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Pure Maths-1

Series (A.P and G.P)
Notes

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Series - A.P and G.P (Notes)

§ Arithmetic Progression (or Arithmetic Sequence) A.P :

Consider a sequence: 3, 7, 11, 15, 19, ---
is called an arithmetic progression.
Here we find that the difference of each term and the previous term is constant,
Called the common difference (of given A.P) and is denoted by 'd'.

$$\begin{cases} 7-3=4 \\ 11-7=4 \\ 15-11=4 \\ \cdots \text{ so on} \end{cases}$$

$(t_{n+1} - t_n = d)$

First term of the sequence is denoted by 'a'
Genereral form of AP:

$$a, \; a+d, \; a+2d, \; a+3d, \; ----$$

10_{th} term $= a+9d \cdots ?$

$$\boxed{n_{th} \text{ term of A.P.} \quad n_{th} \text{ term} = a+(n-1)d}$$

Example 1: Given an A.P. 3, 7, 11, 15, 19, ---
(a) Find the 15_{th} term.
(b) Given the last term of this AP is 83, find the number of term of A.P.

Solution: A.P: 3, 7, 11, 15, 19, ---

(a) First term $a = 3$
Common diff $= 7-3 = 4$

n_{th} term $= a+(n-1)d$

$\therefore 15_{th}$ term $= 3+(15-1)\times 4$
$= 3 \quad 14\times 4$
$= 3+56 = \underline{59}$ ✓

(b) Given n_{th} term $= 83$
or $a+(n-1)d = 83$
$\Rightarrow 3+(n-1)\cdot 4 = 83$
$(n-1)\cdot 4 = 83-3$
$n-1 = \frac{80}{4} = 20$
$\therefore n = 20+1 = 21$
$\therefore \underline{n = 21}$ ✓

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Example 2: An arithmetic progression has first term 7. The n_{th} term is 84 and the $(3n)_{th}$ term is 245. ---[4]
Find the value of n. [SP-20/01/Q3]

Solution: First term $a = 7$

n_{th} term: $a + (n-1)d = 84$

or $7 + (n-1)d = 84$

$(n-1)d = 77$ ----(1)

and $(3n)_{th}$ term,

$7 + (3n-1)d = 245$

or $(3n-1)d = 238$ --(2)

divide (1) ÷ (2)

$\Rightarrow \frac{(n-1)\cancel{d}}{(3n-1)\cancel{d}} = \frac{77}{238} = \frac{11}{34}$

$\Rightarrow 34(n-1) = 11(3n-1)$

$\Rightarrow 34n - 34 = 33n - 11$

$\Rightarrow n = 23$ ✓

Example 3: The first term of an arithmetic progression is $4x$, and second term is x^2, with a common difference is 12. Find the possible values of x and the third term. ---[4]
[SP-17/01/Q8(i)]

Solution: First term $= 4x$

Second term $= x^2$ $(d = t_2 - t_1)$

$\therefore$ Common difference $d = x^2 - 4x = 12$ (Given)

$\Rightarrow x^2 - 4x - 12 = 0$

$\Rightarrow x^2 - 6x + 2x - 12 = 0$

$x(x-6) + 2(x-6) = 0$

$(x-6)(x+2) = 0$

$x = 6; -2$ ✓

Now the third term $= t_2 + d$

$= x^2 + 12$

(i) for $x = 6$, third term $= 6^2 + 12 = 48$ ✓

(ii) for $x = -2$, third term $= (-2)^2 + 12 = 16$ ✓

§ Three terms a, b and c in A.P.:

Given three terms a, b and c in A.P;

Then $b-a=c-b \Rightarrow 2b=a+c$ ✓

or $b=\frac{(a+c)}{2}$ ✓

Example 4: Given three term of a progression:
$1, \cos^2 x$ and $\cos 2x$,
prove that the terms form an A.P.

Solution: Given three term $1, \cos^2 x, \cos 2x$
are in A.P. if

$$2\cos^2 x = 1+\cos 2x$$ ✓

which is true.

$\therefore$ Given three terms are in A.P.

[Three terms a, b and c are in A.P. Then $2b=a+c$]

§ Arithmetic Series (Sum of first n terms of a A.P) S_n:

nth term of A.P $= a+(n-1)d = l$ (let

$$S_n = a+(a+d)+(a+2d)+\cdots(l-2d)+(l-d)+l \quad \text{---(1)}$$

or $$S_n = l+(l-d)+(l-2d)+\cdots(a+2d)+(a+d)+d \quad \text{---(2)}$$

add (1) and (2)

$$2\cdot S_n = (a+l)+(a+l)+(a+l)+\cdots\cdots \; n \text{ terms}$$

$$= n(a+l)$$

or $$S_n = \frac{n}{2}(a+l)$$ ✓

$$S_n = \frac{n}{2}[a+a+(n-1)d]$$

[l = last term = nth term $= a+(n-1)d$]

or $$S_n = \frac{n}{2}[2a+(n-1)d]$$ ✓

Example 5: Given an A.P. 3, 7, 11, 15, 19, ----
(a) Find the sum of first 10 terms.
(b) Given the sum of first n terms is 136, find n.

Solution: Given A.P. 3, 7, 11, 15, 19, --

First term $a = 3$ ✓

Common difference $d = 7-3 = 4$ ✓

$S_n = \frac{n}{2}[2a+(n-1)d]$

(a) Hence for $n = 10$

$S_{10} = \frac{10}{2}[2\times3+(10-1)\times4]$

$= 5[6+36]$

$= 5\times42 = \underline{210}$ ✓

(b) Given $S_n = 136$, $a = 3$, $d = 4$

$\Rightarrow S_n = \frac{n}{2}[2a+(n-1)d] = 136$

$\Rightarrow \frac{n}{2}[2\times3+(n-1)4] = 136$

$\Rightarrow n[6+4n-4] = 272$

$n[4n+2] = 272$

$4n^2+2n-272 = 0$

$2n^2+n-136 = 0$

$\Rightarrow 2n^2+17n-16n-136 = 0$

$n(2n+17)-8(2\ +17) = 0$

$(n-8)(2n+17) = 0$

$\Rightarrow \underline{n = 8}$ ✓ ; $n = -\frac{17}{2}$ ✗

Example 6: In an A.P the sum, S_n, of the first n terms is given by $S_n = 2n^2+8n$. Find the first term and the common difference of the progression. [S-13/13/Q9(a)] --- [3]

Solution: $\boxed{n_{th} \text{ term} = S_n - S_{n-1}}$ and $\boxed{a = S_1}$

Given $S_n = 2n^2+8n$

$n=1 \rightarrow S_1 = 2\times1^2+8\times1 = 10$ $\therefore$ first term $a = S_1 = \underline{10}$ ✓ --- ①

$n=2 \rightarrow S_2 = 2\times2^2+8\times2 = 24$

Second term $= S_2 - S_1 = 24-10 = 14$

or $a+d = 14$

$10+d = 14$ (from ① $a = 10$)

$\therefore d = 14-10 = 4$

or Common difference $= \underline{4}$ ✓

Example 7: An A.P has the first term a and common difference d. It is given that the sum of first 200 terms is 4 times the sum of first 100 terms.

(i) Find d in terms of a. ---[3]

(ii) Find the 100_{th} term in terms of a. ---[2]

[S-14/11/Q5]

Solution: $S_{200} = \frac{200}{2}[2a+(200-1)d]$ $\left[\because S_n = \frac{n}{2}(2a+(n-1)d)\right]$

(i) or $S_{200} = 100(2a+199d)$ --- ①

and $S_{100} = \frac{100}{2}[2a+99d] = 50(2a+99d)$ --- ②

Given $S_{200} = 4 \times S_{100}$

$\Rightarrow 100(2a+199d) = 4\times 50(2a+99d)$

$\Rightarrow 2a+199d = 2(2a+99d)$

$\Rightarrow 2a+199d = 4a+198d$

$\Rightarrow \underline{d = 2a}$ ✓ --- ③

(ii) n_{th} term $= a+(n-1)d$

$\therefore 100_{th}$ term $= a+99d$

$= a+99\times 2a$

$= 199a$ $(\because d = 2a)$

Example 8: The sum S_n, of the first n terms of an arithmetic progression is given by: $S_n = n^2+4n$.

The K_{th} term in the progression is greater than 200. Find the smallest possible value of K. ---[5]

[W-20/12/Q4]

Solution: K_{th} term $= S_K - S_{K-1}$ (Given $S_n = n^2+4n$)

$= (K^2+4K) - [(K-1)^2+4(K-1)]$

$= (K^2+4K) - [K^2-2K+1+4K-4]$

$= K^2+4K-K^2-2K+3$

$= 2K+3$ --- ①

Now given K_{th} term > 200

$\Rightarrow 2K+3 > 200$ (for ①)

$\Rightarrow 2K > 197$

$\Rightarrow K > 98.5$

$\therefore$ The minimum value $\underline{K = 99}$ ✓

§ Geometric Progression (G.P):

Consider a sequence: 2, 6, 18, 54, 162, ---- is called a G.P.

$\frac{6}{2} = 3$

$\frac{18}{6} = 3$

$\frac{54}{18} = 3$

Here we find that the ratio of each term and the previous term is constant, called the common ratio of G.P and is denoted by 'r'. $\left[\frac{t_{n+1}}{t_n} = r\right]$

First term of the sequence is denoted by a.

General form G.P:

$$a,\ ar,\ ar^2,\ ar^3, ----$$

$$10_{th} \text{ term} = ar^9 \text{ --?}$$

$$\boxed{n_{th} \text{ term of G.P} = a.r^{n-1}}$$

Example 9: Given a G.P 2, 6, 18, 54, 162, ---

(a) Find the 10_{th} term of the sequence.

(b) Given the last term of this G.P is 1458, find the number of terms of G.P.

Solution: Given G.P. 2, 6, 18, 54, 162, ----

First term $a = 2$ ✓

Common ratio $r = \frac{6}{2} = 3$

(i) 10_{th} term $= 2 \times (3)^{10-1}$ $\quad [\because n_{th} \text{ term} = a.r^{n-1}]$

$= 2 \times 3^9 = \underline{39366}$ ✓

(ii) Given the last term $= 1458$

n_{th} term $= 2 \times 3^{n-1} = 1458$ $\quad [n_{th} \text{ term} = ar^{n-1}]$

$\Rightarrow 3^{n-1} = \frac{1458}{2} = 729 = 3^6$

$\Rightarrow n - 1 = 6$

$\Rightarrow \underline{n = 7}$ ✓

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Example 10: The third term of a G.P is -108 and sixth term is 32. Find:

(i) Common ratio ---[3]

(ii) the first term ---[1]

(iii) tenth term. [S-13/11/Q4] ---[2]

Solution: 3rd term $= a \cdot r^{3-1} = ar^2 = -108 \dots ①$ [$\because$ nth term of G.P $= ar^{n-1}$]

(i) 6th term $= ar^{6-1} = ar^5 = 32 \dots ②$

divide ② $\div$ ① $\Rightarrow \frac{ar^5}{ar^2} = \frac{32}{-108} = -\frac{8}{27}$

$\Rightarrow r^3 = \left(-\frac{2}{3}\right)^3 \Rightarrow r = -2/3$ ✓

(ii) Put $r = -\frac{2}{3}$ in ① $\Rightarrow a \cdot \left(-\frac{2}{3}\right)^2 = -108$

$\Rightarrow \frac{4}{9}a = -108 \Rightarrow a = -108 \times \frac{9}{4} = -243$ ✓

(iii) 10th term $= ar^9$

$= -243 \times \left(-\frac{2}{3}\right)^9 = -243 \times \left(\frac{-512}{19683}\right) = \frac{512}{81}$ ✓

§ Three terms in G.P:

Given three term; a, b and c are in G.P

$\Rightarrow \frac{b}{a} = \frac{c}{b}$ or $b^2 = ac$ ✓

Example 11: The first, second and the third term of G.P are $2k+6$, $2k$ and $k+2$, where k is a positive constant.

(i) Find the value of k ---[3]

(ii) Find the 6th term of G.P. [S-15/12/Q8] ---[3]

Solution: Three term $(2k+6)$, $2k$ and $(k+2)$ are in G.P (a, b, c are in G.P $\Rightarrow \frac{b}{a} = \frac{c}{b}$)

(i) $\Rightarrow \frac{2k}{(2k+6)} = \frac{k+2}{2k}$

$\Rightarrow 4k^2 = (k+2)(2k+6)$

$\Rightarrow k^2 - 5k - 6 = 0$

$\Rightarrow k = 6$ ✓ ; $k = -1$ ✗

(ii) First term $a = 2k+6 = 2\times 6 + 6 = 18$ ✓

Common ratio $r = \frac{2k}{2k+6} = \frac{12}{18} = \frac{2}{3}$ [$\because k=6$]

$\therefore$ 6th term $= ar^5$

$= 18 \cdot \left(\frac{2}{3}\right)^5 = 18 \times \frac{32}{243}$

$= \frac{64}{27}$ ✓

Example 12: The first three terms of an A.P are 4, x and y respectively. The first three terms of a G.P are x, y and 18 resp. It is given that both x and y are positive.
(i) Find the value of x and the value of y. ---[4]
(ii) Find the fourth term of each progression. ---[3]
W-18/12/Q5

Solution: 4, x, y are in A.P   [a, b, c are in A.P $\Rightarrow b = \frac{a+c}{2}$]
$\Rightarrow x = \frac{4+y}{2}$ ---①

and x, y and 18 are in G.P   [a, b, c are in G.P $\Rightarrow \frac{b}{a} = \frac{c}{b}$]
(i) $\Rightarrow \frac{y}{x} = \frac{18}{y} \Rightarrow y^2 = 18x$ ---②

from ① & ② $y^2 = 18 \times \left(\frac{4+y}{2}\right) \Rightarrow y^2 = 9y + 36$
$\Rightarrow y^2 - 9y - 36 = 0$
$\Rightarrow y^2 - 12y + 3y - 36 = 0$
$y(y-12) + 3(y-12) = 0$
$(y-12)(y+3) = 0$
$y = 12$ ✓, $y = -3$ ✗ ($\because$ y is +ve)
for ① $x = \frac{4+12}{2} = 8$ ✓

(ii) for $x = 8, y = 12$
A.P is 4, 8, 12 $\Rightarrow$ 4th term = 16 ✓ $(d=4)$
G.P is 8, 12, 18 $\Rightarrow$ 4th term = 27 ✓
$(\because r = 3/2)$

§ Geometric Series (Sum of first n term of G.P. 'S_n')
$$S_n = a + ar + ar^2 + \cdots + ar^{n-2} + ar^{n-1} \quad \text{---①}$$
$$r \cdot S_n = \pm ar \pm ar^2 + \cdots \pm ar^{n-1} \pm ar^n \quad \text{---②}$$
Subtracting $S_n(1-r) = a - ar^n$
$$\Rightarrow S_n = \frac{a(1-r^n)}{(1-r)} \quad \text{for } r<1 \checkmark$$
or
$$S_n = \frac{a(r^n-1)}{(r-1)} \quad \text{for } r>1 \checkmark$$

§ Sum to infinity of a Geometric Series:
(Convergent Series)
$$S_\infty = \frac{a}{(1-r)} \checkmark \quad \text{for } |r|<1$$
{$\because n \to \infty$, $r^n \to 0$}

Example 13: Given a G.P. 2, 6, 18, 54, 162, ----
(a) Find the sum of first ten term (S_{10})
(b) Given the sum of n term of this G.P is 6560, find n.

Solution: Given G.P. 2, 6, 18, 54, ---
(a) $a = 2,\ r = 6/2 = 3,\ n = 10$

$\left[S_n = \frac{a(r^n - 1)}{(r-1)}\right]$

$$S_{10} = \frac{2(3^{10} - 1)}{3-1} = \frac{\cancel{2}(59049 - 1)}{\cancel{2}} = 59048$$

(b) Given: $S_n = \frac{a(r^n - 1)}{r-1} = 6560$

$a = 2, r = 3 \Rightarrow \frac{2(3^n - 1)}{(3-1)} = 6560$

$\Rightarrow 3^n - 1 = 6560$

$\Rightarrow 3^n = 6561 = 3^8$

$\Rightarrow n = 8$

Example 14: The second and third terms of a G.P are 48 and 32 respectively. Find the sum to infinity of the progression. [W-13/12/Q7] ---[4]

Solution: Second term $ar = 48$ --- ① [Let G.P. a, ar, ar^2, ar^3 ---]

Third term $ar^2 = 32$ --- ②

Divide ② ÷ ① $\frac{ar^2}{ar} =$

$\Rightarrow r = \frac{2}{3}$,

from ① $a \times \frac{2}{3} = 48 \Rightarrow a = 72$

Hence $S_\infty = \frac{a}{1-r}$

$= \frac{72}{1 - 2/3}$

$\therefore S_\infty = \frac{72 \times 3}{1} = 216$

Example 15: The 1st, 3rd and 13th terms of an A.P. are also the 1st, 2nd and 3rd terms resp. of a G.P. The first term of each progression is 3. Find the common difference of A.P, and the common ratio of G.P. [S-16/13/Q4] --[5]

Solution: Given $a = 3 \Rightarrow$ AP is $3, 3+d,$ ---
and G.P is $3, 3r, 3r^2,$ ---

Now 3rd term of A.P = 2nd term of G.P

$\Rightarrow 3 + 2d = 3r \Rightarrow d = \frac{3(r-1)}{2}$ --- ①

also 13 term of A.P = 3rd term of G.P

$\Rightarrow 3 + 12d = 3r^2 \Rightarrow$

$\Rightarrow d = \frac{3(r^2-1)}{12} = \frac{r^2-1}{4}$ --- ②

from ① and ②

$\frac{3}{2}(r-1) = \frac{(r-1)(r+1)}{4}$

$\Rightarrow r + 1 = 6 \Rightarrow r = 5$

from ① $d = \frac{3}{2}(r-1) = \frac{3}{2}(5-1) = 6$

Example 16: The first term of a G.P is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0<\theta<\pi$. Find the set of value of 'θ' for which the progression is convergent. ---[5]

[S-15/13/Q9(b)]

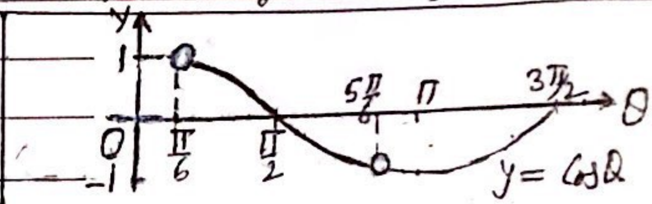
Solution: $a=\sqrt{3}$ and $ar = 2\cos\theta$

[G.P: $a, ar, ar^2, \ldots$ for convergent series; $-1<r<1$]

$\Rightarrow \sqrt{3}r = 2\cos\theta \Rightarrow r = \frac{2}{\sqrt{3}}\cos\theta$

Progression is convergent; $(|r|<1)$

$-1 < \frac{2}{\sqrt{3}}\cos\theta < 1$



$\Rightarrow -\frac{\sqrt{3}}{2} < \cos\theta < \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{6} < \theta < \frac{5\pi}{6}$ ✓

Example 17 (a) A G.P has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. ---[4]

(b) The n_{th} term of a progression is $p+qn$, where p and q are constants and S_n is the sum of the first n terms,

(i) Find an expression, in terms of p, q and n for S_n. ---[3]

(ii) Given $S_4 = 40$ and $S_6 = 76$, find the value of p and q. ---[2]

[S-18/11/Q8]

Solution (a) Let G.P $\rightarrow a, ar, ar^2, \ldots$

Second term $ar = 12 \Rightarrow a = \frac{12}{r}$ --- ①

and $S_\infty = \frac{a}{(1-r)} = 54$ --- ②

from ① and ② $\rightarrow \frac{12/r}{(1-r)} = 54$

$\Rightarrow 12 = r(1-r)54$

$\Rightarrow 9r^2 - 9r + 2 = 0$

$9r^2 - 6r - 3r + 2 = 0$

$3r(3r-2) - 1(3r-2) = 0$

$(3r-1)(3r-2) = 0$

$\Rightarrow r = \frac{1}{3}; \ r = \frac{2}{3}$ ✓

(b)(i) n_{th} term $= p+qn$ --- ③

(which is linear hence A.P)

first term $a = p+q$ (from ③ $n=1$)

Last term $l = p+qn$

$\therefore S_n = \frac{n}{2}[a+l]$

$= \frac{n}{2}[p+q+p+qn]$

$= \frac{n}{2}(2p+q+nq)$ --- ④

(ii) $S_4 = 2(2p+q+4q) = 40$

$\Rightarrow 2p+5q = 20$ --- ⑤

and

$S_6 = 3(2p+q+6q) = 72$

$\Rightarrow 2p+7q = 24$ --- ⑥

Solving ⑤ and ⑥ $p = 5$ ✓

and $q = 2$ ✓

Example 18: A company producing salt from sea water chaged to a new process. The amount of selt each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 Kg salt.

(i) Find the amount of salt obtained in the 12_{th} week after the change ...[3]

(ii) Find the amount of salt obtained in the first 12 weeks after the change. [S-18/12/Q3] ---[2]

Solution: $a = 8000$, $r = 1.02$ [102%]

(i) $n = 12$, $n_{th} = a r^{n-1}$ (G.P)

12_{th} term $= 8000 \times (1.02)^{11}$

$= 8000 \times 1.24337 = 9947$

$= \underline{9950 \text{ Kg}}$ (To the nearest 10)

(ii) $S_n = \dfrac{a(r^n - 1)}{(r-1)} = \dfrac{8000 \times (1.02^{12} - 1)}{(1.02 - 1)}$

$= \dfrac{8000 \times 0.26284}{0.02}$

$= \dfrac{2145.93}{0.02} = 107296$

$= \underline{107000 \text{ Kg}}$ (To the nearest 1000)

Example 19: The common ratio of a G.P is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$ ---[2]

(ii) Find the set of values that S can take. [S-17/13/Q2] --[2]

Solution: G.P.; common ratio $= r$

$a = (r^2 - 3r + 2)$

(i) $S_\alpha = \dfrac{a}{1-r}$ ($\because |r| < 1$)

$= \dfrac{(r^2 - 3r + 2)}{(1-r)}$

$= \dfrac{(r-2)(r-1)}{(1-r)}$

$= \dfrac{-(r-2)\cancel{(1-r)}}{\cancel{(1-r)}}$

$S_\alpha = 2 - r$

(ii)

$S_\alpha = 2 - r$ --① [$|r| < 1$ or $-1 < r < 1$]

$-1 < r < 1$

$\Rightarrow -1 < -r < 1$

$\Rightarrow 1 < (2 - r) < 3$ (add 2)

$\Rightarrow \underline{1 < S_\alpha < 3}$ ✓

Example 20(a) A G.P has first term $3a$ and common ratio r. ---[3]
A second G.P has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r.
(b) The first two terms of an A.P are 15 and 19 respectively. The first two terms of a second A.P are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. $\boxed{W-17/11/Q3}$ ---[3]

Solution: (a) G.P: first term $= 3a$ and common ratio $= r$

for the first G.P, $S_\infty = \dfrac{3a}{(1-r)}$ --- ①

for Second G.P, first term $= a$, common ratio $= -2r$

$$\therefore S_\infty = \frac{a}{1-(-2r)} = \frac{a}{1+2r} \quad \text{②}$$

Given both have the same $S_\infty \Rightarrow \dfrac{3a}{1-r} = \dfrac{a}{1+2r}$

$$\Rightarrow 3(1+2r) = 1-r \Rightarrow 7r = -2 \Rightarrow \underline{r = -\frac{2}{7}} \checkmark$$

(b) For the first AP; $a=15$, $a+d=19 \rightarrow d=4$ $\left[S_n = \frac{n}{2}[2a+(n-1)d]\right]$

$$\therefore S_n = \frac{n}{2}[2\times15+(n-1)\cdot4]$$
$$= \frac{n}{2}(26+4n) \quad \text{③}$$

Now for Second A.P.

$a = 420$; $a+d = 415 \Rightarrow d = -5$

$$\therefore S_n = \frac{n}{2}[2\times420+(n-1)(-5)]$$
$$= \frac{n}{2}[845-5n] \quad \text{④}$$

Given that S_n is same for both.

$$\therefore \frac{\cancel{n}}{\cancel{2}}(26+4n) = \frac{\cancel{n}}{\cancel{2}}(845-5n)$$

$$\Rightarrow 4n+5n = 845-26$$
$$\Rightarrow 9n = 819 \Rightarrow n = \frac{819}{9} = 91$$
$$\Rightarrow \underline{n = 91} \checkmark$$