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FMe

Further Mechanics

Momentum
Notes and Revision

SP-20 | S-20 | W-20 | S-21

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Momentum _ Notes

§ Momentum:

The momentum of a particle of mass m kg and moving with a velocity of $v\ ms^{-1}$ $= mv$

Momentum $P = mv$ $kg\,ms^{-1}$ (or Ns)

momentum is a vector quantity and its direction is same as the direction of the velocity

§ Newton's Second Law of motion:

$$F = \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma \quad [a \text{ is acceleration}]$$

$$\Rightarrow m(v-u) = F \cdot t$$

Impulse I = Change in momentum $= F \cdot t$ Ns

Example 1: A bullet of mass m kg is fired horizontally into a fixed vertical block of material. It enters the block horizontally with speed $250\ ms^{-1}$ and emerges horizontally with speed $70\ ms^{-1}$ after 0.04 s. The block offers a constant horizontal resisting force of magnitude 450N. Find the value of m. [S-18/22/Q1] --[3]

Solution: Change in Momentum $m(v-u) = F \cdot t$ [F is resisting force, $F = -450N$]

$$m(70-250) = -450 \times 0.04$$

$$\Rightarrow \cancel{-}180m = \cancel{-}450 \times 0.04$$

$$m = \frac{450 \times 0.04}{180} = 0.1\ kg$$

Example 2: A bullet of mass 0.08 kg is fired horizontally into a fixed vertical barrier. It enters the barrier horizontally with speed $300\ ms^{-1}$ and emerges horizontally after 0.02 s. There is a constant horizontal resisting force of magnitude 1000 N. Find the speed with which the bullet emerges from the barrier. [S-17/21/Q1] --[3]

Solution: Change in momentum: $m(v-u) = F \cdot t \Rightarrow 0.08(v-300) = -1000 \times 0.02$

$$\Rightarrow v - 300 = -250 \Rightarrow v = 50\ ms^{-1}$$

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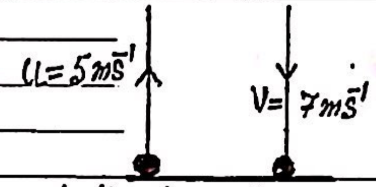
Example 3: A ball of mass 2 kg is thrown upwards with an initial velocity of $5\,ms^{-1}$. It is travelling at $7\,ms^{-1}$ just before it lands. Find the change in its momentum.

Solution:

Change of momentum $= m(V-u)$

$= 2[7-(-5)] = 2\times 12$

$= 24\,Ns$



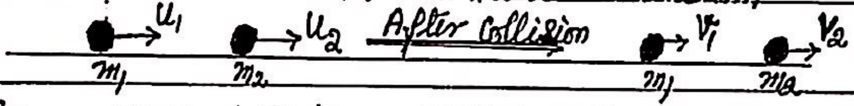
(Downwards direction is +)

§ Collision and conservation of Momentum:

When objects interact, they exert equal and opposite forces on each other.

§ Principle of conservation of momentum:

If two bodies, moving along the same straight line, collide with each other then the total final momentum remains same as the total initial momentum.



Then $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

§ Coalesce:

If the colliding objects stick together as a result of collision (coalesce), they have the same velocity v after collision, then

$$m_1u_1 + m_2u_2 = (m_1+m_2)\cdot v$$

Note 1: Two particles coalesce $\Leftrightarrow e=0$ (Particles are inelastic)

2. If $e=1 \Rightarrow$ particles are perfectly elastic and the law of conservation of K.E is applicable.

(Refer Page-5)

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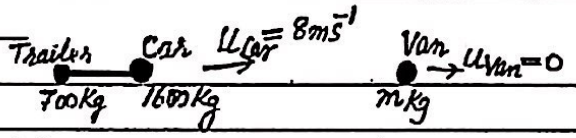
Example 4: On a straight horizontal track, driverless vehicles are tests. A car of mass 1600 kg is towing a trailer of mass 700 kg along a track. A stationary Van is directly in front of the car, Car hits the Van at a speed of $8\,ms^{-1}$. After collision, the van starts to move with speed $5\,ms^{-1}$, and the car and trailer continue moving in the same direction with speed $2\,ms^{-1}$. Find the mass of the van. --[3]

[9709/M-20/42/Q6(a)]

Solution:



Momentum before the car hits the Van $= m(\text{Trailer}+\text{Car}) \times u_{car} + m \times 0$

$= (700+1600) \times 8 + m \times 0.$ --(1)

Momentum after the car hits the Van $= (700+1600) \times 2 + m \times 5$ ---(2)

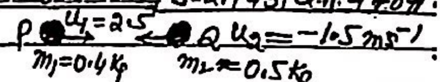
Using the conservation of momentum

$2300 \times 8 = 2300 \times 2 + 5m$ (from (i) & (ii))

$\Rightarrow 5m = 13800 \Rightarrow m = 2760$ kg ✓

Example 5: Particle P of mass 0.4 kg and Q of mass 0.5 kg are free to move on a horizontal plane. P and Q are moving directly towards each other with speeds $2.5\,ms^{-1}$ and $1.5\,ms^{-1}$ respectively. After P and Q collide, the speed of Q is twice the speed of P. Find the two possible values of the speed of P after the collision. --[4]

[S-21/43/Q1/9709]



Solution: for P: $m_1 = 0.4$ kg, $u_1 = 2.5\,ms^{-1}$

Q: $m_2 = 0.5$ kg; $u_2 = -1.5\,ms^{-1}$ / After collision, Let $V_1 = V$, $V_2 = 2V$

Case I when P and Q move in the same direction after collision: P $\rightarrow V_1$ Q $\rightarrow V_2$

$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow 0.4 \times 2.5 + 0.5 \times (-1.5) = 0.4 \times V + 0.5 \times 2V \Rightarrow 1.4V = 0.25 \Rightarrow V = 0.179\,ms^{-1}$ speed of P ✓

Case II when P and Q move in the opposite direction after collision: P $V_1 \leftarrow$ Q $\rightarrow V_2$

$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$

$\Rightarrow 0.4 \times 2.5 + 0.5(-1.5) = 0.4(-V) + 0.5 \times (2V)$

$\Rightarrow 1 - 0.75 = -0.4V + V \Rightarrow 0.25 = 0.6V \Rightarrow V = 0.417\,ms^{-1}$ ✓ Speed of P.

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6. A particle B of mass 5 Kg is at rest on a smooth horizontal table. A particle A of mass 2.5 Kg moves on the table with a speed of $6 m s^{-1}$ and collides directly with B. In the collision the two particles coalesce.

(a) Find the speed on the combined particle after the collision. --[2]

(b) Find the loss of K.E of the system due to the collision. --[3]

[9709/W-20/41/Q1]

Solution:

A ● $u_A = 6 m s^{-1}$ → B ● $u_B = 0$ ⟶ ●● → $V = ?$

$m_A = 2.5 Kg$ $m_B = 5 Kg$ $m_A + m_B$

(a) Using conservation of momentum:

$$m_A \cdot u_A + m_B \cdot u_B = (m_A + m_B) \cdot V$$

$$2.5 \times 6 + 5 \times 0 = (2.5 + 5) \cdot V$$

$$\Rightarrow 15 = 7.5 V \Rightarrow V = \frac{15}{7.5} = \underline{2 m s^{-1}} \checkmark$$

(b) K.E. before collision $= \frac{1}{2} m_A u_A^2 + \frac{1}{2} m_B \cdot u_B^2$

$$= \frac{1}{2} \times 2.5 \times 6^2 + 0 = \underline{45 J} \quad \text{---} (1)$$

K.E. After collision $= \frac{1}{2}(m_A + m_B) \cdot V^2$

$$= \frac{1}{2}(2.5 + 5) \times 2^2$$

$$= \frac{1}{2} \times 7.5 \times 4 = 15 J \quad \text{---} (2)$$

$\therefore$ Loss in K.E $= 45 - 15$ (from (1) & (2))

$$= \underline{30 J} \checkmark$$

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§ Newtons experimental law: (or Newton's Law of Restitution):

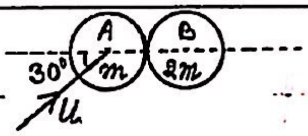
Coefficient of restitution $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$,

or $e = \dfrac{V_2 - V_1}{u_1 - u_2}$; $0 \le e \le 1$

Note:

1. When $e = 0$, the objects colliding have no elasticity (coal
2. When $e = 1$, the objects are said to be perfectly elastic. Then the law of the conservation of K.E is applicable.

Example 7: Two uniform smooth spheres A and B of equal radii have masses m and 2m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u at an angle of 30° to the line of centres of A and B when it collides with B. The coefficient of restitution between the spheres is e.



(a) Show that the speed of B after the collision is $\frac{\sqrt{3}}{6}u(1+e)$ and find the speed of A after the collision. --[6]

(b) Given that $e = \frac{1}{3}$, find the loss of kinetic energy as a result of collision. [SP-20/03/Q4] --[3]

Solution: $m_A = m$, $m_B = 2m$, $u_A = u\cos 30°$, $u_B = 0$;

(a) Using law of conservation of momentum, speeds after collision are V_A and V_B $\Rightarrow m_A u_A + m_B u_B = m_A V_A + m_B V_B$

$\Rightarrow m \cdot u\cos 30° + 0 = m \cdot V_A + 2m \cdot V_B$ --- (1)

Using Law of Restitution $e = \dfrac{V_B - V_A}{u_A - u_B}$

$\Rightarrow e = \dfrac{V_B - V_A}{u\cos 30 - 0}$

$\Rightarrow V_B - V_A = e\,u\cos 30 = \frac{\sqrt{3}}{2}eu$ .. (2)

from (1) $2V_B + V_A = \frac{\sqrt{3}}{2}u$ --- (3)

add (2) & (3) $\Rightarrow V_B = \frac{\sqrt{3}}{6}u(1+e)$ ✓

and along AB $\Rightarrow V_A = \frac{\sqrt{3}}{6}u(1-2e)$

Speed of A, Vertical component of V = $u\sin 30 = \frac{1}{2}u$

$\therefore$ Speed of A, $V_A' = \sqrt{\left(\frac{1}{2}u\right)^2 + \left(\frac{\sqrt{3}}{6}u(1-2e)\right)^2} = u\sqrt{\dfrac{1-e+e^2}{3}}$ ✓

Now given $e = \frac{1}{3}$

(b) $V_A' = u \cdot \sqrt{\frac{7}{27}}$; $V_B = \frac{\sqrt{3}}{6}u \times \frac{4}{3} = \frac{2\sqrt{3}u}{9}$

Loss in K.E

$= \frac{1}{2}mu^2 - \frac{1}{2} \times 2m\left(\frac{2\sqrt{3}u}{9}\right)^2 - \frac{1}{2}m \times \frac{7}{27}u^2$

$= \frac{2}{9}mu^2$ ✓

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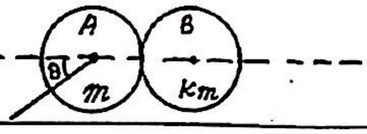
Example 8: Two uniform smooth spheres A and B of equal radii have masses m and km respectively. Sphere A is moving with speed u on a smooth horizontal surface when it collides with sphere B which is at rest. Immediately before collision, A's direction of motion makes an angle θ with the line of centres. The coefficient of restitution between the spheres is $1/3$.



(a) Show that the speed of B after the collision is $\dfrac{4u\cos\theta}{3(1+k)}$ ---[8]

70% of the total K.E of the spheres is lost as a result of collision.

(b) Given that $\tan\theta = \frac{1}{3}$, find the value of k. S-21/33/Q6/ --[6]

Solution: Let the velocities of A and B along the line of centres after collision be v_1 and v_2.

Using law of conservation of momentum

$$m\,u\cos\theta = m v_1 + km\, v_2$$

$$\Rightarrow v_1 + k v_2 = u\cos\theta \quad ---(i)$$

Using the law of restitution:

$$\frac{v_2 - v_1}{u\cos\theta - 0} = e = \frac{1}{3} \text{ (Given)}$$

$$\Rightarrow -v_1 + v_2 = \frac{u\cos\theta}{3} \quad ---(ii)$$

add (i) and (ii) $\quad v_2(1+k) = \dfrac{4u\cos\theta}{3}$

$$\Rightarrow v_2 = \frac{4u\cos\theta}{3(1+k)}$$

(b) from (i) and (ii)

$$v_1 = \frac{(3-k)u\cos\theta}{3(1+k)}$$

and vertical component of the vel of $A = u\sin\theta$

$$\therefore v_A = \sqrt{v_1^2 + (u\sin\theta)^2}$$

Now K.E of the system after collision:

$$= \frac{1}{2}km v_2^2 + \frac{1}{2}m\left(v_1^2 + u^2\sin^2\theta\right)$$

$$= \frac{30}{100}\times\frac{1}{2}mu^2$$

(as 70% of K.E is lost)

$\tan\theta = \frac{1}{3}$, $\sin\theta = \frac{1}{\sqrt{10}}$, $\cos\theta = \frac{3}{\sqrt{10}}$

$$\Rightarrow k\left(\frac{4u\cos\theta}{3(1+k)}\right)^2 + \left[\left\{\frac{(3-k)u\cos\theta}{3(1+k)}\right\}^2 + u^2\sin^2\theta\right] = \frac{3}{10}u^2$$

$$\Rightarrow \frac{16k}{9(1+k)^2}\times\frac{9}{10} + \frac{(3-k)^2}{9(1+k)^2}\times\frac{9}{10} + \frac{1}{10} = \frac{3}{10}$$

$$\Rightarrow \frac{16k}{(1+k)^2} + \frac{(3-k)^2}{(1+k)^2} = \frac{2}{10}$$

$$\Rightarrow k^2 - 6k - 7 = 0 \Rightarrow \underline{k = 7}, \ (k = -1 \text{ as } (k+1) \neq 0)$$

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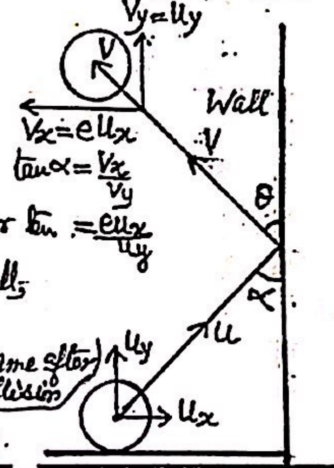


§ Oblique collisions:

Consider a particle travelling towards the wall with velocity u at an angle of ... Component of velocity perpendicular to the wall,

$u_x = u \sin\alpha$,

and parallel to wall $u_y = u\cos\alpha$ (remains same after collision)

After collision, let the velocity of the particle is v and is inclined at an angle θ to the wall, $V_y = u_y$ (same); $V_x = e u_x$ and $\tan\theta = \frac{V_x}{V_y} = \frac{e u_x}{u_y}$

Example 9: A particle P of mass m is moving with speed u on a fixed smooth horizontal surface. The particle strikes a fixed vertical barrier. At the instant of impact the direction of motion of P makes an angle α with the barrier. The coefficient of restitution between P and the barrier is e. As a result of impact, the direction of P is turned through $90°$.

(a) Show that $\tan^2\alpha = \frac{1}{e}$ --[3]

The particle P loses two-third of its K.E. in the impact.

(b) Find the value of α and the value of e. [S-20/31/Q6] --[5]

Solution: Component of u, perpendicular to the barrier $u_x = u\sin\alpha$

(a) and parallel to the barrier $u_y = u\cos\alpha$

Let After collision velocity is V,

Component of V perp. to barrier $V_x = e\,u_x$

$V\sin(90-\alpha) = e u \sin\alpha$

$\Rightarrow V\cos\alpha = e u \sin\alpha$ --(i)

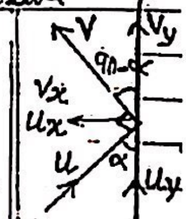
and comp. of V along the barrier $\rightarrow V_y = u_y$

$\Rightarrow V\cos(90-\alpha) = u\cos\alpha$

$\Rightarrow V\sin\alpha = u\cos\alpha$ --(ii)

from (ii) ÷ (i) $\Rightarrow \tan\alpha = \frac{1}{e\tan\alpha}$

$\Rightarrow \tan^2\alpha = \frac{1}{e}$ --(iii)



(b) Now particle loses $\frac{2}{3}$ of K.E after collision.

$\Rightarrow$ K.E after collision $= \frac{1}{3}$ of K.E before collision

$\Rightarrow \frac{1}{2} m V^2 = \frac{1}{3}\times\frac{1}{2} m u^2$

$\Rightarrow V^2 = \frac{1}{3}u^2 \Rightarrow \left(\frac{u\cos\alpha}{\sin\alpha}\right)^2 = \frac{1}{3}u^2$ [$\because$ from (ii) $V = \frac{u\cos\alpha}{\sin\alpha}$]

$\Rightarrow \tan^2\alpha = 3 \Rightarrow \alpha = 60°$ and $e = \frac{1}{\tan^2\alpha} = \frac{1}{3}$ from (iii)

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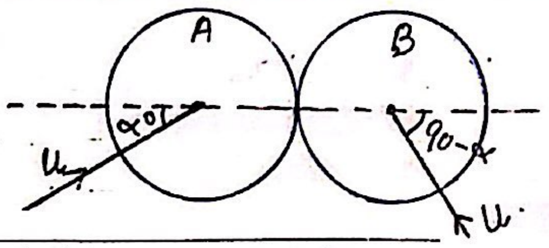

Example 10: Two uniform smooth spheres A and B of equal radii each have mass m. The two spheres are each moving with speed u on a horizontal surface when they collide. Immediately before the collision A's direction of motion makes an angle of α° with the line of centres, and B's direction of motion is perpendicular to that of A. The coefficient of restitution between the spheres is e.

Immediately after the collision, B moves in a direction at right angles to the line of centres.

(a) Show that $\tan\alpha = \frac{1+e}{1-e}$ ---[4]

(b) Given that $\tan\alpha = 2$, find the speed of A after the collision. --[4]

direction → + [S-20/33/Q5]

Solution: Let V be the speed of A along the line of centres after collision.

(a) Law of conservation of momentum along the line of centres.
(as the speed of B along the line of centres is 0: $V_B = 0$, $V_A = -V$ (let))

$$mu\cos\alpha - mu\sin\alpha = -mV + 0 \Rightarrow V = (-u\cos\alpha + u\sin\alpha) \text{ --(i)}$$

Using law of restitution: $\frac{0-(-V)}{u\cos\alpha - (-u\sin\alpha)} = e$ $\left[e = \frac{v_2 - v_1}{u_1 - u_2}\right]$

$$\Rightarrow V = e(u\cos\alpha + u\sin\alpha) \text{ --- (ii)}$$

from (i) and (ii) → $-u\cos\alpha + u\sin\alpha = eu\cos\alpha + eu\sin\alpha$

$$\Rightarrow \sin\alpha(u - eu) = \cos\alpha(u + eu)$$

$$\Rightarrow \tan\alpha = \frac{1+e}{1-e} \checkmark \text{ --(iii)}$$

(b) from (iii) $\frac{1+e}{1-e} = \tan\alpha = 2$ (given)

$\left\{ \tan\alpha = 2 \Rightarrow \left\{ \sin\alpha = \frac{2}{\sqrt5},\ \cos\alpha = \frac{1}{\sqrt5} \right. \right.$

$$\Rightarrow 2(1-e) = 1+e \Rightarrow e = \frac{1}{3} \checkmark$$

from (ii) $V = e(u\cos\alpha + u\sin\alpha)$

$$V = \frac{1}{3}\left(u \times \frac{1}{\sqrt5} + u \times \frac{2}{\sqrt5}\right) = \frac{3u}{3\sqrt5} \checkmark$$ = speed of A along line of centre

$\therefore$ Speed of A $= \sqrt{\left(\frac{u}{\sqrt5}\right)^2 + \left(\frac{2u}{\sqrt5}\right)^2} = u \checkmark$ $\left\{ \text{and speed of A, perp to line of centre} = u\sin\alpha = \frac{2}{\sqrt5}u \text{ (remains same)} \right.$

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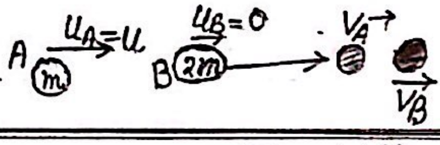
11. Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. ---[3]

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = 3/4$. The coefficient of restitution between B and the wall is $2/3$. Immediately after B collides with the wall, the K.E. of A is $5/32$ of the kinetic energy of B.

(b) Find the possible values of e. [W-20/31/Q6] ---[7]

Solution: using conservation of momentum,



(a) $m \cdot u + 0 = m \cdot V_A + 2m \cdot V_B$

$\Rightarrow \quad V_A + 2V_B = u$ --- (i)

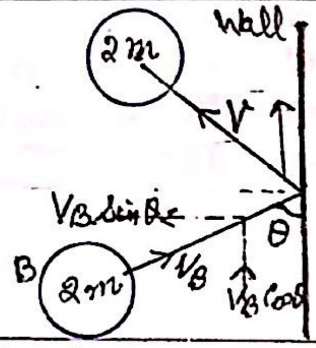
using law of restitution:

$\dfrac{V_B - V_A}{u_A - u_B} = e \Rightarrow \dfrac{V_B - V_A}{u - 0} = e$

$\Rightarrow \quad V_B - V_A = eu$ --- (ii)

$V_B = \dfrac{u}{3}(e+1)$
$V_A = \dfrac{u}{3}(1-2e)$ } from (i) and (ii) --- (iii)

$\tan\theta = \frac{3}{4}$
$\sin\theta = 3/5$
$\cos\theta = \frac{4}{5}$
$e' = 2/3$



Let V is the velocity of B after collision with wall.

(b) Component of V along the wall $= V_B \cos\theta$

Component of V perp. to the wall $= e' V_B \cdot \sin\theta$

$\therefore$ Speed of B after collision with wall

$V = \sqrt{V_B^2 \cos^2\theta + e'^2 V_B^2 \sin^2\theta}$

$= V_B \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{2}{3}\right)^2 \times \left(\frac{3}{5}\right)^2}$

$V = \frac{u}{3}(e+1) \times \frac{2}{\sqrt{5}}$

$\therefore$ K.E of B $= \frac{1}{2} m V^2$

$= \frac{1}{2}(2m) \cdot \frac{u^2}{9}(e+1)^2 \times \frac{4}{5}$ --- (iv)

K.E of A $= \frac{1}{2} m V_A^2$

$= \frac{1}{2} m \cdot \frac{u^2}{9}(1-2e)^2$ --- (v)

Given K.E of A $= \frac{5}{32} \times$ K.E of B

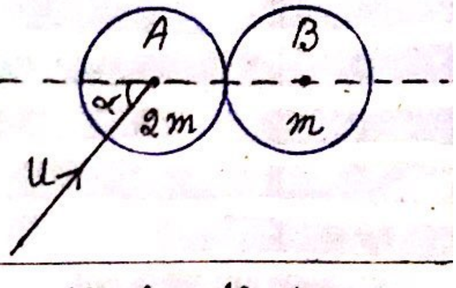
$\Rightarrow \frac{1}{2} m \frac{u^2}{9}(1-2e)^2 = \frac{1}{2} \times 2m \times \frac{4}{5} \times \frac{u^2}{9}(e+1)^2 \times \frac{5}{32}$

$\Rightarrow 4(1-2e)^2 = (e+1)^2$

$\Rightarrow 15e^2 - 18e + 3 = 0$

$\Rightarrow e = 1 \,;\, e = \frac{1}{5}$ ✓

12. Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u, and collides with B. Immediately before collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan\alpha = \frac{4}{3}$. The coefficient of restitution between the spheres is $\frac{1}{3}$.



Find the speed of A after the collision. --[5]

[W-20/32/Q2]

Solution: Let the speeds of A and B are v and w after collision respectively.

Using Law of conservation of momentum: along the lines of centres

$$2m\cdot v + m w = 2m\cdot u\cos\alpha + 0 \quad \text{(along the line of centres)}$$

or $$2v + w = 2u\cos\alpha \quad ---(i)$$

Using law of restitution:

$$\frac{w - v}{u\cos\alpha - 0} = e$$

$$\Rightarrow w - v = eu\cos\alpha \quad --(ii)$$

$\tan\alpha = \frac{4}{3}$, $\sin\alpha = \frac{4}{5}$, $\cos\alpha = \frac{3}{5}$

$e = \frac{1}{3}$

(i) − (ii) $\Rightarrow$ $v = \frac{1}{3}u\cos\alpha(2-e) = \frac{1}{3}u\cdot\frac{3}{5}\left(2-\frac{1}{3}\right) = \frac{1}{3}u$ ✓

and the vertical component of speed of A after the collision = (same as) vertical compnent of u = $u\sin\alpha = u\times\frac{4}{5} = \frac{4u}{5}$ ✓

$$\therefore \text{Speed of } A = \sqrt{\left(\frac{1}{3}u\right)^2 + \left(\frac{4}{5}u\right)^2} = \sqrt{\frac{169}{225}u^2}$$

$$= \frac{13}{5}u \checkmark$$

$$= \underline{0.867u} \checkmark$$

Scanned with CamScanner

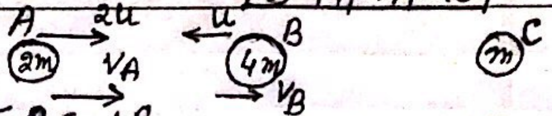
13. Three uniform small spheres A, B and C have equal radii and masses 2m, 4m and m respectively. The spheres are moving in a straight line on a smooth horizontal surface, with 'B' between 'A' and 'C'. The coefficient of restitution between each pair of spheres is 'e'. Spheres 'A' and 'B' are moving towards each other with speeds 2u and u respectively. The first collision is between A and B.

(i) Find the velocity of A and B after this collision. ---[3]

Sphere C is moving toward B with speed $\frac{4}{3}u$ and now collides with it. As a result of this collision, B is brought to rest.

(ii) Find the value of e. ---[4]

(iii) Find the total K.E lost by the three spheres as a result of two collisions. [S-19/21/Q3] ---[3]



Solution: Using conservation of momentum for A and B.

(i) $2m.V_A + 4m.V_B = 2m.2u - 4m.u$

$\Rightarrow V_A + 2V_B = 0$ --- ①

Using New's law of Restitution:

$\dfrac{V_B - V_A}{2u-(-u)} = e \Rightarrow V_B - V_A = 3eu$ --- ②

from ① and ② $V_A = -2eu$; $V_B = eu$ ✓ ✓

(ii) B (4m) $\xrightarrow{eu}$ $\xleftarrow{\frac{4u}{3}}$ (m) C

$\xrightarrow{V_B'}$ $\xrightarrow{V_C}$

Using conservation of momentum

$4m.V_B' + mV_C = 4mV_B - m\cdot\frac{4}{3}u$

$V_C = (4V_B - \frac{4u}{3})$ ... ③ [Given $V_B' = 0$]

Using Restitution: $V_C - V_B' = e(V_B + \frac{4u}{3})$

$\Rightarrow V_C = e(V_B + \frac{4}{3}u)$ --- ④

from ③ & ④ $4V_B - \frac{4}{3}u = eV_B + \frac{4}{3}ue$

from Part (i) $V_B = eu \rightarrow 4eu - \frac{4}{3}u = e.eu + \frac{4}{3}eu$

$\Rightarrow 4e - \frac{4}{3} = e^2 + \frac{4e}{3}$ ✓

$\Rightarrow 3e^2 - 8e + 4 = 0 \Rightarrow e = \frac{2}{3}$; ~~$e = 2$~~

$(\because 0 \le e \le 1)$

(iii) for A, K.E loss

$V_A = -2eu = -2u\times 2/3 = -\frac{4u}{3}$

$= \frac{1}{2}\times 2m(2u)^2 - \frac{1}{2}2m\left(\frac{4u}{3}\right)^2$

$= \frac{1}{2}\times 2m\left[4u^2 - \frac{16u^2}{9}\right]$

$= \frac{20}{9}mu^2$ --- ⑤

for B,

Loss of K.E $= \frac{1}{2}4m.u^2 - 0$

$= 2mu^2$ --- ⑥

for C, Loss of K.E (→ Part (ii))

$= 0$ [as $u_C = V_C = \frac{4u}{3}$] ⊗
(same)

∴ Total Loss of K.E (from ⑤ & ⑥)

$= \frac{20}{9}mu^2 + 2mu^2$

$= \frac{38}{9}mu^2$ ✓

⊗ $V_C = e[V_B + \frac{4}{3}u]$ (from ④)

$= e[eu + \frac{4}{3}u]$ {for ② $V_B = eu$}

$= 2/3.[2/3u + 4/3u] = \frac{4}{3}u$

14. A particle of mass 'm' is able to move in a vertical circle on the Smooth inner surface of a Sphere with centre O and radius 'a'. Points A and B are on the inner surface of the sphere and AOB is a horizontal diameter. Initially, 'P' is projected vertically downwards with speed $\sqrt{\frac{21ag}{2}}$ from A and begin to move in vertical circle.

At the lowest point of its path, vertically below 'O', the particle 'P' collides with a stationary point 'Q', of mass '4m', and rebounds. The speed acquired by Q, as a result of the collision, is just sufficient for it to reach the point B.

(i) Find the speed of P and speed of Q immediately after their collision. ---[7]

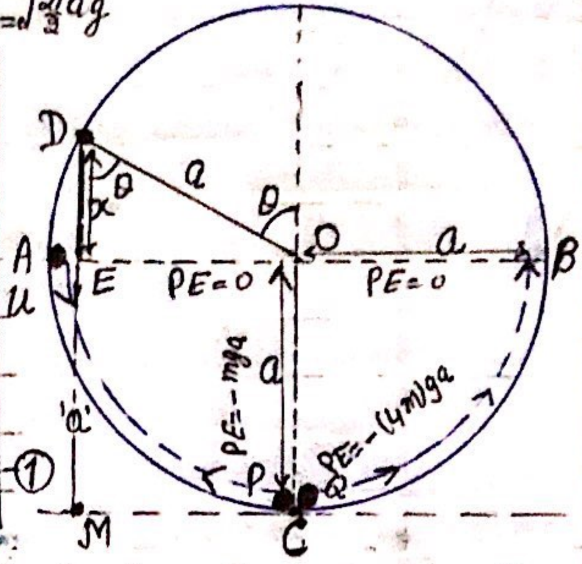
In the subsequent motion, P loses contact with the inner surface of the sphere at the point D, where the angle between OD and the upward vertical through O is θ.

(ii) Find Cosθ. [S-19/21/Q11] ---[5]

Solution on the next Page.

14 → "Continued from P-12"

Solution: $m_P = m$, $m_Q = 4m$, Initial speed of P, $u = \sqrt{\frac{21}{2}ag}$



(i) C is the lowest point of the circle, let the speed of P before collision $= U_P$

Using Energy method for 'P' (A → C):

$\frac{1}{2}mU_P^2 - \frac{1}{2}mu^2 = 0 - (-mga)$

(gain in K.E.) (Loss in P.E)

$\Rightarrow \frac{1}{2}mU_P^2 = \frac{1}{2}mu^2 + mga$

$\Rightarrow \frac{1}{2}U_P^2 = \frac{1}{2}\cdot\frac{21}{2}ag + ga$

$\Rightarrow U_P^2 = \frac{25}{2}ag \Rightarrow U_P = \frac{5}{\sqrt{2}}\sqrt{ag}$ — (1)

Using Energy method for 'Q' (C → B)

Loss in K.E = Gain in P.E. [Let the speeds of P and Q after the collision are V_P and V_Q

$\frac{1}{2}(4m)V_Q^2 = 0 - (-4mga)$

$\Rightarrow V_Q = \sqrt{2ag} = \sqrt{2}\cdot\sqrt{ag}$ — (2)

Now for 'P', using conservation of momentum:

$m\cdot U_P + 4m\times 0 = -m\cdot V_P + 4m\cdot V_Q$

$\Rightarrow V_P = 4V_Q - U_P = (4\sqrt{2} - \frac{5}{\sqrt{2}})\sqrt{ag}$

$V_P = \frac{3}{\sqrt{2}}\sqrt{ag} = 2.12\sqrt{ag}$ ✓ — (3)

(ii) Now let the speed of 'P' at D is 'W_P' when it loses contact with the surface, or Reaction at D = 0

Let $DE = x = a\cos\theta$ ✓

$DM = EM + DE = (a + a\cos\theta)$

Using conservation of energy at D.

$\frac{1}{2}mW_P^2 + (R=0) = \frac{1}{2}mV_P^2 - mg(a + a\cos\theta)$

$\Rightarrow W_P^2 = V_P^2 - 2ga(1+\cos\theta)$

$= \left(\frac{3}{\sqrt{2}}\sqrt{ag}\right)^2 - 2ag(1+\cos\theta)$

$= (\frac{5}{2} - 2\cos\theta)ag$ — (4)

Now Reaction at D, $R_D = \frac{mW_P^2}{a} - mg\cos\theta = 0$ (Loses contact)

$\rightarrow W_P^2 = ag\cos\theta$ — (5)

from (4) and (5)

$(\frac{5}{2} - 2\cos\theta)ag = ag\cos\theta$

$\therefore \cos\theta = \frac{5}{6}$ ✓ (or 0.833)