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Probability and Statistics

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Permutions and Combinations

Notes

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Permutations and Combinations Notes

§ Factorial function:

The product: $5\times4\times3\times2\times1$ in short written as $5!$, called 5 factorial.

Also $7! = 7\times6\times5\times4\times3\times2\times1 = 5040$

In general: factorial $n \to n! = n(n-1)(n-2)\cdots 3\cdot2\cdot1$, for any integer $n>0$

and $0! = 1$.

Now let us see certain calculations:

(i) $6\times5! = 6!$ ($\because 6\times5\times4\times3\times2\times1 = 6!$)

(ii) $\frac{7!}{5!} = \frac{7\times6\times5!}{5!} = 7\times6 = 42$

(iii) $4! + 3! \neq 7!$

(iv) $\frac{3}{n!} - \frac{1}{(n+1)!} = \frac{3(n+1)-1}{(n+1)!} = \frac{3n+2}{(n+1)!}$

(V) Find the smallest value of n: $n! > 100{,}000$

$8! = 40{,}320 < 100{,}000$ but $9! = 362{,}880 > 100{,}000 \Rightarrow n = 9$ ✓

(Vi) Find the largest value of n: $n! < 10{,}00{,}000$

$9! = 3{,}62{,}880$; $10! = 36{,}28{,}880 > 10{,}00{,}000 \Rightarrow n = 9$ ✓

Permutations. (Arrangements)

§ Permutations:

(1) Permutations of n different objection taken all at a time is '$n!$'

(i) How many different words with 2 alphabets A and B $< \begin{matrix} AB \\ BA \end{matrix} = 2\times1$

(ii) How many different words with 3 alphabet (with or without meaning)
A, B, C

$$\left.\begin{matrix} A < \begin{matrix} B - C \\ C - B \end{matrix} \\ B < \begin{matrix} A - C \\ C - A \end{matrix} \\ C < \begin{matrix} A - B \\ B - A \end{matrix} \end{matrix}\right\} 6 \text{ words} = 3\times2\times1 = 3!$$

(iii) Let A, B, C are three friends, how many ways we can have a photograph of A, B, C. all standing in a row.

$$\underset{\text{Ist}}{3} \times \underset{\text{IInd}}{2} \times \underset{\text{3rd}}{1} = 3! = 6 \text{ ways}$$

At the first place any one of the '3' may stand, at the second place any one of the remaining 2, and at the last place the remaing 1.

(iv) In General if 'n' different objects, all of them, to be arranged in a line:

$$\underset{\text{Ist}}{n} \times \underset{\text{IInd}}{(n-1)} \times \underset{\text{IIIrd}}{(n-2)} \times \cdots\cdots \times 3 \times 2 \times \underset{\text{last}}{1} = n!$$

§ Permutation of n different objects taken all at a time: is denoted by: $^nP_n = n!$ ✓

Example 1: How many different way can four girls and three boys sit in row. → $^7P_7 = 7! = 5040$ ✓

§ Permutation of n objects with repetitions:

The number of permutations of n objects of which, p are of one type, q are of another type and r of another type:---

$$= \frac{n!}{p! \times q! \times r! \cdots}$$

Example 2 (i) Find the number of ways in which all nine letters of the word TENNESSEE can be arranged if there is no restriction. ---[1]

(ii) If the 'T' is at one end; there is an 'S' at the other end. --[2]

[SP-17/06/Q5]

Solution: TENNESSEE

(i) Number of arrangements (with no restriction) $= \frac{9!}{4! \times 2! \times 2!} = \underline{3780}$ ✓

E's = 4
N's = 2
S's = 2
T = 1
Total = 9

(ii) T(ENNE~~S~~SEE)S

Now T and S are fixed but may be interchanged S(- - - -)T → 2 way

∴ Number of arrangements;

$$= \frac{7!}{4!\,2!} \times 2 = \underline{210} ✓$$

Rest of the 7 letters.
E's = 4
N's = 2

Example 3: Find the number of ways all 9 letters of the word 'EVERGREEN' can be arranged if; (i) there are no restrictions. --(1)

(ii) the first letter is R and the last letter is G --[2]

[S-16/63/Q6]

Solution (i) Number of arrangement (with no restrictions) $= \frac{9!}{4! \times 2!} = \underline{7560}$ ✓

EVERGREEN
E-4, V=1, G=1, N=1
R-2
Total 9

(ii) R(EVE~~R~~~~G~~REEN).G

(R and G are not interchanged)

$$\frac{7!}{4!} = \underline{210} ✓$$

E's = 4, V, R, N = 7

Example 4: Find the number of different ways the 7 letters of the word BANANAS can be arranged if all the letters A are next to each other. ---[3]

[S-15/62/Q6]

Solution: one AAA BNNS

"BANANAS": A - 3 -1, N - 2, B - 1, S - 1 — 5

All three A's together.

Number of arrangements $= \frac{5!}{2!} = 60$

Example 5: The 11 letters of the word 'REMEMBRANCE' are arranged in a line. Find the number of different arrangements which do not have all 4 vowels (E,E,A,E) next to each other. --[3]

[W-13/62/Q6]

Solution: Arrangements any way:

"REMEMBRANCE": E - 3, R - 2, M - 2; B, N, C, A - 1 each — Total 11

$= \frac{11!}{3! \times 2! \times 2!} = 1,663,200$ ---(i)

Now 4 vowels (E, E, A, E) one together

Then (EEAE) one, R, R, M, M, B, N, C → Total → 8

4 vowels are together → arrangements $= \frac{8!}{2! \times 2!} \times \frac{4!}{3!}$ ← (EEEA) arranged.

$= 40320$ ---(ii)

∴ Number of arrangements in which (AEEE) all four not next to each other $= 1,663200 - 40320$ {from (i) & (ii)

$= 1,622,880$

Example 6: Find the number of different way that the 9 letter of the word 'AGGREGATE' can be arranged if the '3 letters G' are together and 'both the letters E' are together and "both A" also [W-13/63/Q6] --[2]

AGGREGATE - 9: A - 2, G - 3, E - 2, R - 1, T -

Solution: one + one + one. (GGG), (EE), (AA), R, T → Total 5

∴ Number of arrangements $= 5!$

$= 120$

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Example 7: How many different arrangements are there of the 11 letters in the word 'REQUIREMENT', in which there are exactly three letters between two R's. ---[3]

[S-21|53|Q6(C)]

Solution: (R - - - R) - - - - - - -
I II III IV V VI VII

'REQUIREMENT'
R-2 (Total-11)
E-3
Rest 6 one each

Group of five letters (R---R) can take 7 different positions.

Now 2R's are fixed, the other 9 letters in be arranged $= \frac{9!}{3!}$

$\therefore$ Total number of arrangements $= \frac{9!}{3!} \times 7 = 423360$ ✓

Example 8: Find the number of arrangements that can be made out of all 9 letters in the word 'CAMERAMAN', such that there is exactly one letter between two M's. ---[4]

[SM-19|62|Q7(iii)]

Solution: (M - M) - - - - - - (7 Position)
1st II III IV V VI VII

"CAMERAMAN" — 9
A-3
M-2
Rest 4 one each.

here Group of three letters (M-M) can take 7 different positions.

Now 2M's are fixed, the other 7 letters can be arranged in $= \frac{7!}{3!}$ ways

$\therefore$ Total number of arrangements $= \frac{7!}{3!} \times 7 = 5880$ ✓

Example 9: Find the number of different ways that the 9 letters of the word "AGGREGATE" can be arranged in a line if the consonants and vowels occur alternately [W-13|63|Q6(iii)] -[3]

Solution: C $\overset{V}{\uparrow}$ C $\overset{V}{\uparrow}$ C $\overset{V}{\uparrow}$ C $\overset{V}{\uparrow}$ C

"AGGREGATE" — 9
Consonants (5) G-3, R-1, T-1
Vowels E-2, A-2

5 consonants can be arranged $= \frac{5!}{3!}$

4 vowels in the gap between them $= \frac{4!}{2!2!}$

$\therefore$ Total number of arrangements $= \frac{5!}{3!} \times \frac{4!}{2!2!} = 120$ ✓

Example 10: Find the number of numbers of 5 digits that can be formed with the digits 0, 1, 2, 3, 5, if the digits can be repeated in the same number.

Solution: '0' cannot be put in the 'Ten thousands' place → hence 'Ten thousands' place can be filled in 4 ways. Since the repetition of digits is allowed, there for each of the other places can be filled in 5 ways.

$\underset{Ten\,Th.}{4} \times \underset{Th}{5} \times \underset{H}{5} \times \underset{Ten}{5} \times \underset{U}{5}$

∴ Required number of numbers $= 4 \times 5 \times 5 \times 5 \times 5 = \underline{2500}$ ✓

Example 11. Find how many different numbers can be made by arranging all nine digits of the number '223677888' if:

(i) there are no restrictions. --[2]

(ii) the number made is an **even** number. --[4]

[S-15/61/Q7]

Solution: (i) Required Number $= \dfrac{9!}{2! \times 2! \times 3!}$
(No restrictions)

$= \underline{15,120}$ ✓ ---(i)

Number "223677888" has 9 digits
2 – 2 time
8 – 3 times } even digit
6 – 1 time
7 – 2
3 – 1 } odd digit

(ii) Number of even numbers = Total no − odd no's --(ii)

Let us find the no. of odd numbers

(a) 3 in the unit place: $\underbrace{\times \times \times \times \times \times \times \times}_{\text{(8 place)}}\, 3 = \dfrac{8!}{2! \times 2! \times 3!} = 1680$ ✓

(b) odd no's with 7 in unit place. $\underbrace{\times \times \times \times \times \times \times \times}_{\text{(8 Places)}}\, 7 = \dfrac{8!}{2! \times 3!} = 3360$ ✓

∴ Total odd no's = 1680 + 3360
= $\underline{5040}$ ✓ ----(iii)

from (i), (ii), (iii)

∴ Required no. of even numbers = 15,120 − 5040
= $\underline{10,080}$ ✓

Example 12: Find how many five-digit numbers can be made using the digits 2, 3, 4, 5 and 6 once each if:
(a) there are no restrictions
(b) the five-digits number must be:
(i) odd (ii) even (iii) odd and less than 40000.

Solution (a) five digits 2,3,4,5,6 can be arranged any way = $^5P_5 = 5! = \underline{120}$ ✓

(b) (i) odd number: $\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underset{\text{Unit}}{2(3 \text{ or } 5)}$
3 or 5 in unit place; any four in four places = $4! \times 2 = 48$ ✓

(ii) Even number: 2,4,6 unit place. $\underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underset{3}{(2 \text{ or } 4 \text{ or } 6)}$
$= \underline{72}$ ✓

(iii) For odd no – 3 or 5 in the units place.
and must begin with 2 or 3 as number is less than 40,000
and the middle three any way → 3!
Case I (2)(— — —)(3) $= 1 \times 3! \times 1 = 6$ ✓
II (begin 2 or 3) ends with 5 = (3 or 2) $2 \times 3! \times 1$ (5) $= \underline{12}$ ✓ --(ii)
∴ Req. numbers = 6 + 12 = $\underline{18}$ ✓

Example 13: Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5, (i) If digits are not repeated --[2]
(ii) If the digits can be repeated and the number formed is odd. --[3]
[S-17 / b3 / Q6]

Solution: 3000 < Number < 5000; the number has 4 digits.
i) (At thousand's place 3 or 4) → $\underset{Th,}{2 \times} (\underset{H}{4} \times \underset{Ten}{3} \times \underset{Unit}{2})$ ← (Rest 4)
$= \underline{48}$ ✓

(ii) {at units place 1, 3, 5 for an odd No; at Thousand's place (3 or 4) (digits can be repeated)} ⇒ $\underset{Th.}{2} \times \underset{H}{5} \times \underset{T}{5} \underset{U}{3}$ (1,3,5)
$= \underline{150}$ ✓

Example 14: The digits 1, 3, 5, 6, 6, 6, 8 can be arranged to form many different 7-digit numbers.

(i) How many of the 7-digit numbers have all the even digits together and all odd digits together? ---[3]

(ii) How many of the 7-digit numbers are even? ---[3]

[M-18/62/Q6]

Solution: (1, 3, 5) (6, 6, 6, 8)

{Odd → 1, 3, 5
{even – 6, 6, 6, 8

(i) 7-digit No. $= 3! \times \frac{4!}{3!} \times 2$ (even/odd can be interchanged)

$= \underline{48}$ ✓

(ii) For even numbers unit digit should be even

{No. ending 8 $= \frac{6!}{3!} = 120$
{No. ending 6 $= \frac{6!}{2!} = 360$

Total even $= 120 + 360 = \underline{480}$

Example 15: Find the number of different 3-digit numbers greater than 300, that can be made from the digits 1, 2, 3, 4, 6, 8 if

(i) no digits can be repeated ---[3]

(ii) a digit can be repeated and the number made is even. ---[3]

[W-17/62/Q6(a)]

Solution: 3 digit number > 300 | digits → 1, 2, 3, 4, 6, 8

(i) At hundred place digit ≥ 3, (3, 4, 6, 8)

$\underset{H}{4} \times \underset{T}{5} \times \underset{U}{4} = \underline{80}$ ✓

(ii) A unit's place any even digit (2, 4, 6, 8),
At Ten's place → any 6,
At Hundred's place ≥ 3 (3, 4, 5, 6)

$\underset{H}{4} \times \underset{T}{6} \times \underset{U}{4} = \underline{96}$ ✓ (Repetition is allowed)

Example 16: 'Mr and Mrs Keene' and their '5 children' all go to watch a football match, together with their friends 'Mr and Mrs Uzuma' and their '2 children'. Find the number of ways in which all 11 people can line up at the entrance in each of the following cases:

(i) Mr Keene stands at one end of the line and Mr Uzuma stands at the other end. --[2]

(ii) The five Keene children all stand together and the Uzuma children both stand together. --[3]

[S-19/63/Q3]

Solution (i) $K\underbrace{\times \times \times \times \times \times \times \times \times}_{\text{9 people any way}}U$; K and U can interchang

$= 9! \times 2 = 725760$ ✓

(ii) ① — ② — Rest④ as if total six

$(K_1 K_2 K_3 K_4 K_5)$ $(U_1 U_2)$ $\times$ $\times$ $\times$ $\times$

Can be rearranged among them selves.

$\therefore 6! \times 5! \times 2! = 172800$ ✓

Example 17: 'Mr and Mrs Baker' with their three children and 'Mr and Mrs Ahmed' with their two children, are visting an activity centre together. All 9 people stand in a line.

(i) Find the number of arrangements (different) in which Mr Ahmed is not standing next to Mr Baker. ---[3]

(ii) Find the number of different arrangements in which there is exactly one person between Mr Ahmed and Mr Baker. ---[3]

[W-20/52/Q6]

Solution: Total number of different arrangement of 9 persons any way $= 9!$ --(i)

(i) Number of arrangement when Mr Ahmed & Mr Baker are together:

① (Ah & Ba) $\times \times \times \times \times \times \times$ ⑦ more $= 8! \times 2$ (as Ah & Ba can interchange) --(ii)

$\therefore$ No of way when Ah. & Ba. are not together $= 9! - 8! \times 2$

$= 282240$ ✓

(ii) (Ahmed, —, Baker), —, —, —, —, —, — 7 Positions

$= 7! \times 7 \times 2$ (Ah & Ba can interchange) $= 70560$ ✓

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§ Permutation of n distinct object taken r at time: $r \leq n$

Let us see that how many ways seven letters A, B, C, D, E, F and G, can be arranged take 3 at a time.

$$\underline{7} \times \underline{6} \times \underline{5} = \frac{7 \times 6 \times 5 \times 4!}{4!} = \frac{7!}{4!} = \frac{7!}{(7-3)!}$$

denoted by:

$$^{7}P_{3} = \frac{7!}{(7-3)!}$$

In General:

$$^{n}P_{r} = \frac{n!}{(n-r)!} \qquad r \leq n$$

Example 16: Find how many permutations there are of:
(i) five from seven distinct objects.
(ii) four from nine distict objects.

Solution (i) $r = 5,\ n = 7;\ ^{7}P_{5} = \frac{7!}{(7-5)!} = \frac{7!}{2!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1}$
$= 2520$

(ii) $^{9}P_{4} = \frac{9!}{(9-4)!} = \frac{9!}{5!} = 9 \times 8 \times 7 \times 6 = 3024$

Example 17: Numbers are formed using some or all the digits 4, 5, 6, 7, with no digits being used more than once.
(i) How many odd numbers can be formed using exactly 3 digit. --[3]
(ii) Find how many odd numbers altogether can be formed. --[3]
[W-16 | 63 | Q3]

Solution: Using digits 4, 5, 6, 7

(i) For odd numbers - 5 or 7 can be there at the units place.
For 3 digit numbers:

$$\underset{H}{3} \times \underset{T}{2} \times \underset{\substack{U \\ (5 \text{ or } 7)}}{2}$$

$= \underline{12}$ odd numbers

(ii) 1 digit odd number = 2 (5 or 7)
2 digits odd number = $\underset{T}{3} \times \underset{U}{2} = 6$
3 digits odd number = $3 \times 2 \times \underset{(5 \text{ or } 7)}{2} = 12$
4 digits odd Number = $3 \times 2 \times 1 \times 2 = 12$
Total odd Numbers $= 2 + 6 + 12 + 12$
$= 32$

Example 18: Freddie has 6 toy cars and 3 toy buses, all different. Freddie arranges these 9 toys in a line.

(i) Find the number of these arrangements if the buses are all next to each other. ---[3]

(ii) Find the numbers of possible arrangements if there is a car at each end of the line and no buses next to each other. --[3]

S-19/61/Q8

Solution:

(i) $\overset{\text{one}}{(B_1, B_2, B_3)}, C_1, C_2, C_3, C_4, C_5, C_6 \rightarrow$ as if 7 objects

can mutually be arranged.

$\therefore$ Number of arrangements $= 7! \times 3! = \underline{30240}$ ✓

(ii) $C_1 - C_2 - C_3 - C_4 - C_5 - C_6$

Two car at the ends and four in the middle, all '6' can be arranged $= {}^6P_6 = 6!$ ✓ ——(i)

as no buses are next to each other. So buses in the five gaps between the cars $\rightarrow$ 3 buses can be arranged $= {}^5P_3$ ——(ii)

$\therefore$ Total number of arrangements $= 6! \times {}^5P_3 = \underline{43200}$ ✓

Example 19: Rachel has 3 types of ornaments. She has 6 different wooden animals, 4 different sea shells and 3 different pottery ducks.

Rachel displays 10 of the 13 ornaments in a row on her window-sill.

Find the numbers of different arrangements that are possible if:

(i) She has a duck at each end of the row and no ducks anywhere else. --[3]

(ii) She has a duck at each end of the row and wooden animals and sea-shells are placed alternatively in the positions in between. --[3]

S-15/63/Q7

Solution (i) 2 ducks out of 3 at the end.

$\underbrace{D_1 \times \times \times \times \times \times \times \times D_2}_{(8 \text{ out of } 10)}$

$= {}^3P_2 \times {}^{10}P_8 = \underline{10886400}$ ✓

(ii) D S W S W S W S W D

$D \rightarrow {}^3P_2$

$S \rightarrow {}^4P_4$

$W - {}^6P_4$

SW can swap in 2 ways.

Total ways $= {}^3P_2 \times {}^4P_4 \times {}^6P_4 \times 2 = \underline{103680}$ ✓

Example 20(i) Find the number of different arrangements of all nine letters of the word "PINEAPPLE", if no vowel (A,E,I) is next to another vowel. ---[4]

(ii) if all 4 vowels (A,E,E,I) are not next to each other, ---[3]

[W-16/61/Q5]

Solution (i) Vowels (A, E, E, I) are not next to each other.

"PINEAPPLE"
A-1, E-2, I-1 } 4 vowels
P-3, N-1, L-1 } 5 Consonants

• P • N • P • P • L •

• 4 Vowels may occupy the six spaces $= \frac{^6P_4}{2!}$ ----(i)   (2E) → 2!

Consonants may be arranged $= \frac{5!}{3!}$ ---(ii)   3! → (3P)

∴ Total number of arrangements $= \frac{^6P_4}{2!} \times \frac{5!}{3!} = \underline{3600}$ ✓ (from (i) and ii)

(ii) Number of arrangements of all nine letters (any way) $= \frac{9!}{3!\cdot 2!}$ ---(iii)

Number of arrangements when 'all 4' vowels are to together (AEEI) one PNPPL = as if six $= \frac{6!}{3!} \times \frac{4!}{2!}$ ---(iv)

Hence the number of arrangements when 'all 4' vowels are not together $= \frac{9!}{3!\cdot 2!} - \frac{6!}{3!} \times \frac{4!}{2!} = \underline{28800}$ ✓

Example 21: Find the number of ways the 9 letters of the word 'SEVENTEEN' can be arranged (i) one of the E's is in centre with '4' letters on either side. ---[2]

(ii) NO E is next another E. ---[3]

[S-18/63/Q7]

Solution (i) • • • • E • • • •

'SEVENTEEN'-9
E-4
N-2
S, V, T one each.

$\frac{8!}{2!\,3!} = \underline{3360}$ ✓

(ii) ↑•↑•↑•↑•↑•↑
E E E E E E

'N,N,S,V,T' 5 letters can be arranged any way $= \frac{5!}{2!}$ ---(i)

4E can be arranged in 6 alternate places (↑) $= {^6P_4}/4!$ ---(ii)

Total number of arrangements $= \frac{5!}{2!} \times {^6P_4}/4! = \underline{900}$ ✓

§ Combination (or Selection):

$r \le n$

When we select 'r' objects, in no particular order, from 'n' distinct objects, we call it combination of 'n' objects taken 'r' at a time and is denoted by ^nC_r

Now a combination of 'r' objects, which are then arranged in order, is equivalent to permutation.

or $\quad {}^nC_r \cdot r! = {}^nP_r \Rightarrow {}^nC_r = \frac{1}{r!} \times {}^nP_r = \frac{n!}{r!\,(n-r)!}$

Hence, $\quad {}^nC_r = \frac{n!}{r!\,(n-r)!} \quad : r \le n$

Note: (i) ${}^nC_0 = {}^nC_n = 1$ (ii) ${}^nC_1 = n$

(iii) ${}^nC_r = {}^nC_{(n-r)}$ $\left\{ \text{Example: } {}^{10}C_7 = {}^{10}C_3 \right.$

Example: ${}^{10}C_7 = \frac{10!}{7!\,3!} = \frac{10!}{3!\,7!} = {}^{10}C_3$ { as when we select 7 object out of 10, we reject 3 that time.

Example (i) ${}^7C_2 = \frac{7!}{2!\cdot 5!} = \frac{7\times 6}{2\times 1} = 21$ ✓

(ii) ${}^{10}C_3 = \frac{10!}{3!\,7!} = \frac{10\times 9\times 8\times \cancel{7!}}{3!\,\cancel{7!}} = \frac{10\times 9\times 8}{3\times 2\times 1} = 120.$ ✓

Comparing ^nP_r and ^nC_r:

(i) ${}^nC_1 = {}^nP_1 = n$

(ii) ${}^nC_n = 1$ // whereas ${}^nP_n = n!$

Example 22: Four letters are selected from nine letters of the word "VENEZUELA". Find the number of possible selections which contains exactly 1E. --[3]

[SP-17/06/Q5(b)]

Solution:

"VENEZUELA"
E – 3
6 letters all different.

$$4 \text{ letters to be selected} = \underset{\substack{\downarrow \\ 3E}}{1} + \underset{\substack{\downarrow \\ 6\text{ more}}}{3}$$

Now one E is selected

$$3 \text{ more out of } 6 = {}^{6}C_{3} = \frac{6!}{3!\,3!} = \boxed{20}$$

Example 23: A committee of '5' people is to be chosen from 4 men and 6 women. William is one of the 4 men and Mary is one is one of the 6 women. Find the number of different committees that can be chosen if william and Mary refuse to be on the committee together. --[3]

[W-16/63/Q1]

Solution: Men(4) – Women(6) – Total (10) persons

To be selected 5:

Now number of selection of 5 out of 10 any way $= {}^{10}C_{5} = 252$ --(i)

Now the number of selection 5 out of 10 when William and Mary are both on a committee → 3 out 8 $= {}^{8}C_{3} = 56$ --(ii)

∴ Number of committees in which william and Mary both are not together $= 252 - 56$ from (i) & (ii)

$= \underline{196}.$

Example 24: Three letters from the 9 letters of the word 'EVERGREEN' are selected.
(i) Find the number of selections which contain no E's and exactly 1R. --[1]
(ii) Find the number of selections which contain no Es. --[3]
[S-16/63/Q6]

Solution: (i) No E's and exactly one 'R'
To select - 3 → 2 more out of 3 (V, G, N)
$= {}^3C_2 = \underline{3}$ ✓

'EVERGREEN' - 9
E - 4
R - 2
V, G, N - 1 each

(ii) No E's → out of 2R, V, G, N - 1 each.
To be selected - 3; Case I → No R's → ${}^3C_3 = 1$ (out of V, G, N)
Case II → 2R's → ${}^3C_1 = 3$ (")
Case III → 1R → ${}^3C_2 = 3$ (")
Total = $\underline{7}$ ✓
Total number of selections = $\underline{7}$ ✓

Example 25: Four letters are selected from the nine letters of the word "VENEZUELA". Find the number of possible selections which contain exactly one E. ---[3]
[W-15/61/Q5(b)]

Solution: Exactly one E is selected out of 4.
∴ 3 more to be selected out remaining 6.
∴ Number of selections $= {}^6C_3 = \underline{20}$ ✓

"VENEZUELA"
E - 3
Rest 6 all different.

Example 26: 4 letters from the word "REMEMBRANCE" are chosen. Find the number of different selections which contains no M's and no R's and atleast 2E's. [W-13/62/Q6] --[3]

Solution: Now 2E and 2 out of 4 diff $= {}^4C_2 = 6$ ✓
and 3E and 1 out of 4 diff. ${}^4C_1 = \underline{4}$
$\underline{10}$
∴ Total selections = $\underline{10}$ ✓

"REMEMBRANCE"
M - $2^{\times}$
R - $2^{\times}$
E - 3 (atleast 2)
and Remaining 4 one each (B, A, N, C)

Example 27: Donna has '2' necklaces, '8' rings and '4' bracelets, all different. She chooses 4 pieces of jewellery. How many possible selections can she make if she chooses at least '1' necklace and at least 1 bracelet. [S-18/62/Q6]--[4]

Solution: N(2) R(8) Br(4) ; To be chosen 4 → Atleast 1N and Atleast 1B

Total 4:

$1 - 2 - 1 = {}^{2}C_1 \times {}^{8}C_2 \times {}^{4}C_1 = 224$

$1 - 1 - 2 = {}^{2}C_1 \times {}^{8}C_1 \times {}^{4}C_2 = 96$

$1 - 0 - 3 = {}^{2}C_1 \times {}^{8}C_0 \times {}^{4}C_3 = 8$

$2 - 1 - 1 = {}^{2}C_2 \times {}^{8}C_1 \times {}^{4}C_1 = 32$

$2 - 0 - 2 = {}^{2}C_2 \times {}^{8}C_0 \times {}^{4}C_2 = 6$

Total $= 366 = 366$ ✓

Example 28: Three letters are selected from the '9' letters of the word 'CAMERAMAN'.

(i) Find the number of different selections if the three letters include exactly one M and exactly one A. ---[1]

(ii) Find the number of selections if the three letters include at least one M. [M-19/62/Q7]--[3]

Solution (i) 3 letter are selected out of 9
exactly one 'M' & exactly one 'A'
MA → 1 out of remaining $4 = {}^{4}C_1 = 4$ ✓

'CAMERAMAN'
A-3
M-2
C, E, R, N – 1 each.

(ii) Atleast one M

$M - - = {}^{4}C_2 = 6$ (2 out of 4 - C, E, R, N)

$MM - = {}^{4}C_1 = 4$ → (1 out of 4, C, E, R, N)

$MMA = 1$

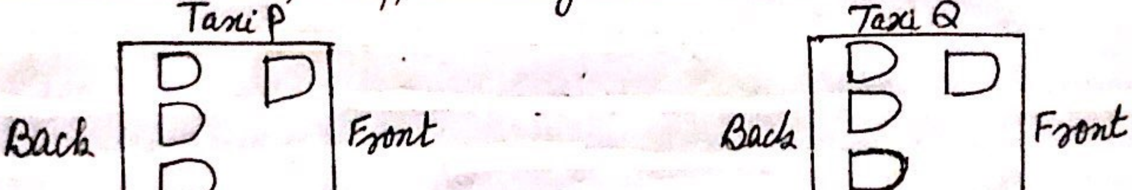
$MAA = 1$

$MA - = {}^{4}C_1 = 4$

Total $= 16$ ✓

Example 29: A group of 8 friends travel to the airport in two taxies, P and Q. Each taxi can take 4 passengers.

(a) The 8 friends divide themselves into two groups of 4, one group for taxi P and group for taxi Q, with Jon and Sarah travelling in the same taxi.
Find the number of different ways in which this can be done --[3]



Each taxi can take 1 passenger in front and 3 passengers in the back, Mark sits the front of taxi P and Jon and Sarah sit in the back of taxi P next to each other.

(b) Find the number of different seating arrangements that are now possible for the 8 friends. [SP-20/05/Q6] --[4]

Solution: Jon and Sarah both may sit in taxi 'P or Q' = 2 ways ---(i)

(a) Now out of remaining 6, two may sit in Jon and Sarah Taxi = ${}^6C_2$ ways.
Rest 4 may sit in another taxi any way = ${}^4C_4$
$\therefore$ The total number of ways $= 2 \times {}^6C_2 \times {}^4C_4 = \underline{30 \text{ ways}}$ ✓

(b) In taxi P → Mark sits on front seat = 1 way ✓
Sarah & Jon sit at back next to each other

may sit in 2 ways ✓
but they can rearrange themselves = 2 ways ✓
and the remaining 1 seat will be taken by one of remaining 5 = ${}^5P_1$ ways ✓
For Taxi P → $1 \times (2 \times 2) \times {}^5P_1 = \underline{20 \text{ ways}}$ ✓

Now the remaining 4 may sit any way in Taxi Q = ${}^4P_4 = 4! = \underline{24}$ ✓
$\therefore$ The total number of seating arrangement $= 20 \times 24$
$= \underline{480}$ ✓

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Example 30: Raman and Sanjay are members of a quiz team which has 9 members in total. Two photographs of the quiz team are to be taken.

For the first photograph, 9 members will stand in a line.

(a) How many different arrangements of the 9 members are possible in which Raman will be at the centre of the line? ---[1]

(b) How many different arrangements of the 9 members are possible in which Raman and Sanjay are not next to each other. ---[3]

For the second photograph, the members will stand in two rows, with 5 in the back row and 4 in the front row.

(c) In how many different ways can the 9 members be divided into a group of 5 and a group of 4.? ---[2]

(d) For a random division into a group of 5 and a group of 4, find the probability of Raman and Sanjay are in the same group as each other

W-21/51/Q5 ---[4]

Solution: (a) × × × × Raman × × × ×

Raman in centre; remaining 8 may be any positions $= 8! = \underline{40320}$ ✓

(b) (Raman & Sanjay) & 7 more = as if 8.

Arrangements if (Raman and Sanjay) are together $= 8! \times 2$ ---(i)

9 members (any way) $= 9!$ ---(ii)

∴ Arrangement if Raman and Sanjay are not together $= 9! - 8! \times 2$

$= \underline{282240}$ ✓

(c) To divide 9 members into two groups of 5 and 4, $= {}^9C_5 \times {}^4C_4$

$= \underline{126}$ ✓

(d)

Case I: (Raman & Sanjay) are the Group of 5 $\rightarrow {}^7C_3 \times {}^4C_4 = 35$

Case II when (Raman & Sanjay) are in Group of 4 $\rightarrow {}^7C_2 \times {}^5C_5 = 21$

∴ Total $= 35 + 21 = 56$

∴ P(Sanjay & Raman are in one Group) $= \frac{56}{126} = \underline{0.444}$ ✓

Example 31: A group of 6 people is to be chosen from 4 men and 11 women.
(a) In how many different ways a group of 6 be chosen if it must contain exactly 1 man? --[2]
Two of the 11 women are sisters Jane and Kate.
(b) In how many different ways can a group of 6 be chosen if Jane and Kate cannot both be in the group. --[3]
[W-21|52|Q2]

Solution: M(4) and W(11) ; a Group of 6 to be chosen
(a) with exactly 1M & 5W = $^4C_1 \times {}^{11}C_5 = \underline{1848}$ ✓

(b) In a group of 11W & 4M (both Kate & Jane both are not in a Grp. of 6)
(i) None of the two ~~the two~~ sisters = (11+4−2) = $^{13}C_6 = 1716$
Jane is in the grp of 15 (but not Kate)
out 13 to be selected 5 = $^{13}C_5 = 1287$
Now Kate is in the grp (but Jane is not there) = $^{13}C_5 = 1287$
∴ Total number of possible grps = 1716 + 1287 + 1287 = $\underline{4290}$ ✓

Example 32: The 26 members of the local sports club include Mr and Mrs Khan and their son Abad. The club is holding a party to celebrate Abad's birthday, but there is only room for 20 people to attend.
In how many ways can the 20 people be chosen from the 26 members of the club, given that Mr and Mrs Khan and Abad must be included? [W-21|53|Q1] --[2]

Solution: out of 26 to be selected 20.
but 3 members "Mr and Mrs Khan with their son Abad" are there. Hence out of (26−3) to be selected are (20−3)
$= {}^{23}C_{17} = \underline{100947}$ ✓