction at A;

$X = T\sin\theta = 3W \times \frac{\sqrt{7}}{4}$

Vertically $Y = 3W - T\cos\theta = 3W - 3W \times \frac{3}{4}$
$= 3W/4$

$\therefore F = \sqrt{X^2 + Y^2} = \sqrt{\left(\frac{3\sqrt{7}}{4}W\right)^2 + \left(\frac{3W}{4}\right)^2}$

$F = \frac{3\sqrt{2}}{2}W$ ✓